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Hochschild Cohomology of the Algebra of Conformal Endomorphisms

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Abstract: It was proved by I. Dolguntseva (St. Peterburg Math. J., 2010) that second Hochschild cohomology groups for the associative conformal algebra $Cend_k$ with coefficients in an arbitrary conformal bimodule are trivial. In this work, we prove the same for all higher Hochschild cohomologies of $Cend_k$ by means of algebraic discrete Morse theory applied to the bar complex of the 1st Weyl algebra.

Keywords: conformal algebra, Hochschild cohomology, Groebner–Shirshov basis, Morse matching

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Научная статья

Когомологии Хохшильда алгебры конформных эндоморфизмов

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Аннотация: И. А. Долгунцевой (St. Petersburg Math. J., 2010) доказано, что вторые группы когомологии Хохшильда для ассоциативной конформной алгебры $Cend_k$ с коэффициентами в произвольном конформном бимодуле тривиальны. Доказывается то же самое для всех высших когомологий Хохшильда $Cend_k$ с помощью алгебраической дискретной теории Морса, применённой к bar-комплексу первой алгебры Вейля.

Ключевые слова: конформная алгебра, когомологии Хохшильда, базис Грёбнера – Ширшова, соответствие Морса

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1. Introduction

The notion of a conformal (Lie) algebra emerged in [17] as a tool in the theory of vertex algebras which goes back to mathematical physics [6] and representation theory (see, e.g., [8]). From the algebraic point of view, the structure of a vertex algebra is a breed of two structures: a differential left-symmetric algebra and a Lie conformal algebra [4].

The structure theory of (finite) Lie conformal algebra was developed in [10], irreducible representations of simple and indecomposable semisimple finite Lie conformal algebras were described in [9]. Given a finite conformal module M over a Lie conformal algebra L, the representation of L on M is a homomorphism from L to the Lie conformal algebra of conformal endomorphisms $\mathrm{gc}\,(M)$, see [17, Ch. 2]. The latter is an analogue of the "ordinary" Lie algebra $\mathrm{gl}\,(V)$ of a linear space V in the category of conformal algebras. As in ordinary algebras, $\mathrm{gc}\,(M)$ is the commutator algebra of an *associative* conformal algebra $Cend(M)$. Thus the study of associative conformal algebras (and $Cend(M)$, in particular) is essential for representation theory of Lie conformal algebras and, as a corollary, for vertex algebras theory. A systematic study of $Cend(M)$ was performed in [7], its simple subalgebras were described in [19]. The most interesting case is when M is a free H-module of rank k, then $Cend(M)$ is denoted $Cend_k$. This system plays the same role in the theory of conformal algebras as the matrix algebra $M_k(\Bbbk)$ does in the ordinary algebra.

The homological studies for conformal algebras starts from the paper [3]. Conceptually, to define (co)chains, (co)cycles, and (co)boundaries for a particular class of algebras over a field $\Bbbk$, one needs to know what a multilinear mapping is, how to combine such mappings, and how symmetric groups act on multilinear mappings. All these notions have their analogues in the category of modules over cocommutative bialgebras, that is, these are pseudo-tensor categories [5]. In particular, the definition of Hochschild cohomologies for an associative algebra in the pseudo-tensor category over the polynomial bialgebra $H = \Bbbk[\partial]$, where ∂ is a primitive element, coincides with the definition of Hochschild cohomology of associative conformal algebras in [3].

It is well-known since [15] that for the associative algebra $End(V)$ of linear transformations of a finite-dimensional space V all nth Hochschild cohomology groups are trivial for $n \geq 1$. The problem of description of conformal Hochschild cohomologies of $Cend_k(M)$ for a finite free H-module M was stated in [3]. In [11], it was shown that the second Hochschild cohomology group of $C = Cend_k(M)$ is trivial for all conformal bimodules over C when M is a free H-module, which was a partial solution of the problem from [3]. The purpose of this paper is to complete solving this problem without the freeness assumption and prove that all nth Hochschild cohomology groups of $Cend_k(M)$ for $n \geq 2$ with coefficients in all conformal bimodules over $Cend_k(M)$ are trivial.

Note that the classical triviality argument (see [15]) based on the isomorphism $\mathrm{H}^n(A, M) \simeq \mathrm{H}^{n-1}(A, \mathrm{Hom}\,(A, M))$ does not work for conformal algebras since the pseudo-tensor analogue of Hom denoted Chom (see [17]) does not carry a structure of conformal bimodule due to locality issues.

As shown in [3], the calculation of conformal Hochschild cohomology $\mathrm{H}^\bullet(C, M)$ of an associative conformal algebra C with coefficients in a conformal bimodule M over C is based on the ordinary Hochschild cohomology $\mathrm{H}^\bullet(\mathcal{A}_+(C), M)$, where $\mathcal{A}_+(C)$ is the positive part of the coefficient algebra of C.

For $C = Cend_k$, the positive part $\mathcal{A}_+(Cend_k)$ of its coefficient algebra is isomorphic to the matrix algebra over the first Weyl algebra W_1, i.e., the unital associative algebra generated by two elements p, q such that $qp - pq = 1$.

The series of Weyl algebras (and, in particular, the first one) is under intensive study in various areas of mathematics. Homological properties of these algebras were considered, for example, in [13; 14; 22]. For instance, the global dimension of the Weyl algebra W_n, $n \geq 1$, essentially depends on the characteristic of the base field. One of the by-products of this paper is an explicit computation of the 3rd Hochschild cohomology group of the first Weyl algebra by means of the Anick resolution. We apply the Morse matching method to transform a bar-resolution of the first Weyl algebra into its Anick resolution and calculate explicitly $\mathrm{H}^3(W_1, M)$ for an arbitrary W_1-bimodule M.

As a result, we solve a problem stated in [3] on the computation of Hochschild cohomologies of the conformal algebra $Cend_k$:
we prove $\mathrm{H}^n(Cend_k, M) = 0$ for all $n \geq 2$ and for all conformal bimodules M over $Cend_k$ without the freeness assumption.

2. Morse matching method for constructing the Anick resolution

The idea of D. Anick on the construction of a relatively small free resolution for an augmented algebra has shown its effectiveness in a series of applications [1;21]. Let us briefly observe the main points of this construction and its application to the computation of Hochschild cohomologies of associative algebras. Suppose Λ is a unital associative algebra equipped with a homomorphism $\varepsilon : \Lambda \to \Bbbk$, $\varepsilon(1) = 1$ (augmentation). Denote by A the cokernel $\Lambda/\Bbbk$ of the inverse embedding $\eta : \Bbbk \to \Lambda$ and consider the two-sided bar resolution of free Λ-bimodules

$$0 \leftarrow \Bbbk \leftarrow \mathrm{B}_0 \leftarrow \mathrm{B}_1 \leftarrow \dots \leftarrow \mathrm{B}_n \leftarrow \mathrm{B}_{n+1} \leftarrow \dots,$$

where $\mathrm{B}_0 = \Lambda \otimes \Lambda$, $\mathrm{B}_n = \Lambda \otimes A^{\otimes n} \otimes \Lambda$ for $n \geq 1$. We will denote a tensor $a_1 \otimes \dots \otimes a_n \in A^{\otimes n}$ as $[a_1|\dots|a_n]$ and omit the tensor product signs between Λ and $A^{\otimes n}$. The arrows $d_{n+1} : \mathrm{B}_{n+1} \to \mathrm{B}_n$ are Λ-bimodule homomorphisms given by

$$d_{n+1}[a_1|\dots|a_{n+1}] = a_1[a_2|\dots|a_{n+1}] + \sum_{i=1}^{n}(-1)^i[a_1|\dots|a_ia_{i+1}|\dots|a_{n+1}] \\ +(-1)^{n+1}[a_1|\dots|a_n]a_{n+1},$$

for $n > 0$, and

$$d_1 : [a] \mapsto a \otimes 1 - 1 \otimes a, \quad d_0 : a \otimes b \mapsto \varepsilon(ab).$$

If M is an arbitrary unital Λ-bimodule then

$$\mathrm{Hom}_{\Lambda-\Lambda}(\mathrm{B}_n, M) \simeq \mathrm{Hom}(A^{\otimes n}, M)$$

as linear spaces, and for every $\varphi \in \mathrm{Hom}_{\Lambda-\Lambda}(\mathrm{B}_n, M)$ the composition $\varphi d_{n+1} : \mathrm{B}_{n+1} \to M$ corresponds to the $\Bbbk$-linear map $\Delta^n(\varphi) : A^{\otimes(n+1)} \to M$ which is given by the Hochschild differential formula.

Therefore, if we start with an associative algebra A, join an exterior identity to get $\Lambda = A \otimes \Bbbk 1$ with $\varepsilon(A) = 0$, then the cochain complex

$$\big(\operatorname{Hom}_{\Lambda-\Lambda}(\mathrm{B}_\bullet, M), \Delta^\bullet\big)$$

coincides with Hochschild complex $\mathrm{C}^\bullet(A, M)$.

The bar resolution $(\mathrm{B}_\bullet, d_\bullet)$ is easy to construct but it is too large for practical computations. Therefore, it is reasonable to replace $(\mathrm{B}_\bullet, d_\bullet)$ with a smaller but homotopy equivalent resolution, e.g., the *Anick resolution* $(\mathrm{A}_\bullet, \delta_\bullet)$,

$$0 \leftarrow \Bbbk \leftarrow \mathrm{A}_0 \leftarrow \mathrm{A}_1 \leftarrow \cdots \leftarrow \mathrm{A}_n \leftarrow \mathrm{A}_{n+1} \leftarrow \dots, \quad \delta_{n+1} : \mathrm{A}_{n+1} \to \mathrm{A}_n.$$

Then, given an A-bimodule (hence, a unital Λ-bimodule), the cohomologies of the complex

$$\big(\operatorname{Hom}_{\Lambda-\Lambda}(\mathrm{A}_\bullet, M), \Delta^\bullet\big), \quad \Delta^n\varphi = \varphi\delta_{n+1},\ \varphi \in \operatorname{Hom}_{\Lambda-\Lambda}(\mathrm{A}_n, M),$$

coincide with the Hochschild cohomologies $\mathrm{H}^\bullet(A, M)$.

Suppose X is a set of generators of the algebra A. Denote by X^* the set of nonempty words in X, and let $\Bbbk\langle X\rangle$ stand for the linear span of X^*, this is the free associative algebra generated by X.

Let $\Sigma \subset \Bbbk\langle X\rangle$ be a Groebner–Shirshov basis of A relative to an appropriate monomial order (e.g., deg-lex order). We will denote by $V = \overline{\Sigma}$ the set of principal parts of relations from Σ (called *obstructions*). Recall that $\mathrm{A}_0 = \mathrm{B}_0 = \Lambda \otimes \Lambda$, $\mathrm{A}_n = \Lambda \otimes \Bbbk V^{(n-1)} \otimes \Lambda$, where $V^{(k)}$ stands for the set of *Anick k-chains*. By definition (see [2]), $V^{(0)} = \{[x] \mid x \in X\}$, $V^{(1)} = \{[x|s] \mid x \in X, s \in X^*, xs \in V\}$, and for $k \geq 2$ the set $V^{(k)}$ is constructed on the words in X^* obtained by consecutive “hooking” of the words from $\overline{\Sigma}$.

This definition becomes transparent in the case when the defining relations Σ contain at most quadratic monomials, so that all words in V are of length two. For $n \geq 1$, an Anick n-chain is a word $v = [x_1|\dots|x_{n+1}] \in X^*$ such that $x_i x_{i+1} \in V$ for $i = 1, \dots, n$.

Example 1. Let $\mathfrak{g}$ be a Lie algebra over $\Bbbk$ with a linearly ordered basis X. Denote $[x, y] \in \Bbbk X$, $x, y \in X$, the Lie product in $\mathfrak{g}$. Set $\Sigma = \{xy - yx - [x, y] \mid x, y \in X, x > y\}$, $A = \Bbbk\langle X\rangle/(\Sigma)$. Then $\Lambda = A \oplus \Bbbk 1$ is exactly the universal enveloping associative algebra $U(\mathfrak{g})$. Then $V^{(k)} = \{[x_1|x_2|\dots|x_{k+1}] \mid x_1 > x_2 > \dots > x_{k+1}, x_i \in X\}$. The elements of $V^{(k)}$ are in obvious one-to-one correspondence with the basis of $\wedge^{k+1}\mathfrak{g}$.

The Anick differentials were computed in [2] by means of a complicated inductive procedure. In order to make this computation easier, in [16] and, independently, in [23], it was proposed to use algebraic discrete Morse theory developed in [12] to construct a smaller complex (of free modules) which is homotopy equivalent to a given one. In particular, given a bar resolution of an augmented algebra Λ, the resulting complex is the Anick resolution.

The Morse matching method for computing the Anick resolution [16], [23] is also described in [1; 21]. In a few words, the problem is to choose an appropriate set of edges in the weighted directed graph describing the structure of the bar resolution. Then one has to transform the graph by means of inverting the matched edges. Inverting means not only switch of direction, but also replacing the weight c of the matched edge with $-c^{-1}$. In the resulting graph, the non-matched vertices (critical cells) are exactly the Anick chains. Finally, in order to calcuate the Anick differential δ_{n+1} on a chain w from $V^{(n)}$ one has to track all paths from w to vertices from $V^{(n-1)}$. The weight of each path is equal to the product of the weights of all its edges.

Let us consider the construction of the Anick resolution $(\mathrm{A}_\bullet, \delta_\bullet)$ by means of the Morse matching method in details. Suppose $\mathrm{B}_\bullet = (\mathrm{B}_n, d_n)_{n\ge 0}$ is a chain complex of free (left) Λ-modules. Suppose Y_n is a fixed basis of B_n as of Λ-module. Then for each $b \in Y_n$ there is a unique presentation

$$d_n(b) = \sum_{b' \in Y_{n-1}} [b : b']b', \quad [b : b'] \in \Lambda.$$

Define a weight of a potential edge from b to b' as the coefficient $[b : b']$. Construct a directed weighted graph $\Gamma(\mathrm{B}_\bullet) = (Y, E)$ considering $Y = \cup_{n>0} Y_n$ as the set of vertices. The edges in $\Gamma(\mathrm{B}_\bullet)$ are defined as follows:

$$Y_n \ni b \xrightarrow{[b:b']} b' \in Y_{n-1}$$

whenever $[b : b'] \neq 0$.

Definition 1 ([16; 23]). *Let M be a subset of E. Then M is called a* Morse matching *if and only if*

- *Each vertex $y \in Y$ lies in at most one edge $e \in M$;*
- *For all edges $(b, b', [b : b']) \in M$, the weight $[b : b']$ is an invertible element in the center of Λ;*
- *The graph $\Gamma_M = (Y, E_M)$ has no directed cycles, where E_M is given by*
$$E_M = (E \setminus M) \cup \{(b', b, -[b : b']^{-1}) \mid (b, b', [b : b']) \in M\}.$$

A vertex $b \in Y$ is critical *with respect to M if b does not belong to an edge $e \in M$; we denote by Y_n^M the set of critical edges from Y_n.*

Suppose p is a path $p = b_1 \to \ldots \to b_r$ in a weighted directed graph with vertices X. Then

$$\omega(p) := \prod_{i=1}^{r-1} \lambda_i, \quad e_i = (b_i, b_{i+1}, \lambda_i).$$

Denote by $\Gamma(b,b')$, $b,b' \in Y$, *the sum of weights of all paths from* b *to* b'.

Given a Morse mathcing in the graph $\Gamma(\mathrm{B}_\bullet)$, construct a smaller complex of free Λ-modules $\mathrm{B}_\bullet^M$ as follows. Let

$$\mathrm{B}_n^M = \bigoplus_{b\in Y_n^M} \Lambda b,$$

and

$$d_n^M : \mathrm{B}_n^M \to \mathrm{B}_{n-1}^M; \quad d_n^M(b) = \sum_{b'\in Y_{n-1}^M} \Gamma(b,b')b',$$

where $\Gamma(b,b')$ is calculated in the graph $\Gamma_M = (Y, E_M)$.

Theorem 1 ([16]). *A chain complex of free* Λ*-modules* $(\mathrm{B}_n, d_n)_{n\geq 0}$ *is homotopy equivalent to the complex* $(\mathrm{B}_n^M, d_n^M)_{n\geq 0}$.

Let $\mathrm{B}_\bullet = (\mathrm{B}_n, d_n)_{n\geq 0}$ be the bimodule bar resolution for an augmented algebra Λ. Assume the latter is defined by a set of generators X and a family of defining relations that form a Groebner–Shirshov basis with a set of obstructions V. We may consider $\mathrm{B}_\bullet$ as a complex of left $\Lambda\otimes\Lambda^{op}$-modules, the basis of B_n is formed by the elements $[u_1|u_2|\dots|u_n]$ where u_i are V-reduced words in the alphabet X. Then there is a general way how to choose a Morse matching in $\Gamma(\mathrm{B}_\bullet)$ described in [16;23], see also [1, Theorem 2.2].

In the case when the Groebner–Shirshov basis of Λ consists of quadratic-linear relations, we may assume that each element of V is a two-letter word. Then the general construction of a Morse matching becomes quite simple: the set M consists of all edges

$$[x_1|\dots|x_i|x'_{i+1}|x''_{i+1}|x_{i+2}|\dots|x_n] \to [x_1|\dots|x_i|w_{i+1}|x_{i+2}|\dots|x_n], \quad i\geq 0$$

for $x_j, x'_{i+1}, x''_{i+1} \in X$, where $x'_{i+1}x''_{i+1} = w_{i+1}$ in Λ (here w_{i+1} is actually a linear combination of V-reduced words) and $x_i x'_{i+1} \in V$ (or $i=0$).

The smaller chain complex $(\mathrm{B}_n^M(\Lambda), d_n^M)_{n\geq 0}$ obtained in this way from the bar complex of Λ is the Λ-free Anick resolution $\mathrm{A}_\bullet = (\mathrm{A}_n, \delta_n)_{n\geq 0}$ [16].

Example 2. Let $\mathfrak{g} = H_3$ be the Heisenberg Lie algebra. The universal enveloping algebra $U(H_3)$ is generated by the elements x, y, z, relative to the following relations:

$$xy = yx + z, \quad xz = zx, \quad yz = zy.$$

Assume $x > y > z$. Then the Anick n-chains are:

$$V^{(1)} = \{[x|y],[x|z],[y|z]\}, \quad V^{(2)} = \{[x|y|z]\}, \quad V^{(n)} = \emptyset,\ n\geq 3.$$

In order to compute $\delta_3[x|y|z]$, consider a fragment of the bar resolution graph and choose a Morse matching Tracking the paths and collecting similar terms lead to the following answer:

$$\delta_3[x|y|z] = x[y|z] - [y|z]x + [x|z]y - y[x|z] + z[x|y] - [x|y]z.$$

3. Conformal endomorphisms and the 1st Weyl algebra

From now on, $\Bbbk$ is a field of characteristic zero, $H = \Bbbk[\partial]$ is the polynomial algebra in one variable.

Suppose V and M are two H-modules. A *conformal homomorphism* φ (see [17]) from V to M is a $\Bbbk$-linear map

$$\varphi_\lambda : V \to M[\lambda] = \Bbbk[\partial, \lambda] \otimes_H M$$

such that

$$\varphi_\lambda(f(\partial)v) = f(\partial + \lambda)\varphi_\lambda(v)$$

for all $v \in V$, $f = f(\partial) \in H$. Here λ is a formal variable. All conformal homomorphisms from V to M form a linear space denoted $Chom(V, M)$. This is also an H-module: the action is given by

$$(\partial\varphi)_\lambda = -\lambda\varphi_\lambda.$$

If $M = V$ then the space of all conformal homomorphisms from V to M is denoted $Cend(V)$. If V is a finitely generated H-module then $Cend(V)$ is an *associative conformal algebra* [17]: for every $\varphi, \psi \in Cend(V)$ we have

$$(\varphi \circ_\lambda \psi) \in Cend(V)[\lambda]$$

defined by the rule

$$(\varphi \circ_\lambda \psi)_\mu = \varphi_\lambda \psi_{\mu-\lambda}.$$

If V is a free H-module of rank $k \in \mathbb{N}$ then $Cend(V)$ is denoted $Cend_k$.

Up to an isomorphism (see [7; 19]), one may identify $Cend_k$ with the space of all $(k \times k)$-matrices over the polynomial ring $\Bbbk[\partial, x]$ equipped with the operation

$$f(\partial, x) \circ_\lambda g(\partial, x) = f(-\lambda, x)g(\partial + \lambda, x + \lambda),$$

$f, g \in \Bbbk[\partial, x]$. For matrices, the operation $(\cdot \circ_\lambda \cdot)$ is extended by the ordinary row-column rule.

Let H act from the right on the Laurent polynomials $\Bbbk[t, t^{-1}]$ in such a way that $\partial = -d/dt$. For every conformal algebra C in the sense of [17], one may define the *coefficient algebra* $\mathcal{A}(C)$ as the linear space $\Bbbk[t,t^{-1}]\otimes_H C$ equipped with the multiplication

$$a(n)b(m) = \sum_{s\ge 0}\binom{n}{s}(a\circ_s b)(n+m-s) \tag{3.1}$$

where $t^n\otimes_H a = a(n)$ for $a\in C$, $n\in\mathbb{Z}$, and $(a\circ_s b)$ stands for the coefficient at $\lambda^s/s!$ of $(a\circ_\lambda b)$, $a,b\in C$. For polynomials from $Cend_1$, for example, we have

$$f(x)\circ_s g(x) = f(x)\frac{d^s}{dx^s}g(x)$$

by the Taylor formula.

The subspace of $\mathcal{A}(C)$ spanned by all $a(n)$, $n\ge 0$, $a\in C$, is a subalgebra of $\mathcal{A}(C)$ denoted $\mathcal{A}_+(C)$. For instance, $\mathcal{A}(Cend_1)=\Bbbk[t,t^{-1},x]$ as a linear space, the isomorphism identifies $t^n\otimes_H x^m$, $n\in\mathbb{Z}$, $m\in\mathbb{Z}_+$, with $x^m t^n\in\Bbbk[t,t^{-1},x]$. The product of two such monomials is calculated via (3.1). For example,

$$t^n\cdot xt^m = (1\circ_0 x)t^{n+m} + n(1\circ_1 x)t^{n+m-1} = xt^{n+m}+nt^{n+m-1},$$

so $tx = xt+1$, $t^{-1}x = xt^{-1}-t^{-2}$, etc. Hence, $\mathcal{A}(Cend_1)$ is isomorphic to the localization of the first Weyl algebra $W_1=\Bbbk\langle p,q\mid qp-pq=1\rangle$ relative to the multiplicative set $\{q^s\mid s\ge 0\}$. The positive part $\mathcal{A}_+(Cend_1)$ is isomorphic to the Weyl algebra itself, so $\mathcal{A}_+(Cend_k)\simeq M_k(W_1)$.

Let C be an associative conformal algebra, and let M be a conformal bimodule over C. Then M is a bimodule over the ordinary associative algebra $A=\mathcal{A}_+(C)$, the action is given by

$$a(n)\cdot u = a\circ_n u,\quad u\cdot a(n) = \{u\circ_n a\} = \sum_{s\ge 0}(-1)^{n+s}\frac{1}{s!}\partial^s(u\circ_{n+s} a),$$

for $u\in M$, $a\in C$, $n\in\mathbb{Z}_+$.

The *basic Hochschild complex* [3] of C with coefficients in M is isomorphic to the Hochschild complex of $A=\mathcal{A}_+(C)$ with coefficients in the same bimodule M. There is a linear map

$$D_n: C^n(A,M)\to C^n(A,M)$$

given by

$$\begin{aligned}(D_nf)(a_1(m_1),\dots,a_n(m_n)) &= \partial f(a_1(n_1),\dots,a_n(m_n))\\ &+\sum_{i=1}^n m_i f(a_1(n_1),\dots,a_i(m_i-1),\dots,a_n(m_n)),\end{aligned}$$

for $f \in C^n(A, M)$. The maps D_n are induced by the derivation $\partial : a(m) \mapsto -ma(m-1)$ on the algebra A. Since $D_{n+1}\Delta^n = \Delta^n D_n$, the image $D_\bullet C^\bullet(A, M)$ is a subcomplex of $C^\bullet(A, M)$, and the quotient

$$\overline{C}^\bullet(A, M) = C^\bullet(A, M)/D_\bullet C^\bullet(A, M)$$

is isomorphic to the *reduced Hochschild complex* of the conformal algebra C (see [3, Theorem 6.1, Corollary 6.1]).

Proposition 1. *If C is an associative conformal algebra, $A = \mathcal{A}_+(C)$, M is a conformal bimodule over C, and $\mathrm{H}^q(A, M) = 0$ for all $q \geq 3$, then $\mathrm{H}^q(\overline{C}^\bullet(A, M)) = 0$ for all $q \geq 3$.*

Proof. The short exact sequence

$$0 \to D_\bullet C^\bullet(A, M) \to C^\bullet(A, M) \to \overline{C}^\bullet(A, M) \to 0$$

gives rise to the long exact sequence of cohomologies

$$\begin{aligned} \dots \to \mathrm{H}^q(D_\bullet C^\bullet(A, M)) \to \mathrm{H}^q(C^\bullet(A, M)) \to \mathrm{H}^q(\overline{C}^\bullet(A, M)) \\ \to \mathrm{H}^{q+1}(D_\bullet C^\bullet(A, M)) \to \mathrm{H}^{q+1}(C^\bullet(A, M)) \to \mathrm{H}^{q+1}(\overline{C}^\bullet(A, M)) \to \dots \end{aligned}$$

By [3, Proposition 2.1], the complexes $C^\bullet = C^\bullet(A, M)$ and $D_\bullet C^\bullet$ are isomorphic in positive degrees. Hence, under the conditions of the statement, $\mathrm{H}^q(\overline{C}^\bullet(A, M))$, $q \geq 3$, is clamped between zeros, thus it is zero itself. □

4. Two-sided Anick resolution for the first Weyl algebra

In this section, we apply the Morse matching method described in Section 2 to compute the 3rd Hochschild cohomology of the first Weyl algebra with coefficients in an arbitrary bimodule. The Weyl algebra W_1 is generated by the elements q, p, e, relative to the following relations:

$$qp = pq + e, \quad pe = p, \quad qe = q, \quad eq = q, \quad ep = p, \quad ee = e.$$

Assume $q > p > e$. Then the sets of Anick n-chains for $n = 1, 2, 3$ are easy to find:

$$\begin{aligned} V^{(1)} &= \{[q|p], [q|e], [p|e], [e|q], [e|p], [e|e]\}, \\ V^{(2)} &= \{[q|p|e], [e|q|p], [q|e|p], [p|e|q], [q|e|e], [p|e|e], [e|e|q], [e|e|p], [e|q|e], \\ &\quad [e|p|e], [q|e|q], [p|e|p], [e|e|e]\}, \\ V^{(3)} &= \{[q|p|e|e], [e|q|p|e], [q|e|p|e], [p|e|q|e], [q|e|e|e], [p|e|e|e], \\ &\quad [e|q|e|e], [e|p|e|e], [q|e|q|e], [p|e|p|e], [e|e|e|e], [e|e|q|p], [e|q|e|p], \\ &\quad [e|p|e|q], [e|e|e|q], [e|e|e|p], [e|e|q|e], [e|e|p|e], [e|q|e|q], \\ &\quad [e|p|e|p], [q|e|e|p], [p|e|e|q], [q|e|e|q], [p|e|e|p], [q|p|e|q], [q|p|e|p]\}. \end{aligned}$$

In order to compute $\mathrm{H}^3(W_1, M)$ for an arbitrary W_1-bimodule M we need to know the Anick differentials on $V^{(2)}$ and $V^{(3)}$.

For example, consider a fragment of the graph constructed from the bar resolution of $\Lambda = W_1 \oplus \Bbbk 1$ with the vertex $[q|p|e]$ with a matched edge $[p|q|e] \to [pq|e]$, see Fig. 1 a. Note that $[p|q]$ is not an Anick chain thus should not be a critical cell. Indeed, the vertex $[p|q]$ belongs to another matched edge $[p|q] \to [pq]$ which also appears in the bar resolution graph, see Fig. 1 b. In a similar way, construct a fragment with the vertex $[e|q|p]$ on Fig. 2 a: all ending vertices of this fragment are either Anick chains or $[p|q]$ which is already matched. Note that the vertices $[e|p|q]$ and $[q|p|e]$ belong to matched edges. As a final example, consider the fragment with $[e|q|p|e]$ (Fig. 2 b): all ending vertices of this graph are either Anick chains or already matched ones.

In the sequel, we will often omit symbols $|$ in the elements of $V^{(n)}$.

In the same way, one may compute Anick differentials on the other chains from $V^{(2)}$ and $V^{(3)}$. As a resul, we get the following statements.

Lemma 1. *The differential $\delta_3 : \mathrm{A}_3 \to \mathrm{A}_2$ in the Anick resolution for W_1 is given by the following formulas (where we write $[xyz]$ instead of $[x|y|z]$ for brevity):*

$$\begin{gathered}
\delta_3[eqe] = e[qe] - [qe] + [eq] - [eq]e, \\
\delta_3[epe] = e[pe] - [pe] + [pe] - [ep]e, \\
\delta_3[qpe] = q[pe] - p[qe] - [ee] + [qp] - [qp]e, \\
\delta_3[eqp] = e[qp] - [qp] + [ep]q + [ee] - [eq]p, \\
\delta_3[qep] = q[ep] - [qe]p, \quad \delta_3[peq] = p[eq] - [pe]q, \\
\delta_3[qee] = q[ee] - [qe]e, \quad \delta_3[pee] = p[ee] - [pe]e, \\
\delta_3[eeq] = e[eq] - [ee]q, \quad \delta_3[eep] = e[ep] - [ee]p, \\
\delta_3[qeq] = q[eq] - [qe]q, \quad \delta_3[pep] = p[ep] - [pe]p, \\
\delta_3[eee] = e[ee] - [ee]e.
\end{gathered}$$

Lemma 2. *The differential $\delta_4 : \mathrm{A}_4 \to \mathrm{A}_3$ in the Anick resolution for W_1 is given by:*

$$\begin{aligned}
\delta_4[qeep] &= q[eep] - [qep] + [qee]p, & \delta_4[peeq] &= p[eeq] - [peq] + [pee]q, \\
\delta_4[qeee] &= q[eee] - [qee] + [qee]e, & \delta_4[peee] &= p[eee] - [pee] + [pee]e, \\
\delta_4[eeeq] &= e[eeq] - [eeq] + [eee]q, & \delta_4[eeep] &= e[eep] - [eep] + [eee]p, \\
\delta_4[eeqe] &= e[eqe] - [eeq] + [eeq]e, & \delta_4[eepe] &= e[epe] - [eep] + [eep]e, \\
\delta_4[qeeq] &= q[eeq] - [qeq] + [qee]q, & \delta_4[peep] &= p[eep] - [pep] + [pee]p, \\
\delta_4[qepe] &= q[epe] - [qep] + [qep]e, & \delta_4[peqe] &= p[eqe] - [peq] + [peq]e,
\end{aligned}$$

a)

$$\begin{tikzcd}
{[p|e]} & {[q|p|e]} \arrow[l, "q\otimes 1"'] \arrow[r, "-1\otimes e"] \arrow[d, "-1"'] \arrow[dr, "+1"] & {[e|q]} \\
 & {[pq|e]} \arrow[d, bend right, "+1"'] & {[q|p]} \\
{[q|e]} & {[p|q|e]} \arrow[l, "p\otimes 1"'] \arrow[r, "-1\otimes e"] \arrow[u, dotted, "-1"'] \arrow[d, "1"'] & {[p|q]} \\
 & {[p|q]} &
\end{tikzcd}$$

b)

$$\begin{tikzcd}
{[p]} & {[q|p]} \arrow[l, "q\otimes 1"'] \arrow[r, "1\otimes p"] \arrow[dl, "-1"'] \arrow[d, "-1"] & {[q]} \\
{[e]} & {[pq]} \arrow[d, bend right, "+1"'] & \\
{[q]} & {[p|q]} \arrow[l, "p\otimes 1"'] \arrow[r, "1\otimes q"] \arrow[u, dotted, "-1"'] & {[p]}
\end{tikzcd}$$

Figure 1. Calculating the Anick differential of $[q|p|e]$ and $[q|p]$

$$\begin{aligned}
&\delta_4[epee] = e[pee] - [pee] + [epe]e, \quad \delta_4[qeqe] = q[eqe] - [qeq] + [qeq]e,\\
&\delta_4[pepe] = p[epe] - [pep] + [pep]e, \quad \delta_4[eeee] = e[eee] - [eee] + [eee]e,\\
&\delta_4[eqep] = e[qep] - [qep] + [eqe]p, \quad \delta_4[epeq] = e[peq] - [peq] + [epe]q,\\
&\delta_4[epep] = e[pep] - [pep] + [epe]p, \quad \delta_4[eqeq] = e[qeq] - [qeq] + [eqe]q,\\
&\delta_4[eeqp] = e[eqp] - [eep]q - [eee] + [eeq]p,\\
&\delta_4[eqpe] = e[qpe] - [qpe] + [eee] - [eqp] + [eqp]e,\\
&\delta_4[qpee] = q[pee] - p[qee] - [eee] + [qpe]e,\\
&\delta_4[qpeq] = q[peq] - [eeq] - p[qeq] + [qpe]q,\\
&\delta_4[qpep] = q[pep] - [eep] - p[qep] + [qpe]p,\\
&\delta_4[eqee] = e[qee] - [qee] + [eqe]e.
\end{aligned}$$

Theorem 2. *For an arbitrary W_1-bimodule M, the Hochchild cohomology group $\mathrm{H}^3(W_1, M)$ is trivial.*

Proof. It is enough to find the respective cohomology group of the complex $\mathrm{Hom}_{\Lambda-\Lambda}(\mathrm{A}_\bullet, M)$, where $\Lambda = W_1 \oplus \Bbbk 1$, as above.

Note that an arbitrary bimodule M over W_1 is a direct sum of four components:

$$M = M_{1,1} \oplus M_{0,1} \oplus M_{1,0} \oplus M_{0,0},$$

where the identity element $e \in W_1$ act on $M_{i,j}$ in such a way that $eu = iu$, $ue = ju$, for $u \in M_{i,j}$, $i, j \in \{0, 1\}$. Hence, we may consider cohomologies with coefficients on the summands $M_{i,j}$ separately.

First, assume $M = M_{1,1}$, i.e., $eu = ue = u$ for all $u \in M$. Suppose $\varphi : \mathrm{A}_3 \to M$ is a cocycle, i.e., $\Delta^3(\varphi) = \varphi\delta_4 = 0$. Apply φ to all relations in Lemma 2: since zero emerges in all right-hand sides, we get the following *a*)

$$\begin{tikzcd}
{[q|p]} & {[e|q|p]} \arrow[l, "e\otimes 1"'] \arrow[r, "-1\otimes p"] \arrow[dl, "-1"'] \arrow[d, "1"'] & {[e|q]} \\
{[q|p]} & {[e|pq]} \arrow[d, bend right, "-1"'] & \\
{[p|q]} & {[e|p|q]} \arrow[l, "e\otimes 1"'] \arrow[r, "-1\otimes q"] \arrow[u, dotted, "+1"'] \arrow[d, "1"'] & {[e|p]} \\
 & {[p|q]} &
\end{tikzcd}$$

b)

$$\begin{tikzcd}
{[q|p|e]} & {[e|q|p|e]} \arrow[l, "e\otimes 1"'] \arrow[r, "1\otimes e"] \arrow[dl, "-1"'] \arrow[d, "+1"'] \arrow[dr, "-1"] & {[e|q|p]} \\
{[q|p|e]} & {[e|pq|e]} \arrow[d, bend right, "-1"'] & {[e|q|p]} \\
{[p|q|e]} & {[e|p|q|e]} \arrow[l, "e\otimes 1"'] \arrow[r, "1\otimes e"] \arrow[u, dotted, "+1"'] \arrow[dl, "-1"'] \arrow[dr, "-1"] & {[p|q|e]} \\
{[p|q|e]} & & {[e|p|q]}
\end{tikzcd}$$

Figure 2. Calculating the Anick differential of $[e|q|p]$ and $[e|q|p|e]$

relations on the values of φ on the basis of A_3 as of a free Λ-bimodule:

$$\begin{aligned}
\varphi[qpe] &= -q\varphi[pee] + p\varphi[qee] + \varphi[eee],\\
\varphi[eqp] &= \varphi[eep]q + \varphi[eee] - \varphi[eeq]p,\\
\varphi[qep] &= q\varphi[eep] + \varphi[qee]p,\\
\varphi[peq] &= p\varphi[eeq] + \varphi[pee]q,\\
\varphi[qeq] &= q\varphi[eeq] + \varphi[qee]q,\\
\varphi[pep] &= p\varphi[eep] + \varphi[pee]p,\\
q\varphi[eee] &= p\varphi[eee] = \varphi[eee]q = 0,\\
\varphi[eee]p &= \varphi[eqe] = \varphi[epe] = 0.
\end{aligned} \tag{4.1}$$

As a corollary,

$$\varphi[eee] = e\varphi[eee] = q(p\varphi[eee]) - p(q\varphi[eee]) = 0.$$

Hence, φ is completely determined by its values

$$\varphi[eeq],\ \varphi[eep],\ \varphi[qee],\ \varphi[pee].$$

Let us define $\psi \in \mathrm{Hom}_{\Lambda-\Lambda}(\mathrm{A}_2, M)$ in such a way that

$$\psi[eq] = \varphi[eeq],\ \psi[ep] = \varphi[eep],\ \psi[qe] = -\varphi[qee],\ \psi[pe] = -\varphi[pee],$$

and $\psi[ee] = \psi[qp] = 0$. Then $\Delta^2(\psi) = \psi\delta_3$ is a coboundary, and by Lemma 1 we have:

$$\begin{aligned}
(\psi\delta_3)[eeq] &= e\psi[eq] - \psi[ee]q = \varphi[eeq] + 0 = \varphi[eeq],\\
(\psi\delta_3)[eep] &= e\psi[ep] - \psi[ee]p = \varphi[eep] + 0 = \varphi[eep],\\
(\psi\delta_3)[qee] &= q\psi[ee] - \psi[qe]e = 0 + \varphi[qee] = \varphi[qee],\\
(\psi\delta_3)[pee] &= p\psi[ee] - \psi[pe]e = 0 + \varphi[pee] = \varphi[pee].
\end{aligned}$$

Hence, $\Delta^2(\psi)=\varphi$, i.e., every 3-cocycle is a coboundary, so $\mathrm{H}^3(W_1,M)=0$ for every bimodule M over W_1.

Next, assume $M=M_{1,0}$, i.e., $eu=u$ and $ue=0$ for all $u\in M$. It follows from (4.1) that

$$q\varphi[pee]-p\varphi[qee]-\varphi[eee]=0,\quad \varphi[qep]=q\varphi[eep],\quad \varphi[peq]=p\varphi[eeq],$$
$$\varphi[qee]=q[eee],\quad \varphi[pee]=p[eee],\quad \varphi[eqe]=[eeq],\quad \varphi[epe]=[eep],$$
$$\varphi[qeq]=q\varphi[eeq],\quad \varphi[pep]=p\varphi[eep],\quad \varphi[eqp]=\varphi[eee].$$

Therefore, φ is completely determined by its values $\varphi[eeq]$, $\varphi[eep]$, $\varphi[eee]$, $\varphi[qpe]$. Let us define $\psi\in\mathrm{Hom}_\Lambda(\mathrm{A}_2,M)$ in such a way that

$$\psi[eq]=\varphi[eeq],\ \psi[ep]=\varphi[eep],\ \psi[qe]=\varphi[qee],$$
$$\psi[pe]=\varphi[pee],\ \psi[ee]=\varphi[eee],\ \psi[qp]=\varphi[qpe].$$

Then $\Delta^2(\psi)=\psi\delta_3$ is a coboundary, and

$$\begin{aligned}(\psi\delta_3)[eeq]&=e\psi[eq]=\psi[eq]=\varphi[eeq],\\(\psi\delta_3)[eep]&=e\psi[ep]=\psi[ep]=\varphi[eep],\\(\psi\delta_3)[eee]&=e\psi[ee]=\varphi[eee]=\varphi[eee],\\(\psi\delta_3)[qpe]&=q\psi[pe]-p\psi[qe]-\psi[ee]+\psi[qp]\\&=q\varphi[pee]-p\varphi[qee]-\varphi[eee]+\varphi[qpe]\\&=0+\varphi[qpe]=\varphi[qpe].\end{aligned}$$

Hence, $\Delta^2(\psi)=\varphi$, i.e., every 3-cocycle is a coboundary, so $\mathrm{H}^3(W_1,M)=0$.

The cases of right-unital ($M_{0,1}$) and trivial ($M_{0,0}$) modules are completely analogous. □

Since for every associative algebra A and for every A-bimodule M we have $\mathrm{H}^{n+1}(A,M)=\mathrm{H}^n(A,\mathrm{Hom}(A,M))$, all higher cohomologies (for $n\geq 3$) also vanish, where $\mathrm{Hom}(A,M)$ carries the $A-$bimodule structure from [15].

Corollary 1. *For every $n\geq 3$ we have $\mathrm{H}^n(W_1,M)=0$.*

The Hochschild cohomology is invariant under Morita equivalence of algebras, and it is known that an algebra A is Morita equivalent to the algebra of matrices $M_n(A)$ [18], [20, Chapter 7], so $\mathrm{H}^n(M_k(W_1),M)=\mathrm{H}^n(W_1,M)=0$.

As a corollary, we obtain the following description of conformal Hochschild cohomologies of the associative conformal algebra $Cend_k$.

Theorem 3. *Let M be a conformal bimodule over $Cend_k$, $k\geq 1$. Then $\mathrm{H}^n(Cend_k,M)=0$ for $n\geq 2$.*

Proof. Recall that $A = \mathcal{A}_+(Cend_k) \simeq M_k(W_1)$, where W_1 is the first Weyl algebra. By Corollary 1 (which is a direct consequence of Theorem 2), we have $\mathrm{H}^n(W_1, N) = 0$ for all $n \geq 3$ and any W_1-bimodule N. Since Hochschild cohomology is invariant under Morita equivalence, it follows that $\mathrm{H}^n(M_k(W_1), M) = 0$ for all $n \geq 3$ and any $Mk(W1)$-bimodule M (which is a conformal bimodule over $Cend_k$).

Therefore, the conditions of Proposition 1 are satisfied, and it implies $\mathrm{H}^n(Cend_k, M) = 0$ for $n \geq 3$. For $n = 2$, the result was obtained in [11]. $\square$

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