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Ferrohydrodynamic Instability of a Viscoelastic Magnetic Fluid Layer under the Influence of Time-varying Magnetic Field

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Abstract. The current work concerns the impact of a time-varying magnetic field on the threshold of Oldroyd-B viscoelastic magnetic fluid convection in the presence of both modulated and unmodulated magnetic fields. A time-varying magnetic field plays a crucial role in creating circulation in small passages where the effect of gravity is absent or ineffective. The resulting eigenvalue problem is obtained by a regular perturbation expansion under the assumption of a small modulation amplitude. The impact of the magnetic parameter, Prandtl number, stress relaxation parameter, strain retardation parameter and magnetic field modulation frequency, were discussed. The study shows that, the magnetic field modulation has a destabilizing impact on the system with convection occurs faster.

Keywords: magnetic fluid, time-varying (modulation) magnetic field, viscoelasticity

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Научная статья

Феррогидродинамическая неустойчивость слоя вязкоупругой магнитной жидкости под действием

переменного во времени магнитного поля

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Аннотация. Изучается влияние изменяющегося во времени магнитного поля на порог конвекции вязкоупругой магнитной жидкости Олдройда-Б при наличии как модулированных, так и немодулированных магнитных полей. Изменяющееся во времени магнитное поле играет решающую роль в создании циркуляции в небольших проходах, где действие гравитации отсутствует или неэффективно. Результирующая задача на собственные значения получается путем регулярного разложения по возмущениям в предположении малой амплитуды модуляции. Обсуждается влияние магнитного параметра, числа Прандтля, параметра релаксации напряжений, параметра замедления деформации и частоты модуляции магнитного поля. Показано, что модуляция магнитного поля оказывает дестабилизирующее воздействие на систему, при этом конвекция происходит быстрее.

Ключевые слова: магнитная жидкость, изменяющееся во времени (модуляционное) магнитное поле, вязкоупругость

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1. Introduction

Ferrohydrodynamics is the study of the movement mechanisms of fluids driven by strong magnetic polarization forces. Ferrohydrodynamics addresses issues bearing non-electrically conducting fluids with magnetic characteristics; the most widely used fluids are ferromagnetic fluids (magnetic fluids or ferrofluids). Magnetic fluids have demonstrated great potential in areas such as biomedicine, aeronautical engineering, rotary seals in computer storage drives, magnetic resonance photography as contrast material and thermal engineering [11, 23, 19]. The concept of magnetic fluid convection toward the thermal expansion in a layer comprising magnetic fluid is similar to Chandrasekhar's [4] classical Rayleigh–Benard instability. Magnetic fluid garners much attention in the literature because it has a potential to serve as a mechanism for exchanging heat. Finlayson [7] was the one who meticulously contemplated Chandrasekhar's [4] classical study regarding the Rayleigh-Benard problem for magnetic fluid through the normal mode technique by means of an infinite amplitude disturbance. As a result of this research, various aspects of ferroconvection models have been analyzed by many researchers over the years as well [1, 13, 14, 20, 22, 24, 25].

Modulating an appropriate parameter (gravity, temperature, electric field [18] etc.) can have a considerable influence on the motion of a variety of domains; varying these parameters with respect to time can enhance the stability of the system. The present work focuses on magnetic field modulation; it plays a crucial role in creating circulation in small passages where the effect of gravity is absent or ineffective. One can easily regulate convective motion in the system using magnetic field modulation. It has many engineering applications, such as magnetic field detection devices, charged particles in electrode materials, modulators, magnetic resonators, and optical devices, to name a few. The time-varying magnetic field at the commencement of magnetic fluid convection and the conflict between harmonic and sub-harmonic modes using Floquet theory, the Chebyshev pseudo-procedure, and the QZ method have been well documented by eminent researchers in detail [3, 6, 10, 15]. Widening the focus of interest, the effect of magnetic field modulation with couple stresses and rotation were discussed in the theoretical work [5, 2].

A non-Newtonian fluid does not obey Newton's viscosity law, which implies that its viscosity varies with stress. Non-Newtonian fluids, in particular, tend to change their viscosity when exposed to force. When the imposed stress is removed from some fluids, they partially revert to their original shape. Such fluids are known as viscoelastic fluids. Viscoelastic fluids have gained popularity in recent years as a result of their uses in industry, medical science, and technology. Polymer solutions, DNA suspensions, biological fluids, cosmetic goods, colloidal solutions, oils, lubricants, and clay coatings are some examples of viscoelastic fluids. For the classical Rayleigh-Benard issue the overstabilities in the Oldroyd-B fluid are thoroughly investigated in the case of a fixed lower wall temperature [21] and later the work extends to a fixed heat flux in the bounding surfaces [12]. Since then, several researchers have made significant contributions to this field, both theoretically and experimentally [8, 9, 16, 17, 26].

The available literature clearly shows that no one has previously examined the impact of a magnetic field with time-varying characteristics on the starting point of convection in a viscoelastic magnetic fluid. As a result, the current work focuses on the ferrohydrodynamic instability of viscoelastic magnetic fluids in the presence of a time-varying magnetic field. The direction of this work is to determine the base state and its stability through a linear analysis and the eigenvalue of the problem by means of a regular perturbation approach. Further, the study is based on the assumption that the convective currents are minimal and the frequency of the time-varying magnetic field is extremely small. As a result, depending on the frequency of the time-varying magnetic field, the advent of viscoelastic magnetic fluid convection can be either hastened or postponed.

2. Mathematical Model

This model is examined with a Cartesian reference frame $(x,\ y,\ z)$, where the horizontal directions are denoted by the $x-$and $y-$axes and the $z-$axis denotes the vertical upward direction. The system is comprised of a thin surface of viscoelastic magnetic fluid in the horizontal directions (i.e., in the $x-y$ plane), which is heated from the underside and cooled from the overside as shown in Fig. 1. The region is supposed to have two surfaces, say $z \geq 0$ as the bottom surface of magnetic fluid and $z \leq d$ as the top surface of magnetic fluid. The Oldroyd-B model, which comprises the time derivative, is used as a momentum equation due to the introduction of viscoelasticity. The system is working under the impact of a vertically upward, external time-varying magnetic field $\vec{H}_0^{ext}(t) = H_0^{ext}(t) = H_0\left(1+\varepsilon\cos\omega t\right)\hat{k}$ due to magnetic field modulation and acceleration due to vertically downward gravitational force $\vec{g} = -g\hat{k}$. Here, H_0, ε, ω, t and g stand for uniform magnetic field, amplitude, frequency, time and gravitational acceleration, respectively. The bottom and top surfaces are kept at different uniform temperatures with a gradient ΔT. The Boussinesq approximation is applied to account for the influence of density variations. With these assumptions, the relevant equations governing the flow of an incompressible viscoelastic magnetic fluid with time-varying magnetic field impacts are as follows [7, 12]

$$\nabla\cdot\vec{q} = 0. \tag{2.1}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\begin{pmatrix}\rho_0\left[\dfrac{\partial\vec{q}}{\partial t}+\left(\vec{q}\cdot\nabla\right)\vec{q}\right]-\rho\vec{g}\\ +\nabla p-\nabla\cdot\left(\vec{H}\,\vec{B}\right)\end{pmatrix} = \left(1+\lambda_2\frac{\partial}{\partial t}\right)\mu\nabla^2\vec{q}, \tag{2.2}$$

$$\rho = \rho_0\left[1-\alpha\left(T-T_a\right)\right], \tag{2.3}$$

$$C_1\left[\frac{\partial T}{\partial t}+\left(\vec{q}\cdot\nabla\right)T\right]+\mu_0\,T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\cdot\left[\frac{\partial\vec{H}}{\partial t}+\left(\vec{q}\cdot\nabla\right)\vec{H}\right] = K_1\nabla^2 T, \tag{2.4}$$

$$M = M_0+\chi_m\left(H-H_0\right)-K_m\left(T-T_a\right), \tag{2.5}$$

where λ_1, $\vec{q}$, p, $\vec{H}$, $\vec{B}$, λ_2 T, T_a, α, ρ, ρ_0, M_0, χ_m, K_m and μ stand for relaxation time, velocity, pressure, magnetic field, magnetic induction, retardation time, temperature, reference temperature, coefficient of thermal

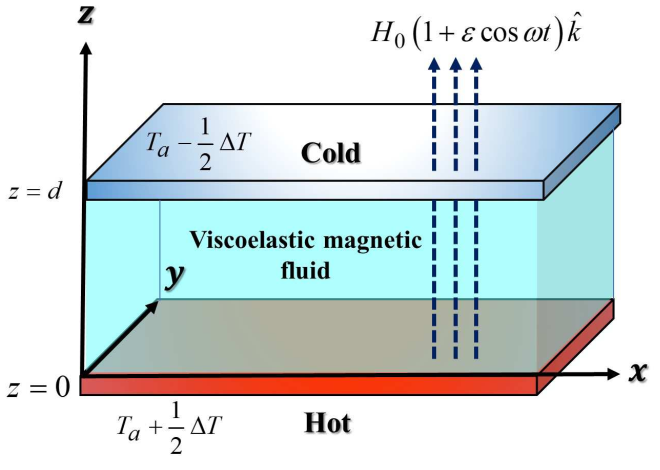

Figure 1. Sketch of the physical system.

expansion, density, reference density, reference magnetization, magnetic susceptibility, pyromagnetic coefficient and viscosity, respectively.
$C_1 = \left(\rho_0 C_{V,H} - \mu_0 \vec{H} \cdot \left((\partial \vec{M})/(\partial T)\right)_{V,H}\right)$, $C_{V,H}$, $\vec{M}$, μ_0 and K_1 describe specific heat at constant volume and magnetic field, magnetization, magnetic permeability and thermal conductivity, respectively.

Maxwell's equations applicable to the problem at hand are

$$\nabla \cdot \vec{B} = 0, \ \nabla \times \vec{H} = 0, \tag{2.6}$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M}\right), \tag{2.7}$$

$$\vec{M} = \frac{\vec{H}}{H} M(H, T). \tag{2.8}$$

3. Linear Stability

On applying the method of small perturbation and introducing the magnetic potential ϕ, we obtain the following stability equations [2, 5, 18]

$$\left(1 + F_1 \frac{\partial}{\partial t}\right) \begin{pmatrix} \frac{1}{\Pr} \frac{\partial}{\partial t} \left(\nabla^2 W\right) - [R + C_2] \nabla_1^2 T \\ + C_2 \frac{\partial}{\partial z} \left(\nabla_1^2 \phi\right) \end{pmatrix} = \left(1 + F_2 \frac{\partial}{\partial t}\right) \nabla^4 W, \tag{3.1}$$

$$\left(\begin{array}{l}\left(\frac{\partial T}{\partial t}-W\right)-M_2\left(\frac{(1+\varepsilon L)^2}{\chi_0\,(1+\chi_0)}\right)\left(\frac{\partial T}{\partial t}-W\right)\\ -\,M_2\frac{1}{H_0}\frac{\partial}{\partial t}\frac{H_0\,(1+\varepsilon L)}{(1+\chi_0)}\left(\frac{\partial\phi}{\partial z}\right)-\,M_2\left(\frac{(1+L)^2}{(1+\chi_0)}\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right)\\ -\,M_2\frac{1}{H_0}T\frac{\partial}{\partial t}H_0\,(1+\varepsilon L)+\,M_2\left(\frac{(1+\varepsilon L)^2}{(1+\chi_0)}\right)W\end{array}\right)=\nabla^2 T, \tag{3.2}$$

$$\nabla^2\phi=\frac{\partial T}{\partial z}, \tag{3.3}$$

where $C_2 = RM_1(1+\varepsilon L)^2$; $L = \mathrm{Re}\left\{e^{-i\omega t}\right\} = \cos\omega t$; $\nabla_1^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$; $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$ and various non-dimensional parameters are $\mathrm{Pr} = \frac{\mu}{\rho_0\kappa}$; $R = \frac{\alpha\rho_0 g\Delta T d^3}{\mu\kappa}$; $\kappa = \frac{K_1}{C_{V,H}}$; $F_1 = \frac{\lambda_1\kappa}{d^2}$; $F_2 = \frac{\lambda_2\kappa}{d^2}$; $M_2 = \frac{\mu_0\chi_0^2 H_0^2}{C_1(1+\chi_0)T_a}$; $M_1 = \frac{\mu_0\Delta T\chi_0^2 H_0^2}{T_a^2(1+\chi_0)^3\alpha\rho_0 g d}$; $RM_1 = \frac{\mu_0\,\chi_0^2(\Delta T)^2 d^2 H_0^2}{\mu\kappa(1+\chi_0)^3 T_a^2}$.

Here, ∇^2 is the Laplacian differential operator, Pr is the Prandtl number, R is the thermal Rayleigh number, κ is the effective thermal diffusivity, F_1 is the stress relaxation parameter, F_2 is the strain retardation parameter, M_2 is the magnetic parameter, M_1 is the magnetic number and RM_1 is the magnetic Rayleigh number.

Since the typical value of M_2 is equivalent to the order of 10^{-6} [5, 2] for magnetic fluid with different base liquids and hence its effect is neglected as compared to unity. Equations (3.1) – (3.3) are to be solved using suitable boundary conditions [4]

$$W=\frac{\partial^2 W}{\partial z^2}=T=\frac{\partial\phi}{\partial z}=0 \text{ at } z=0\,,\,1. \tag{3.4}$$

It is suitable to rewrite the whole problem in terms of the vertical component of the velocity W. Upon combining Eqs. (3.1) – (3.3), we obtain the following equations

$$G_1\nabla^4 W = G_2\nabla^2\nabla_1^2 W + G_2 M_1(1+\varepsilon L)^2\nabla_1^4 W + G_3\nabla^6 W \tag{3.5}$$

where $G_1 = \frac{1}{\mathrm{Pr}}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}-\nabla^2\right)\left(1+F_1\frac{\partial}{\partial t}\right)$; $G_2 = R\left(1+F_1\frac{\partial}{\partial t}\right)$;
$G_3 = \left(1+F_2\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial t}-\nabla^2\right)$.

The boundary conditions in Eq. (3.4) can also be rearranged in terms of W in the form (Chandrasekhar [4]).

$$W=\frac{\partial^2 W}{\partial z^2}=\frac{\partial^4 W}{\partial z^4}=0 \text{ at } z=0,\,1 \tag{3.6}$$

4. Method of Solution

The eigenfunctions, W and the eigenvalues, R associated with the above eigenvalue problem for a modulated magnetic field that is different from the constant magnetic field by means of small quantity of order ε. Therefore, we assumed the solution of Eq. (3.5) in the form [31, 32]

$$\begin{aligned} W &= W_0 + \varepsilon W_1 + \varepsilon^2 W_2 + \cdots\cdots\cdots \\ R &= R_0 + \varepsilon R_1 + \varepsilon^2 R_2 + \cdots\cdots\cdots\cdots \end{aligned} \tag{4.1}$$

Following the analysis of [18, 2], we get the expression for the Rayleigh number R_0 for the viscoelastic magnetic fluid in the absence of time-varying magnetic field

$$R_0 = \frac{\left(\pi^2 + \alpha^2\right)^4}{\alpha^2 \left[\pi^2 + (1 + M_1)\,\alpha^2\right]}, \tag{4.2}$$

where $\alpha = \sqrt{\alpha_x^2 + \alpha_y^2}$ is the overall horizontal wavenumber. Equation (4.2) is the expression for the thermal Rayleigh number as a function of the wavenumber and the magnetic force for the unmodulated viscoelastic magnetic fluid convection (i.e., when $M_1 = 0$), the expression for R_0 reduces to that of Chandrasekhar [7]. The plot of R_0 versus α is delineated in the section 5.

Following the analysis of [18, 2], we get the following expression for R_2

$$R_2 = -\frac{2R_0^2 M_1^2 \alpha^6}{\left[\pi^2 + (1 + M_1)\,\alpha^2\right]} \sum_{n=1}^{\infty} \frac{C_n}{D_n} \tag{4.3}$$

where $C_n = A\left(1 + 2\omega^2 F_1^2\right) - \omega^2 F_1 B$; $D_n = A^2 + (\omega B)^2$
with

$$A = \begin{pmatrix} -\dfrac{1}{\mathrm{Pr}}\omega^2\left(1 + F_1\left(n^2\pi^2 + \alpha^2\right)\right)\left(n^2\pi^2 + \alpha^2\right)^2 + \left(n^2\pi^2 + \alpha^2\right)^4 \\ -\,\omega^2 F_2\left(n^2\pi^2 + \alpha^2\right)^3 - R_0\alpha^2\left[n^2\pi^2 + (1 + M_1)\,\alpha^2\right] \end{pmatrix}$$

$$B = \begin{pmatrix} \dfrac{1}{\mathrm{Pr}}\left(\omega^2 F_1 - \left(n^2\pi^2 + \alpha^2\right)\right) - \left(n^2\pi^2 + \alpha^2\right)^3 - F_2\left(n^2\pi^2 + \alpha^2\right)^4 \\ +\,R_0\alpha^2 F_1\left[n^2\pi^2 + (1 + M_1)\,\alpha^2\right] \end{pmatrix}$$

5. Results and Discussion

The stability analysis of ferromagnetic viscoelastic liquids with a magnetic field effect on the onset of ferroconvection is meticulously examined for the subsequent pair of cases: unmodulated magnetic field and modulated magnetic field. These cases are discussed and analysed with the help of suitable graphs (refer to Figures 2 through 6).

5.1. Onset of ferroconvection (unmodulated case)

This section analyzes the influence of the buoyancy-magnetization parameter M_1 for the Rayleigh-Benard ferroconvective instability in the absence of magnetic field modulation [7]. In Figure 2, the effect of M_1 (the ratio of magnetic force to gravitational force) is depicted and the graph is plotted between the stationary thermal Rayleigh number denoted by R_0 and the wave numbers denoted by α. The parameter M_1 bears the values

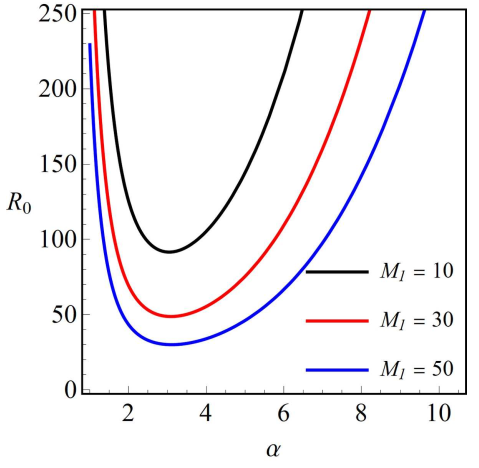

Figure 2. Variation in R_0 versus α_c on increasing M_1

as $M_1 = 10$, 30, 50. It is noticed that magnetic mechanism M_1 is showing the destabilizing impact for the Rayleigh-Bénard ferroconvection without magnetic field modulation. Because as the values of M_1 are increased, the critical values R_{0c} of thermal Rayleigh number is diminished and from the definition of M_1 it follows that an increase in the magnetic force has a destabilizing effect on the flow. Further, the critical values of α_c wave number corresponding to R_0, decreases with an increase in M_1.

5.2. Onset of ferroconvection (modulated case)

The effect of a time-varying magnetic field on the commencement of ferroconvection in a viscoelastic ferromagnetic smart liquid is inspected. Adopting the normal mode technique, the subsequent eigenvalue problem is attacked by means of the regular perturbation method. The principle behind the stability analysis is based on the assumption of minimal amplitude of modulation. The expression for the correction Rayleigh number R_2 is found to be proportional to the modulation frequency ω, the buoyancy-magnetization parameter M_1, Prandtl number Pr, stress relaxation parameter F_1 and strain retardation parameterF_2. We observed that the starting point of convection in the electrically non-conducting fluid is affected by time-varying magnetic field. This is due to the suspended microscopic magnetic particles in the non-magnetic liquid carrier medium which are coated with a surfactant. The microscopic magnetic particles make the fluid magnetically-responding in addition to being thermally responding. We notice that the presence of suspended microscopic magnetic particles in the liquid carrier increases the viscosity of ferromagnetic fluid. In the absence of magnetic field modulation, the viscosity of ferromagnetic fluid depends on the concentration of magnetic particles and surfactant. Whereas, under the influence of magnetic field which is varied sinusoidally with respect to time, the viscosity of ferromagnetic fluid depends on the modulation frequency. If the modulation frequency is less, then the applied magnetic field modulation increases the viscosity of ferromagnetic fluid and for moderate and large range of values of frequency it reduces the viscosity of ferromagnetic fluid. Further, in the case of viscoelastic ferromagnetic fluids due to the presence of long molecules, the viscosity of the fluid is much more than that of a Newtonian fluid. In view of this we consider values of the Prandtl number of ferromagnetic fluid higher than those of carrier liquid without suspended particles. Figures 3 through 6 are being used to summarize the results of the current investigation. The sign of critical correction thermal Rayleigh number R_{2c} is accountable for the stabilizing or destabilizing impact of magnetic field fluctuation on the stability of the system. A positive R_{2c} implies that the magnetic field modulation effect is stabilizing, whereas a negative R_{2c} indicates that the magnetic field modulation effect is destabilizing.

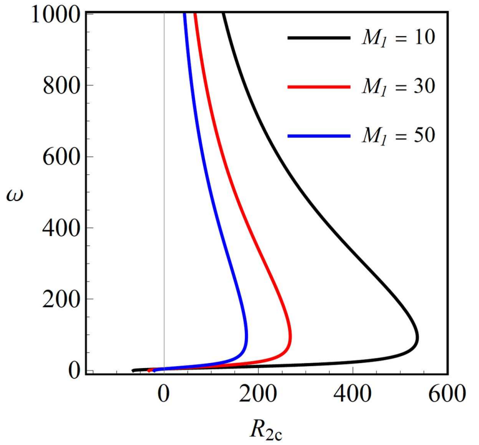

Figure 3. Variation in R_{2c} versus ω on increasing M_1

In Fig. 3, to investigate the effect of magnetic mechanism , all parameters are fixed except M_1 and the graph is plotted between correction to the critical Rayleigh number denoted by R_{2c} and the modulation frequency denoted by ω. The magnetic parameter M_1 bears the values as $M_1 = 10, 30, 50$ and the other parameters are supposed to be constant as $\mathrm{Pr} = 10$, $F_1 = 0.2$ and $F_2 = 0.5$. The general influence of M_1 is to make the system unstable, it can be inferred from Fig. 2 (unmodulated case). It is noticed that the magnetic mechanism M_1 is showing the destabilizing impact on the configuration for the viscoelastic ferroconvective instability over a moderate and large range of frequency (i.e., $20 < \omega \leq 1000$), because as the values of M_1 are increased, the critical values R_{2c} of correction Rayleigh number are diminished. Physically, the magnitude of the magnetic mechanism M_1 depends on the magnetic and gravitational forces, it is clear that when M_1 increases, either magnetic force increases or gravitational force decreases. Thus, the destabilizing impact of magnetic mechanism M_1 is quite significant. Magnetic mechanism playing a significant role in amplifying the onset of viscoelastic ferroconvection by providing a destabilizing force that makes the system more unstable in the interval of the frequency $10 < \omega \leq 1000$. On the other hand, stabilization occur at very small range of frequency (i.e., $0 < \omega \leq 10$), when the values of M_1 are increased. This is observed because, as mentioned above that for very small value ω of modulation frequency the viscosity is high in ferrofluids due to the suspended microscopic magnetic particles in the liquid carrier wrapped with surfactant. Ultimately, for very small ω the suspended magnetic particles in the ferrofluid takes time to expand in the fluid and hence delays the onset of viscoelastic ferroconvection. Thus, system exhibits the stability in dual modes i.e. subcritical instability on increasing M_1 when ω is moderate and large and supercritical instability on increasing M_1 when ω is very small.

Fig. 4 shows the effect of Prandtl number with variations as $\mathrm{Pr} = 5, 10, 15$ and fixing the values of other parameters as $M_1 = 30$, $F_1 = 0.2$ and $F_2 = 0.5$. The parameter Pr is the ratio of the kinematic viscosity to that of thermal diffusivity. In general, the kinematic viscosity varies much more widely than that of thermal diffusivity, so high Prandtl number fluids are very viscus ones, for instance, ferromagnetic fluids are of this type. It should be observed that the expression for R_0 does not involve the Prandtl number Pr and the Prandtl number Pr affects only R_{2c}. It is evident from Fig. 4 that an increase in Prandtl number is stimulating the stabilization within the system at moderate and large values of frequencies (i.e., $150 < \omega \leq 1000$), because as the values of Pr are increased, the critical values R_{2c} of correction Rayleigh number are increased. This is observed because, in the case of viscoelastic ferromagnetic fluids due to the presence of long molecules, the viscosity of the fluid is much more than that of a Newtonian fluid. Therefore, Prandtl number Pr is acting to maximize the stabilizing effect of magnetic field modulation over a moderate and a large

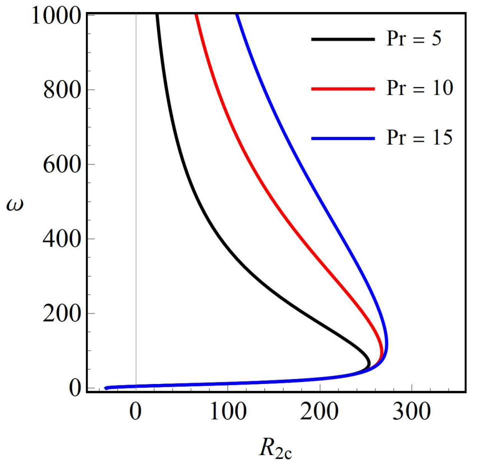

Figure 4. Variation in R_{2c} versus ω on increasing Pr

value of frequency, which leads to make the system more stable. On the other hand, the impact of magnetic field modulation disappears altogether for small range of values ω of frequency (i.e., $0 < \omega \leq 150$).

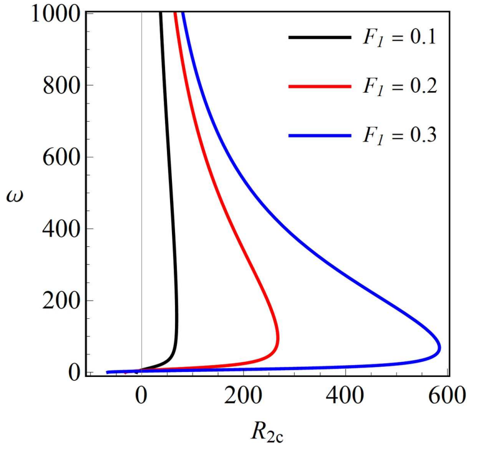

Figure 5. Variation in R_{2c} versus ω on increasing F_1

Fig. 5 corresponds to the change in values of critical correction Rayleigh number R_{2c} with the change in frequency ω for fixed values of parameters $M_1 = 30$, $\Pr = 10$, $F_2 = 0.5$ with the variation in stress relaxation parameter by taking its different values as $F_1 = 0.1, 0.2, 0.3$. The parameter F_1 stands for the non-dimensional relaxation time (elastic parameter), which specifies the time it takes for the viscoelastic ferromagnetic fluid to recover from its distorted condition to its initial state of equilibrium. It is found that the elastic parameter plays a vital role in the stability of the system when it is varied. The system is highly stabilizing for increasing values of elastic parameter over a wide range of values of frequency (i.e., $10 < \omega < 1000$). This is noticeable because, in the case of viscoelastic ferromagnetic fluids due to the presence of long molecules, the viscosity of the fluid is much more than that of a Newtonian fluid. As a result, viscoelastic ferromagnetic fluids take time to reach an equilibrium state and hence the elastic parameter imparting the significant impact on decelerating the onset of viscoelastic ferroconvective instability. On the other hand, the system is destabilizing for increasing the values of F_1 elastic parameter for a very small values of frequency (i.e., $0 < \omega \leq 10$).

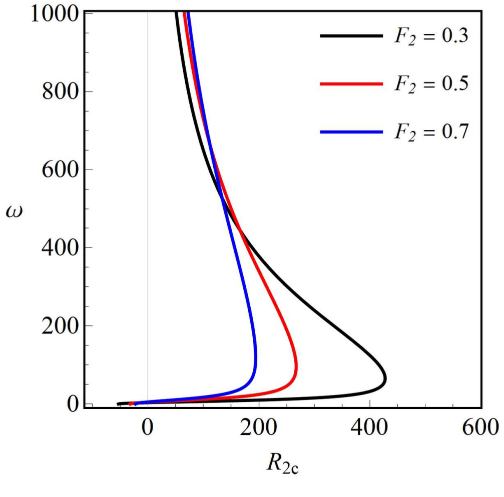

Figure 6. Variation in R_{2c} versus ω on increasing F_2

In the Fig. 6, the critical correction Rayleigh number R_{2c} is plotted with the modulation frequency ω when the value of strain retardation parameter is varied. The retardation parameter bears the value as $F_2 = 0.3$, 0.5, 0.7 and the other parameters are supposed to be constant as $M_1 = 30$, $\mathrm{Pr} = 10$ and $F_1 = 0.2$. The parameter F_2 indicates the non-dimensional retardation time, it refers to the delayed response to an applied force, often known as the delay of elasticity. On increasing F_2, Fig. 6 shows that the critical values R_{2c} of the correction Rayleigh number are decreasing for small and moderate frequency (i.e., $0 < \omega \leq 500$) and hence quickens the onset of viscoelastic ferromagnetic fluid convection. On the other hand, stabilization occur for higher values of frequency (i.e., $500 < \omega < 1000$), when F_2 is increased. Therefore, the strain retardation parameter exhibits a dual impact on the stability of the magnetic field modulated viscoelastic ferromagnetic fluid layer. In addition, the effect of strain retardation parameter F_2 is to augment the destabilizing nature of magnetic field modulation when ω is small and moderate and to amplify the stabilizing influence of magnetic field modulation when ω is large.

6. Conclusion

The Rayleigh-Benard problem of viscoelastic ferromagnetic fluid is investigated for the following cases: unmodulated and modulated magnetic fields. Small linear perturbations are introduced to the system, and the resultant equations are solved using the normal mode approach. Analytical solutions are obtained by means of regular perturbation method. The impacts of the buoyancy-magnetization parameter, Prandtl number, stress relaxation parameter, and strain retardation parameter are explained in graphs, and the following conclusions are drawn:

- i The buoyancy-magnetization parameter M_1 shows destabilizing impact on the system for both the modulated and unmodulated cases of magnetic field.
- ii The impact of Prandtl number Pr is to stabilize the system when the modulation frequency is moderate and large. However, for small values of frequency, the effect of Prandtl number Pr on the modulated magnetic field completely vanishes.
- iii The relaxation parameter F_1 plays a vital role in slowing down the process of viscoelastic ferroconvective instability.
- iv The strain retardation parameter F_2 tends to destabilize the system when the modulation frequency is small and moderate and to stabilize the system with large values of frequency. Thus, it is inferred that the strain retardation parameter exhibits a dual impact on the stability of the magnetic field modulated viscoelastic ferromagnetic fluid layer.

In a nutshell, the impact of a time-varying magnetic field can accelerate or decelerate the thresholds for the onset of viscoelastic magnetic fluid convective instability, depending on the frequency of the modulation. The effect of a time-varying magnetic field could be used to control the convection in a viscoelastic magnetic fluid. Time-varying magnetic field plays a crucial role in creating circulation in small passages where the effect of gravity is absent or ineffective. One can easily regulate convective motion in the system using time-varying (modulating) magnetic field. The results obtained here can be useful for magnetic field detection devices, charged particles in electrode materials, modulators, magnetic resonators, optical devices, dynamic loudspeakers, polymeric solutions, paints, DNA replication, diagnostic systems, in-line polarized fibre modulators, treatment of tumour cells, and zero-gravity situations involving a viscoelastic magnetic fluid as the working medium. On the other hand, the floquet theory could be used to attack the current problem in an alternative way. The Landau-Ginzburg and Lorenz techniques are some of the useful nonlinear approaches to the solution of the problem, which offer reliable information about the stability boundaries.

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