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Formulas and Properties for Families of Theories of Abelian Groups *

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Abstract. First-order formulas reflect an information for semantic and syntactic properties. Links between formulas and properties define their existential and universal interrelations which produce both structural and topological possibilities for characteristics classifying families of semantic and syntactic objects. We adapt general approaches describing links between formulas and properties for families of Abelian groups and their theories defining possibilities for characteristics of formulas and properties including rank values. This adaptation is based on formulas reducing each formula to an appropriate Boolean combination of given ones defining Szmielew invariants for theories of Abelian groups. Using this basedness we describe a trichotomy of possibilities for the rank values of sentences defining neighbourhoods for the set of theories of Abelian groups: the rank can be equal -1, 0, or ∞. Thus the neighbourhoods are either finite or contain continuum many theories. Using the trichotomy we show that each sentence defining a neighbourhood either belongs to finitely many theories or it is generic. We introduce the notion of rich property and generalize the main results for these properties.

Keywords: formula, property, elementary theory, abelian group, rank.

1. Introduction

The class of Abelian groups has very good and productive elementary classification by Szmielew invariants [1; 2; 16] reducing Abelian groups to standard ones which are represented by direct sums of a given collection. These invariants lead to efficient closure control for families of theories of Abelian groups [7], for counting ranks for these families [8], for characterizing approximability over given families [8; 9].

In this paper we apply a general approach connecting formulas and properties, their characteristics [11] for natural families of theories of Abelian groups and describe some forms of these connections.

The paper is organized as follows. In Section 2 we consider general notions for families of elementary theories including definable subfamilies with respect to formulas and sets of formulas, ranks, degrees and possibilities of their values, characterizations for totally transcendental families, spectra of values for subfamilies of given families, characterizations for disjoint E-closures and for existence of least generating sets. In Section 3 notions and characteristics for families of Abelian groups including Szmielew invariants and RS-ranks are given. Possibilities for ranks of sentences for properties in the set of theories of Abelian groups are obtained in Section 4, including generic sentences which produce maximal values of RS-ranks as well as sentences defining totally transcendental properties with given rank and degree values. In Section 5, we introduce the notion of rich property, characterize the richness, show that there are many rich properties, and describe rank characteristics of these properties.

2. Properties for theories and their ranks

Let Σ be a language. If Σ is relational we denote by $\mathcal{T}_\Sigma$ the family of all complete first-order theories of the language Σ. If Σ contains functional symbols f then $\mathcal{T}_\Sigma$ is the family of all theories of the language Σ', which is obtained by replacements of all n-ary symbols f with $(n+1)$-ary predicate symbols R_f interpreted by $R_f = \{(\bar{a}, b) \mid f(\bar{a}) = b\}$.

Following [12] we define the *rank* $\mathrm{RS}(\cdot)$ for properties $P \subseteq \mathcal{T}_\Sigma$, similar to Morley rank for a fixed theory, and a hierarchy with respect to these ranks in the following way.

By $F(\Sigma)$ we denote the set of all formulas in the language Σ and by $\mathrm{Sent}(\Sigma)$ the set of all sentences in $F(\Sigma)$.

For a sentence $\varphi \in \mathrm{Sent}(\Sigma)$ we denote by P_φ the set of all theories $T \in P$ with $\varphi \in T$.

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Any set P_φ is called the *φ-neighbourhood*, or simply a *neighbourhood*, for P, or the (φ-)*definable* subset of P. The set P_φ is also called (*formula-* or *sentence-*)*definable* (by the sentence φ) with respect to P, or (*sentence-*)*P-definable*, or simply *s-definable*.

Definition 1. [12]. For the empty property P we put the rank $\mathrm{RS}(P) = -1$, for finite nonempty properties P we put $\mathrm{RS}(P) = 0$, and for infinite properties P — $\mathrm{RS}(P) \geq 1$.

For a property P and an ordinal $\alpha = \beta+1$ we put $\mathrm{RS}(P) \geq \alpha$ if there are pairwise inconsistent $\Sigma(P)$-sentences φ_n, $n \in \omega$, such that $\mathrm{RS}(P_{\varphi_n}) \geq \beta$, $n \in \omega$.

If α is a limit ordinal then $\mathrm{RS}(P) \geq \alpha$ if $\mathrm{RS}(P) \geq \beta$ for any $\beta < \alpha$.

We set $\mathrm{RS}(P) = \alpha$ if $\mathrm{RS}(P) \geq \alpha$ and $\mathrm{RS}(P) \not\geq \alpha + 1$.

If $\mathrm{RS}(\mathcal{T}) \geq \alpha$ for any α, we put $\mathrm{RS}(\mathcal{T}) = \infty$.

A property P is called *e-totally transcendental*, or *totally transcendental*, if $\mathrm{RS}(P)$ is an ordinal.

If P is e-totally transcendental, with $\mathrm{RS}(P) = \alpha \geq 0$, we define the *degree* $\mathrm{ds}(P)$ of P as the maximal number of pairwise inconsistent sentences φ_i such that $\mathrm{RS}(P_{\varphi_i}) = \alpha$.

Definition 2. [13]. An infinite property P is called *e-minimal* if for any sentence $\varphi \in \Sigma(P)$, P_φ is finite or $P_{\neg\varphi}$ is finite.

By the definition a property P is e-minimal iff $\mathrm{RS}(P) = 1$ and $\mathrm{ds}(P) = 1$ [12], and iff P has a unique accumulation point [13].

In the paper [14] the notion of E-closure was introduced and characterized as follows:

Proposition 1. *If $P \subseteq \mathcal{T}_\Sigma$ is an infinite property and $T \in \mathcal{T}_\Sigma \setminus P$ then $T \in \mathrm{Cl}_E(P)$ (i.e., T is an accumulation point for P with respect to E-closure Cl_E) if and only if for any sentence $\varphi \in T$ the set P_φ is infinite.*

The following theorem characterizes the property of e-total transcendency for countable languages.

Theorem 1. [12] *For any property $P \subseteq \mathcal{T}_\Sigma$ with $|\Sigma(\mathcal{T})| \leq \omega$ the following conditions are equivalent:*

(1) $|\mathrm{Cl}_E(P)| = 2^\omega$;

(2) e-$\mathrm{Sp}(P) = 2^\omega$;

(3) $\mathrm{RS}(P) = \infty$.

Definition 3. cf. [6]. For a property $P \subseteq \mathcal{T}_\Sigma$ a 2-*tree* $\mathrm{Tree}(P)$ is a family $\{P_{\varphi_\delta} \mid \delta \in {}^{<\omega}2\}$ of nonempty sets P_{φ_δ}, for $\varphi_\delta \in \mathrm{Sent}(\Sigma)$, satisfying the following conditions:

a) $\varphi_{\delta\frown i} \vdash \varphi_\delta$ for any $\delta \in {}^{<\omega}2$, $i \in \{0, 1\}$;

b) $\varphi_{\delta\frown 0}$ and $\varphi_{\delta\frown 1}$ are $\mathcal{T}$-inconsistent for any $\delta \in {}^{<\omega}2$.

The set $\{\varphi_\delta \mid \delta \in {}^{<\omega}2\}$ is also called the 2-*tree* for the property P.

The following theorem extends criteria above for $\mathrm{RS}(P)=\infty$.

Theorem 2. *For any property P with $|\Sigma(P)|\le\omega$ the following conditions are equivalent:*
(i) $\mathrm{RS}(P)=\infty$;
(ii) *there exists a 2-tree of sentences φ for s-definable properties P_φ.*

Proof. (i) $\Rightarrow$ (ii). Let $\mathrm{RS}(P)=\infty$. It suffices to show that there is a sentence $\varphi\in\mathrm{Sent}(\Sigma)$ with $\mathrm{RS}(P_\varphi)=\infty$ and $\mathrm{RS}(P_{\neg\varphi})=\infty$. Assuming on contrary that the required sentence φ does not exist we enumerate $\mathrm{Sent}(\Sigma)=\{\varphi_i\mid i\in\omega\}$ and step-by-step form a unique ultrafilter T consisting of sentences φ_i with $\mathrm{RS}(P_{\varphi_i})=\infty$. It means that each $\mathrm{RS}(P_{\neg\varphi_i})$ is a countable ordinal such that $P_{\neg\varphi_i}$ consists of at most countably many theories. These theories have the unique accumulation point T outside the union $\bigcup\limits_i \mathrm{Cl}_E(P_{\neg\varphi_i})$ contradicting $|\mathrm{Cl}_E(P)|=2^\omega$ that is asserted in Theorem 1.

(ii) $\Rightarrow$ (i). If P has a 2-tree $\{\varphi_\delta\mid\delta\in{}^{<\omega}2\}$ each $\Delta\in 2^\omega$ produces an ultrafilter T_Δ for the set of φ_δ, where δ is in initial segment of Δ, such that T_Δ is an accumulation point for P. By the construction of 2-tree if $\Delta'\in 2^\omega$ with $\Delta'\ne\Delta$ then $T_{\Delta'}\ne T_\Delta$. Thus $|\mathrm{Cl}_E(P)|=2^\omega$ producing $\mathrm{RS}(P)=\infty$ by Theorem 1. □

Theorem 3. [3] *For any language Σ either $\mathrm{RS}(\mathcal{T}_\Sigma)$ is finite, if Σ consists of finitely many 0-ary and unary predicates, and finitely many constant symbols, or $\mathrm{RS}(\mathcal{T}_\Sigma)=\infty$, otherwise.*

For a language Σ we denote by $\mathcal{T}_{\Sigma,n}$ the family of all theories in $\mathcal{T}_\Sigma$ having n-element models, $n\in\omega$, as well as by $\mathcal{T}_{\Sigma,\infty}$ the family of all theories in $\mathcal{T}_\Sigma$ having infinite models.

Theorem 4. [3] *For any language Σ either $\mathrm{RS}(\mathcal{T}_{\Sigma,n})=0$, if Σ is finite or $n=1$ and Σ has finitely many predicate symbols, or $\mathrm{RS}(\mathcal{T}_{\Sigma,n})=\infty$, otherwise.*

Theorem 5. [3] *For any language Σ either $\mathrm{RS}(\mathcal{T}_{\Sigma,\infty})$ is finite, if Σ is finite and without predicate symbols of arities $m\ge 2$ as well as without functional symbols of arities $n\ge 1$, or $\mathrm{RS}(\mathcal{T}_{\Sigma,\infty})=\infty$, otherwise.*

By the definition the families $\mathcal{T}_\Sigma$, $\mathcal{T}_{\Sigma,n}$, $\mathcal{T}_{\Sigma,\infty}$ are E-closed. Thus, combining Theorem 1 with Theorems 3–5 we obtain the following possibilities of cardinalities for the families $\mathcal{T}_\Sigma$, $\mathcal{T}_{\Sigma,n}$, $\mathcal{T}_{\Sigma,\infty}$ depending on Σ and $n\in\omega$:

Proposition 2. *For any language Σ either $\mathcal{T}_\Sigma$ is countable, if Σ consists of finitely many 0-ary and unary predicates, and finitely many constant symbols, or $|\mathcal{T}_\Sigma|\ge 2^\omega$, otherwise.*

Proposition 3. *For any language Σ either $\mathcal{T}_{\Sigma,n}$ is finite, if Σ is finite or $n=1$ and Σ has finitely many predicate symbols, or $|\mathcal{T}_{\Sigma,n}|\ge 2^\omega$, otherwise.*

Proposition 4. *For any language Σ either $\mathcal{T}_{\Sigma,\infty}$ is at most countable, if Σ is finite and without predicate symbols of arities $m \geq 2$ as well as without functional symbols of arities $n \geq 1$, or $|\mathcal{T}_{\Sigma,\infty}| \geq 2^\omega$, otherwise.*

Definition 4. [4] If P is a property for theories and Φ is a set of sentences, then we put $P_\Phi = \bigcap\limits_{\varphi\in\Phi} P_\varphi$ and the set P_Φ is called (*type-*) or (*diagram-*)*definable* (by the set Φ) with respect to P, or (*diagram-*)P*-definable*, or simply d*-definable*.

Clearly, finite unions of d-definable sets are again d-definable. Considering infinite unions P' of d-definable sets P_{Φ_i}, $i \in I$, one can represent them by sets of sentences with infinite disjunctions $\bigvee\limits_{i\in I} \varphi_i$, $\varphi_i \in \Phi_i$. These unions P' are called d_∞*-definable* properties.

Definition 5. [4] Let P be a property for theories, Φ be a set of sentences, α be an ordinal $\leq \mathrm{RS}(P)$ or -1. The set Φ is called α*-ranking* for P if $\mathrm{RS}(P_\Phi) = \alpha$. A sentence φ is called α*-ranking* for P if $\{\varphi\}$ is α*-ranking* for P.

The set Φ (the sentence φ) is called *ranking* for P if it is α-ranking for P with some α.

Theorem 6. [4] *For any ordinals $\alpha \leq \beta$, if $\mathrm{RS}(P) = \beta$ then $\mathrm{RS}(P_\varphi) = \alpha$ for some (α-ranking) sentence φ. Moreover, there are $\mathrm{ds}(P)$ pairwise P-inconsistent β-ranking sentences for P, and if $\alpha < \beta$ then there are infinitely many pairwise P-inconsistent α-ranking sentences for P.*

Theorem 7. [4] *Let P be a property for a countable language Σ and with $\mathrm{RS}(P) = \infty$, α be a countable ordinal, $n \in \omega \setminus \{0\}$. Then there is a d_∞-definable subproperty $P^* \subset \mathcal{T}$ such that $\mathrm{RS}(P^*) = \alpha$ and $\mathrm{ds}(P^*) = n$.*

Now using Theorems 1, 2, 6, 7 we consider a general assertion for the family $\mathcal{T}_\Sigma$ of all theories in a countable language Σ, an arbitrary property $P \subseteq \mathcal{T}_\Sigma$ with $\mathrm{RS}(P) = \infty$, and a set $\Phi \subseteq \mathrm{Sent}(\Sigma)$ of sentences in the language Σ.

Proposition 5. *For any at most countable set X of finite or countable ordinals, a set $Y = \{(\alpha, n_\alpha) \mid \alpha \in X, n_\alpha \in \omega \setminus \{0\}\}$, and a property $P \subseteq \mathcal{T}_\Sigma$ with $\mathrm{RS}(P) = \infty$ there exist $P' \subseteq P$ and a set $\Phi \subseteq \mathrm{Sent}(\Sigma)$ such that P'_φ are pairwise disjoint for $\varphi \in \Phi$ and $\{(\mathrm{RS}(P'_\varphi), \mathrm{ds}(P'_\varphi)) \mid \varphi \in \Phi\} = Y$ (respectively, the set of pairs $(\mathrm{RS}(P'_\varphi), \mathrm{ds}(P'_\varphi))$ with totally transcendental P'_φ, and values $\mathrm{RS}(P'_\varphi) = \infty$ with non-totally transcendental P'_φ, for $\varphi \in \Phi$, equals $Y \cup \{\infty\}$).*

Proof. Since $\mathrm{RS}(P) = \infty$ we have, in view of Theorems 1 and 2, that there exists a 2-tree of sentences φ for s-definable families P_φ.

Now using the 2-tree we can find countably many disjoint s-definable families P_{φ_k} with $\mathrm{RS}(P_{\varphi_k})=\infty$, $k\in\omega$. By Theorems 6, 7 the families P_{φ_k} contain subfamilies P'_k with given values $(\mathrm{RS}(P'_k),\mathrm{ds}(P'_k))\in Y$. We denote by P' the union $\bigcup\limits_k P'_k$. This P' is required for Y since $P'_{\varphi_k}=P'_k$, $k\in\omega$.

Considering characteristics in $Y\cup\{\infty\}$, we put some $P'_k=P_{\varphi_k}$ witnessing $\mathrm{RS}(P'_{\varphi_k})=\infty$. $\Box$

Theorem 8. [15] *For any two disjoint subproperties P_1 and P_2 of an E-closed property P the following conditions are equivalent:*

(1) *P_1 and P_2 are separated by some sentence φ: $P_1\subseteq P_\varphi$ and $P_2\subseteq P_{\neg\varphi}$;*
(2) *E-closures of P_1 and P_2 are disjoint in P: $\mathrm{Cl}_E(P_1)\cap\mathrm{Cl}_E(P_2)\cap P=\emptyset$;*
(3) *E-closures of P_1 and P_2 are disjoint: $\mathrm{Cl}_E(P_1)\cap\mathrm{Cl}_E(P_2)=\emptyset$.*

Definition 6. [14] Let P_0 be a property for theories. A subset $P'_0\subseteq P_0$ is said to be *generating* if $P_0=\mathrm{Cl}_E(P'_0)$. The generating property P'_0 (for P_0) is *minimal* if P'_0 does not contain proper generating subsets. A minimal generating property P'_0 is *least* if P'_0 is contained in each generating property for P_0.

Theorem 9. [14] *If P'_0 is a generating property for a E-closed set P_0 then the following conditions are equivalent:*

(1) *P'_0 is the least generating property for P_0;*
(2) *P'_0 is a minimal generating property for P_0;*
(3) *any theory in P'_0 is isolated by some set $(P'_0)_\varphi$, i.e., for any $T\in P'_0$ there is $\varphi\in T$ such that $(P'_0)_\varphi=\{T\}$;*
(4) *any theory in P'_0 is isolated by some set $(P_0)_\varphi$, i.e., for any $T\in P'_0$ there is $\varphi\in T$ such that $(P_0)_\varphi=\{T\}$.*

3. Families of theories of Abelian groups and their characteristics

Following [7; 9] we denote by $\overline{\mathcal{TA}}$ the family of all theories of Abelian groups in a relativized group language $\Sigma_0=\{+,-,0\}$.

Let $\mathcal{A}$ be an Abelian group in the language Σ_0. Then $k\mathcal{A}$ denotes its subgroup $\{ka\mid a\in A\}$ and $\mathcal{A}[k]$ denotes the subgroup $\{a\in A\mid ka=0\}$. Let P^* be the set of all prime numbers. If $p\in P^*$ and $pA=\{0\}$ then $\dim\mathcal{A}$ denotes the dimension of the group $\mathcal{A}$, considered as a vector space over a field with p elements. The following numbers, for arbitrary $p\in P^*$ and $n\in\omega\setminus\{0\}$ are called the *Szmielew invariants* for the group A [2; 16]:

$$\alpha_{p,n}(\mathcal{A})=\min\{\dim((p^n\mathcal{A})[p]/(p^{n+1}\mathcal{A})[p]),\omega\},$$

$$\beta_p(\mathcal{A})=\min\{\inf\{\dim((p^n\mathcal{A})[p]\mid n\in\omega\},\omega\},$$

$$\gamma_p(\mathcal{A}) = \min\{\inf\{\dim((\mathcal{A}/\mathcal{A}[p^n])/p(\mathcal{A}/\mathcal{A}[p^n])) \mid n \in \omega\}, \omega\},$$

$$\varepsilon(\mathcal{A}) \in \{0, 1\},$$

$$\text{and } \varepsilon(\mathcal{A}) = 0 \Leftrightarrow (n\mathcal{A} = \{0\} \text{ for some } n \in \omega, n \neq 0).$$

It is known [2, Theorem 8.4.10] that two Abelian groups are elementarily equivalent if and only if they have the same Szmielew invariants. In addition, the following proposition holds.

Proposition 6. [2, Proposition 8.4.12] *Let for any p and n the cardinals $\alpha_{p,n}$, β_p, $\gamma_p \leq \omega$, and $\varepsilon \in \{0, 1\}$ be given. Then there is an Abelian group $\mathcal{A}$ such that the Szmielew invariants $\alpha_{p,n}(\mathcal{A})$, $\beta_p(\mathcal{A})$, $\gamma_p(\mathcal{A})$, and $\varepsilon(\mathcal{A})$ are equal to $\alpha_{p,n}$, β_p, γ_p, and ε, respectively, if and only if the following conditions hold:*

(1) *if for prime p the set $\{n \mid \alpha_{p,n} \neq 0\}$ is infinite then $\beta_p = \gamma_p = \omega$;*

(2) *if $\varepsilon = 0$ then for any prime p, $\beta_p = \gamma_p = 0$ and the set $\{\langle p, n\rangle \mid \alpha_{p,n} \neq 0\}$ is finite.*

We denote by $\mathbf{Q}$ the additive group of rational numbers, $\mathbf{Z}_{p^n}$ — the cyclic group of the order p^n, $\mathbf{Z}_{p^\infty}$ — the quasi-cyclic group of all complex roots of 1 of degrees p^n for all $n \geq 1$, R_p — the group of irreducible fractions with denominators which are mutually prime with p. The groups $\mathbf{Q}$, $\mathbf{Z}_{p^n}$, R_p, $\mathbf{Z}_{p^\infty}$ are called *basic*. Below the notations of these groups will be identified with their universes.

Since Abelian groups with the same Szmielew invariants have the same theories, any Abelian group $\mathcal{A}$ is elementarily equivalent to a group

$$\oplus_{p,n}\mathbf{Z}_{p^n}^{(\alpha_{p,n})} \oplus \oplus_p \mathbf{Z}_{p^\infty}^{(\beta_p)} \oplus \oplus_p R_p^{(\gamma_p)} \oplus \mathbf{Q}^{(\varepsilon)}, \tag{3.1}$$

where $\mathcal{B}^{(k)}$ denotes the direct sum of k subgroups isomorphic to a group $\mathcal{B}$. Thus, any theory of an Abelian group has a model represented by a direct sum of based groups. The groups of form (3.1) are called *standard*.

Recall that any complete theory of an Abelian group is based by the set of positive primitive formulas [2, Lemma 8.4.5], reduced to the set of the following formulas:

$$\exists y(m_1x_1 + \ldots + m_nx_n \approx p^k y), \tag{3.2}$$

$$m_1x_1 + \ldots + m_nx_n \approx 0, \tag{3.3}$$

where $m_i \in \mathbf{Z}$, $k \in \omega$, p is a prime number [1], [2, Lemma 8.4.7]. Formulas (3.2) and (3.3) witness that Szmielew invariants define theories of Abelian groups modulo Proposition 6.

In view of Proposition 6 and equations (3.2) and (3.3) we have the following:

Remark 1. [9] Theories of Abelian groups are forced by sentences implied by formulas of form (3.2) and (3.3) and describing dimensions with respect to $\alpha_{p,n}$, β_p, γ_p, ε as well as bounds for orders p^k of elements and possibilities for divisions of elements by p^k. Moreover, various values of Szmielew invariants are separated by some sentences modulo Proposition 6.

Thus, definable properties for families of theories of Abelian groups are described by dimensions with respect to Szmielew invariants, possibilities for orders and divisions by prime numbers.

In view of Proposition 6 following [8] we observe that all dependencies between values of Szmielew invariants in a given theory of an Abelian group are exhausted by ones given by infinite $\{n \mid \alpha_{p,n} \neq 0\}$ implying $\beta_p = \gamma_p = \omega$ as well as by infinite $\{\langle p, n\rangle \mid \alpha_{p,n} \neq 0\}$ implying $\varepsilon = 1$. It means that Szmielew invariants, for a fixed theory and for a property, can not force positive values $\alpha_{p,n}$, β_p, γ_p using positive values $\alpha_{p',n}$, $\beta_{p'}$, $\gamma_{p'}$ for different prime p' and/or ε. Besides, all values $\alpha_{p,n}$ and natural values β_p, γ_p do not forced by other Szmielew invariants. Moreover, finite values $\alpha_{p,n}$, β_p, γ_p, for theories in $\mathrm{Cl}_E(P)$, can not be forced by other finite or infinite values of these invariants. Thus, as noticed in [8], all dependencies between distinct Szmielew invariants $\alpha^T_{p,n}$, β^T_p, γ^T_p, ε^T, for theories $T \in \mathrm{Cl}_E(P) \setminus P$, are exhausted by the following ones for sequences $(T_k)_{k\in\omega}$ of theories in P:

1) $\alpha^T_{p,n} = \lim\limits_{k\to\infty} \alpha^{T_k}_{p,n}$,
2) $\beta^T_p = \lim\limits_{k\to\infty} \beta^{T_k}_p$,
3) $\gamma^T_p = \lim\limits_{k\to\infty} \gamma^{T_k}_p$,
3) $\varepsilon^T = \lim\limits_{k\to\infty} \varepsilon^{T_k}$,
4) $\beta^T_p = \gamma^T_p = \omega = \lim\limits_{n} \alpha^{T_k}_{p,n}$,
5) $\varepsilon^T = 1 = \lim\limits_{p,n} \alpha^{T_k}_{p,n}$.

The items 1)–5) show that limit values for Szmielew invariants are independent modulo $\alpha^{T_k}_{p,n}$, i.e., the limits of $\beta^{T_k}_p$, $\gamma^{T_k}_p$, ε^{T_k} can produce only β^T_p, γ^T_p, ε^T, respectively, whereas $\alpha^{T_k}_{p,n}$ can generate both $\alpha^T_{p,n}$, $\beta^T_p = \gamma^T_p = \omega$ and $\varepsilon^T = 1$.

Using the limit values above we can control both Szmielew invariants for the E-closure of a given property $P \subseteq \overline{\mathcal{TA}}$ and sentences forming theories in $\mathrm{Cl}_E(P)$.

In particular, the limit values allow to strengthen Theorems 6 and 7 attracting various families of Szmielew invariants.

Theorem 10. [8] *Let α be at most countable ordinal, $n \in \omega \setminus \{0\}$. Then there is a d-definable property $P = \left(\overline{\mathcal{TA}}\right)_\Phi$ such that $\mathrm{RS}(P) = \alpha$ and $\mathrm{ds}(P) = n$.*

4. Ranks of sentences and generic sentences

Definition 7. [11] For a sentence $\varphi \in \mathrm{Sent}(\Sigma)$ and a property $P = P_t \subseteq \mathcal{T}_\Sigma$ we put $\mathrm{RS}_P(\varphi) = \mathrm{RS}(P_\varphi)$, and $\mathrm{ds}_P(\varphi) = \mathrm{ds}(P_\varphi)$ if $\mathrm{RS}(P_\varphi)$ is defined.

If $P = \mathcal{T}_\Sigma$ then we omit P and write $\mathrm{RS}(\varphi)$, $\mathrm{ds}(\varphi)$ instead of $\mathrm{RS}_P(\varphi)$ and $\mathrm{ds}_P(\varphi)$, respectively.

Definition 8. [11] (cf. [10; 17; 18]) For a property $P \subseteq \mathcal{T}_\Sigma$, a sentence $\varphi \in \mathrm{Sent}(\Sigma)$ is called *P-generic* if $\mathrm{RS}_P(\varphi) = \mathrm{RS}(P)$, and $\mathrm{ds}_P(\varphi) = \mathrm{ds}(P)$ if $\mathrm{ds}(P)$ is defined.

If $P = \mathcal{T}_\Sigma$ then we omit P and a P-generic sentence is called *generic*.

Remark 2. As shown in [8], $\mathrm{RS}(\overline{\mathcal{TA}}) = \infty$, with $|\overline{\mathcal{TA}}| = 2^\omega$ in view of Theorem 1. Since the language Σ_0 is finite and the representation (3.1) for a theory $T\overline{\mathcal{TA}}$ with a finite model is unique, assertions 3–4 have the following modification for the family $\overline{\mathcal{TA}}$: any restriction $\overline{\mathcal{TA}} \cap \mathcal{T}_{\Sigma_0,n}$, for $n \in \omega \setminus \{0\}$, is finite and the cardinality of this restriction is defined by the possibilities of representations of n by multiplications of p^m for prime p and $m \in \omega \setminus \{0\}$.

For instance, if $n = p^2 q$, for prime p and q, there are two possibilities for $\overline{\mathcal{TA}} \cap \mathcal{T}_{\Sigma_0,n}$: $\mathrm{Th}(\mathbf{Z}_{p^2} \times \mathbf{Z}_q)$ and $\mathrm{Th}(\mathbf{Z}_p \times \mathbf{Z}_p \times \mathbf{Z}_q)$.

Lemma 1. *If a $\overline{\mathcal{TA}}$-consistent sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ does not belong to theories in $P = \overline{\mathcal{TA}}$ with infinite models then $\mathrm{RS}_P(\varphi) = 0$.*

Proof. If $\varphi \in \mathrm{Sent}(\Sigma_0)$ does not belong to theories in P then by compactness and Remark 2, φ belongs to finitely many theories $T_1, \ldots, T_n$ in $\overline{\mathcal{TA}}$, and all these theories have finite models only. It means that φ is represented as a disjunction of complete sentences for the theories $T_1, \ldots, T_n$. Thus, P_φ is finite and $\mathrm{RS}_P(\varphi) = 0$, with $\mathrm{ds}_P(\varphi) = n$. □

Lemma 2. *If a sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ belongs to a theory in $P = \overline{\mathcal{TA}}$ with infinite models then $\mathrm{RS}_P(\varphi) = \infty$.*

Proof. If φ belongs to a theory $T \in \overline{\mathcal{TA}}$ with an infinite model $\mathcal{M}$ then by Remark 1, φ describes possibilities on divisibility for finitely many p^n with finitely many prime numbers p, possibilities for elements of finitely many orders p^n, as well as finitely many possibilities of linear (in)dependence. Since $\mathcal{M}$ is infinite the information above is consistent with a similar information for other prime numbers q. Varying this information for distinct q we obtain a 2-tree refining φ and witnessing $\mathrm{RS}_P(\varphi) = \infty$. □

By Lemmas 1 and 2 we have the dichotomy for $\overline{\mathcal{TA}}$-consistent sentences φ: either $\mathrm{RS}_{\overline{\mathcal{TA}}}(\varphi) = 0$ or $\mathrm{RS}_{\overline{\mathcal{TA}}}(\varphi) = \infty$. Moreover, we obtain the following:

Theorem 11. *For any sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ and $P = \overline{\mathcal{TA}}$ the following possibilities hold:*

(1) $\mathrm{RS}_P(\varphi) = -1$, *if φ is $\overline{\mathcal{TA}}$-inconsistent;*

(2) $\mathrm{RS}_P(\varphi) = 0$, *if φ is $\overline{\mathcal{TA}}$-consistent and belongs to (finitely many) theories in $\overline{\mathcal{TA}}$ with finite models only;*

(3) $\mathrm{RS}_P(\varphi) = \infty$, *if φ belongs to a theory $T \in \overline{\mathcal{TA}}$ with an infinite model.*

By the definition each neighbourhood $\overline{\mathcal{TA}}_\varphi$ is E-closed. Applying Theorems 1 and 11, and similar to Theorem 3, we have:

Theorem 12. *For any sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ and $P = \overline{\mathcal{TA}}$ either P_φ is E-closed and finite or P_φ is E-closed with $|P_\varphi| = 2^\omega$.*

Theorem 12 immediately implies:

Corollary 1. *For any sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ and $P = \overline{\mathcal{TA}}$ either φ is represented by a disjunction of finitely many sentences φ_i isolating theories $T_i \in \overline{\mathcal{TA}}$ with finite models, or φ is P-generic.*

Corollary 2. *For any sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ and $P = \overline{\mathcal{TA}}$ either φ is P-generic or $\neg\varphi$ is P-generic.*

Now applying assertions above we consider RS-spectra for subfamilies of $\overline{\mathcal{TA}}$ similar to spectra [11] for families of theories in a general case.

Taking a generic sentence $\varphi_0 \in \mathrm{Sent}(\Sigma_0)$ and its family $(\overline{\mathcal{TA}})_{\varphi_0}$ we have $\mathrm{RS}\left((\overline{\mathcal{TA}})_{\varphi_0}\right) = \infty$ by Theorem 11. Thus by Proposition 5 we have the following modification of Theorem 10:

Theorem 13. *For any at most countable set X of finite or countable ordinals, a set $Y = \{(\alpha, n_\alpha) \mid \alpha \in X, n_\alpha \in \omega \setminus \{0\}\}$, and a property $P = (\overline{\mathcal{TA}})_{\varphi_0}$, with a generic sentence $\varphi_0 \in \mathrm{Sent}(\Sigma_0)$, there exist $P' \subseteq P$ and a set $\Phi \subseteq \mathrm{Sent}(\Sigma_0)$ such that P'_φ are pairwise disjoint for $\varphi \in \Phi$ and $\{(\mathrm{RS}(P'_\varphi), \mathrm{ds}(P'_\varphi)) \mid \varphi \in \Phi\} = Y$ (respectively, the set of pairs $(\mathrm{RS}(P'_\varphi), \mathrm{ds}(P'_\varphi))$ with totally transcendental P'_φ, and values $\mathrm{RS}(P'_\varphi) = \infty$ with non-totally transcendental P'_φ, for $\varphi \in \Phi$, equals $Y \cup \{\infty\}$).*

Remark 3. The set $\overline{\mathcal{TA}}$ admits special series of properties satisfying Theorem 13. Taking, for instance, an arbitrary infinite set Z of prime numbers p and the subfamily $\overline{\mathcal{TA}}(Z)$ of $\overline{\mathcal{TA}}$ consisting of all theories with $\alpha_{p,n} = \beta_p = \gamma_p = 0$ for prime $p \notin Z$ we have $\mathrm{RS}(\overline{\mathcal{TA}}(Z)) = \infty$. Now we can apply Proposition 5 obtaining the assertion of Theorem 13 both using a general approach and dividing Z into countably many disjoint infinite parts Z_i and reducing $\overline{\mathcal{TA}}(Z_i)$ till totally transcendental subfamilies with given RS-ranks and ds-degrees.

Remark 4. Applying Theorem 8, s-definable subsets P'_φ in Theorem 13 have disjoint E-closures such that if P'_φ are totally transcendental then they have least generating sets in view of Theorem 9, and if P'_φ are not totally transcendental then they can be constructed both with and without least generating sets, which are controlled by limit values of Szmielew invariants following links described in Section 3. In the latter case, with $\mathrm{RS}(P'_\varphi) = \infty$ one can form $\mathrm{RS}(P'_\varphi)$ approximating theories in P'_φ by theories of finite Abelian groups and obtaining either $\alpha_{p,n} = \omega$ or $\beta_p = \gamma_p = \omega$.

5. Rich properties and their characteristics

Definition 9. A property $P \subseteq \overline{\mathcal{TA}}$ is called *rich* if $P \cap P' \neq \emptyset$ for each nonempty property $P' = \left(\overline{\mathcal{TA}}\right)_\varphi$ defined by a sentence φ locally describing linear (in)dependence, (in)divisibilities and orders of elements.

Proposition 7. *A property $P \subseteq \overline{\mathcal{TA}}$ is rich if and only if* $\mathrm{Cl}_E(P) = \overline{\mathcal{TA}}$.

Proof. Let $P \subseteq \overline{\mathcal{TA}}$ is rich. Since each $\overline{\mathcal{TA}}$-consistent sentence φ is reduced, for theories of Abelian groups, to sentences ψ locally describing linear (in)dependence, (in)divisibilities and orders of elements, we have $P_\varphi \neq \emptyset$. Thus each theory $T \in \overline{\mathcal{TA}}$ either belongs to P or each $\chi \in T$ belongs to infinitely many theories in P. Applying Proposition 1 we obtain $\mathrm{Cl}_E(P) = \overline{\mathcal{TA}}$.

Conversely if $\mathrm{Cl}_E(P) = \overline{\mathcal{TA}}$, we again apply Proposition 1 obtaining $P_\psi \neq \emptyset$ for each sentence ψ locally describing linear (in)dependence, (in)divisibilities and orders of elements, i.e., P is rich. □

Proposition 8. $|\{P \subseteq \overline{\mathcal{TA}} \mid P$ *is rich*$\}| = 2^\omega$, *moreover,* $|\{P \subseteq \overline{\mathcal{TA}} \mid P$ *is rich and countable*$\}| = 2^\omega$.

Proof. Since $|\overline{\mathcal{TA}}| = 2^\omega$ with 2^ω nonisolated points T, each $\overline{\mathcal{TA}} \setminus \{T\}$ is rich by Proposition 7, producing $|\{P \subseteq \overline{\mathcal{TA}} \mid P \text{ is rich}\}| = 2^\omega$. Since each rich property is approximated by countable one, we obtain $|\{P \subseteq \overline{\mathcal{TA}} \mid P$ is rich and countable$\}| = 2^\omega$. □

Applying Proposition 7 we obtain the following generalization of Theorem 11:

Theorem 14. *For any sentence $\varphi \in \mathrm{Sent}(\Sigma_0)$ and a rich property $P \subseteq \overline{\mathcal{TA}}$ the following possibilities hold:*

(1) $\mathrm{RS}_P(\varphi) = -1$, *if φ is $\overline{\mathcal{TA}}$-inconsistent;*

(2) $\mathrm{RS}_P(\varphi) = 0$, *if φ is $\overline{\mathcal{TA}}$-consistent and belongs to* (*finitely many*) *theories in $\overline{\mathcal{TA}}$ with finite models only;*

(3) $\mathrm{RS}_P(\varphi) = \infty$, *if φ belongs to a theory $T \in \overline{\mathcal{TA}}$ with an infinite model.*

Corollary 3. *For any sentence* $\varphi \in \mathrm{Sent}(\Sigma_0)$ *and rich* $P \subseteq \overline{\mathcal{TA}}$ *either* φ *is represented by a disjunction of finitely many sentences* φ_i *isolating theories* $T_i \in \overline{\mathcal{TA}}$ *with finite models, or* φ *is* P*-generic.*

Corollary 4. *For any sentence* $\varphi \in \mathrm{Sent}(\Sigma_0)$ *and rich* $P \subseteq \overline{\mathcal{TA}}$ *either* φ *is* P*-generic or* $\neg\varphi$ *is* P*-generic.*

Remark 5. The assertions 14–4 can fail if $P \subseteq \overline{\mathcal{TA}}$ is not rich. Indeed, in view of Theorem 13 there are properties $P \subseteq \overline{\mathcal{TA}}$ whose neighbourhoods P_φ produce arbitrary ordinal ranks $\mathrm{RS}(P_\varphi)$. Thus, the values $\mathrm{RS}(P_\varphi)$ can have many possibilities, with non-P-generic sentences φ having $\mathrm{RS}(P_\varphi) \geq 1$.

6. Conclusion

We characterized and described possibilities for rank characteristics of formulas and properties of theories of Abelian groups, both for the family of all theories of Abelian groups and for its subfamilies including rich ones. The machinery of these characteristics in terms of Szmielew invariants as well as E-closures are clarified. Generations of properties by least generating sets including theories of finite Abelian groups are shown. Some illustrations for characteristics of properties for theories of Abelian groups are given. It would be interesting to describe the dynamics of rank characteristics for natural properties of (in)complete theories of Abelian groups using a general topological approach for families of theories [5].

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Формулы и свойства для семейств теорий абелевых групп

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Аннотация. Формулы первого порядка отражают информацию о семантических и синтаксических свойствах. Связи между формулами и свойствами определяют их экзистенциальные и универсальные взаимосвязи, которые создают как структурные, так и топологические возможности для характеристик, позволяющих классифицировать семейства семантических и синтаксических объектов. Адаптируются общие подходы, описывающие связи между формулами и свойствами для семейств абелевых групп и их теорий, определяя возможности характеристик формул и свойств, включая значения рангов. Эта адаптация основана на формулах, сводящих каждую формулу к подходящей булевой комбинации формул, определяющих шмелевские инварианты для теорий абелевых групп. Используя эту базируемость, описывается трихотомия возможностей значений ранга для предложений, определяющих окрестности для множества теорий абелевых групп: ранг может быть равен -1, 0 или ∞. Тем самым, окрестности либо конечны, либо содержат континуальное число теорий. Используя трихотомию, показывается, что каждое предложение, определяющее окрестность, либо принадлежит конечному множеству его теорий, либо является генерическим. Также вводится понятие богатого свойства и обобщаются основные результаты для таких свойств.

Ключевые слова: формула, свойство, элементарная теория, абелева группа, ранг.

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