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Groups with a Strongly Embedded Subgroup Saturated with Finite Simple Non-abelian Groups *

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Abstract. An important concept in the theory of finite groups is the concept of a strongly embedded subgroup. The fundamental result on the structure of finite groups with a strongly embedded subgroup belongs to M. Suzuki. A complete classification of finite groups with a strongly embedded subgroup was obtained by G. Bender. Infinite periodic groups with a strongly embedded subgroup were first investigated by V. P. Shunkov and A. N. Izmailov under certain restrictions on the groups in question. The structure of a periodic group with a strongly embedded subgroup saturated with finite simple non-abelian groups is developed. The concepts of a strongly embedded subgroup and a group saturated with a given set of groups do not imply the periodicity of the original group. In this connection, the question arises of the location of elements of finite order both in groups with a strongly embedded subgroup and in groups saturated with some set of groups. One of the interesting classes of mixed groups (i.e., groups containing both elements of finite order and elements of infinite order) is the class of Shunkov groups. It is proved that a Shunkov group with a strongly embedded subgroup saturated with finite simple non-abelian groups has a periodic part.

Keywords: a periodic group, Shunkov group, groups saturated with a given set of groups, strongly embedded subgroup, Bender's theorem.

1. Introduction

The concept of a strongly embedded subgroup is fundamental in the theory of finite groups [5]. A subgroup R of a group G is called *strongly embedded in G* if R contains an involution (a second order element) and for any element $g \in G \setminus R$, $R \cap R^g$ does not contain an involution. The fundamental result on the structure of finite groups with a strongly embedded

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subgroup belongs to M. Suzuki. A complete classification of finite groups with a strongly embedded subgroup belongs to G. Bender [1; 5]. Infinite periodic groups with a strongly embedded subgroup were first investigated by V.P. Shunkov and A.N. Izmailov under certain restrictions on the groups in question [6; 7]. In the groups $L_2(P)$ and $Sz(Q)$ (P, Q are locally finite fields of characteristic 2) the normalizers of Sylow 2 subgroups are strongly embedded and are Frobenius groups. V.P. Shunkov posed question 10.76 in the Kourovka notebook:

Let G be a periodic group with an infinite Sylow 2 subgroup of S , which either

a) elementary abelian, or

b) Suzuki 2 group, moreover, the normalizer $N_G(S)$ is strongly embedded in G and is a Frobenius group with locally cyclic complement. Should the group G be locally finite?

In the case of condition a), the question was resolved by A.I. Sozutov [15], in the case of condition b) the question was resolved by A.I. Sozutov and N.M. Suchkov [16]. It was established that the group G is isomorphic to one of the groups of the set $\{L_2(P), Sz(Q)\}$.

From the G. Bender research mentioned above, it follows, in particular, that finite simple groups with a strongly embedded subgroup, up to isomorphism, are groups from the set $\{L_2(2^{n+1}), Sz(2^{2n+1}), U_3(2^{n+1}) \mid n = 1, 2, \dots\}$. In this paper we prove Theorems 1, 2, "transferring" this result to periodic groups and Shunkov groups saturated with finite simple non-abelian groups. Let \mathfrak{X} be some set of groups. Group G is saturated with groups from the set \mathfrak{X} if any finite subgroup of G is contained in a subgroup of G , isomorphic to some group from \mathfrak{X} [13].

The concepts of a strongly embedded subgroup and a group saturated with a given set of groups do not imply the periodicity of the original group. In this connection, the question arises of the location of elements of finite order both in groups with a strongly embedded subgroup and in groups saturated with some set of groups. One of the interesting classes of mixed groups (that is, groups containing both elements of finite order and elements of infinite order) is the class of Shunkov groups [12].

A group G is called a Shunkov group if, for any finite subgroup H from G in the quotient group $N_G(H)/H$, any two conjugate elements of prime order generate a finite group.

It is well known that in the general case the Shunkov group does not have to have the periodic part [2] (the *periodic part* $T(G)$ of the group G means the whole set of its elements of finite order, provided that it forms the group [8]).

2. Definitions, known facts, auxiliary statements

Definition 1. *In this article, the symbol 1 will be used to denote a single element of a group or a single group. From the context it will always be clear what kind of situation is involved.*

Definition 2. [10] *If the group G is saturated with groups from the set of groups \mathfrak{X} , then this set is called saturation set for the group G .*

Definition 3. [10] *Let G be a group, K be a subgroup of G , \mathfrak{X} be a set of groups. By $\mathfrak{X}_G(K)$ we denote the set of all subgroups of G , containing K and isomorphic to groups from \mathfrak{X} . In particular, if 1 is the identity subgroup of G , then $\mathfrak{X}_G(1)$ denotes the set of all subgroups of the group G , isomorphic to groups from \mathfrak{X} . If it is clear from the context which group we are talking about, then instead of $\mathfrak{X}_G(K)$ we will write $\mathfrak{X}(K)$.*

Definition 4. *For a group G and a set of groups \mathfrak{X} , the notation $G \tilde{\in} \mathfrak{X}$ means that G is isomorphic to some group from \mathfrak{X} . Respectively, the notation $G \not\tilde{\in} \mathfrak{X}$ means that G is not isomorphic to any group from \mathfrak{X} .*

Proposition 1. [5] *Let $G = L_2(q)$, where q is an odd number, greater than 3 . Then the following statements are true :*

1. *The Sylow 2 subgroup of the group G is a dihedral group.*
2. *If a is an involution from G , then $C_G(a)$ is a dihedral group.*
3. *$Z(C_G(a)) = \langle a \rangle$.*

Proposition 2. [1] *Suppose that a finite group G has a strongly embedded subgroup. Then one of the following statements holds:*

- (1) *A Sylow 2 subgroup from G is cyclic or a group of quaternions.*
- (2) *$O^2(G/O(G)) \tilde{\in} \{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\}$.*

Proposition 3. [11] *Let G be a finite simple non-abelian group, and all involutions from its Sylow 2 subgroup S lie in the center of S . Then G is isomorphic to one of the groups of the following set :*

$$\{J_1; L_2(q), q > 3, q \equiv 3, 5 \pmod{8}; L_2(2^n); U_3(2^n); Sz(2^{2n+1}); Re(3^{2n+1})\}.$$

Moreover, G does not contain elements of order 8 , and all involutions from G are conjugate.

Proposition 4. [13, Lemma 1.3] *If in G periodic group some Sylow 2 subgroup is finite, then all Sylow 2 subgroups of G are finite and conjugate.*

Proposition 5. [4] *Let a periodic group G be saturated with finite simple non-abelian groups, and in any of its finite 2 -subgroups K all involutions from K lie in the center K . Then G is isomorphic to one of the groups of the following set :*

$$\{J_1, L_2(P), Re(W), U_3(F), Sz(Q) \mid W, F, Q, P - \text{locally finite fields}\}.$$

Proposition 6. [14] *Suppose that the Shunkov group G is saturated with finite simple non-abelian groups, and in any of its finite 2-subgroups K all involutions from K lie in the center of K . Then G has a periodic part that is isomorphic to one of the groups of the following set : $\{J_1, L_2(P), Re(W), U_3(F), Sz(Q)\}$ for suitable locally finite fields P, W, F, Q .*

3. Proof of the main results

Theorem 1. *Let G be a periodic group with a strongly embedded subgroup saturated with finite simple non-abelian groups. Then G is isomorphic to one of the groups of the following set:*

$$\{L_2(P), Sz(Q), U_3(F)\},$$

where P, Q, W are locally finite fields of characteristic two.

Proof. Let G be a counterexample to the theorem, R a strongly embedded subgroup of G , \mathfrak{M} the set of all finite simple non-abelian groups.

Lemma 1. *If for some element $x \in G$ the intersection $R \cap R^x$ contains an involution, then $x \in R$.*

Proof. An immediate consequence of the definition of a strongly embedded subgroup given above.

The Lemma is proved. □

Lemma 2. *Let K be a finite 2-subgroup of G . Then in G there is an element g such that $K^g \leq R$.*

Proof. Let the involution $z \in Z(K)$ and $v \in R$ is considering. Since G is a periodic group, then $\langle z, v \rangle = \langle d \rangle \rtimes \langle z \rangle = \langle d \rangle \rtimes \langle v \rangle$ is infinite dihedral group. If the element d is of odd order, then $\langle z \rangle$ and $\langle v \rangle$ are Sylow 2 subgroups of second order of $\langle z, v \rangle$ group. Therefore for some $g \in \langle z, v \rangle$ equality $\langle z \rangle^g = \langle v \rangle$ occurs. Since $\langle z \rangle^g = \{1, z\}^g = \{1^g, z^g\} = \{1, z^g\} = \langle v \rangle = \{1, v\}$, than $z^g = v$. Further, from the fact that $z \in Z(K)$, it follows that $z^y \in Z(K^y)$ for any $y \in G$. Therefore, $z^g = v \in Z(K^g)$ for any $x \in K^g$, $v \in R \cap R^x$. From the above and from Lemma 1 follows that $K^g \leq R$, in which case Lemma is proved. Let the element d have even order. Consider $w \in \langle d \rangle$ involution. It is obvious that w in the center of a group $\langle z, v \rangle$. Therefore, $v \in R \cap R^w$, by the Lemma 1 $w \in R$. Then $w \in R \cap R^z$ by Lemma 1 $z \in R$. Since $z \in Z(K)$, then for any $x \in K$, $z \in R \cap R^x$, by Lemma 1 $K < R$, that as g element you can take a 1 single element of the group.

The Lemma is proved. □

Lemma 3. *Let $M \in \mathfrak{M}(1)$. Then*

$$M \cong \{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\}.$$

Proof. Let S_M be a certain Sylow 2 subgroup from M . Due to Lemma 2 there may be considered to be such an element $g \in G$, that $S_M^g \leq R \cap M^g$. Note that $R_{M^g} = R \cap M^g$ is a strongly M^g -embedded subgroup. Since R_{M^g} include S_M^g , then R_{M^g} contain involution. Suppose that for some $x \in M^g \setminus R_{M^g}$, $R_{M^g} \cap R_{M^g}^x$ contain involution y . Then $y \in R_{M^g} \leq R$ and $y \in R_{M^g}^x \leq R^x$. Therefore, $y \in R \cap R^x$ and $x \in R$ due to Lemma 1. In this case $x \in R \cap M^g = R_{M^g}$. Contradict with choice of x . So, R_{M^g} – a strongly embedded subgroup in M^g . It is clear that $R_{M^g}^{g^{-1}}$ – is strongly embedded subgroup in M . Since M is finite, simple, non-abelian group, then $O(K) = 1$, $O^2(M/O(M)) \simeq M$. At the suggestion of the 2 $M \cong \{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\}$.

The Lemma is proved. □

Therefore identifying the set $\mathfrak{M}(1)$, the latest can be taken as a saturated set for the group G . Then by Lemma 3 as a saturated set for a group G can be taken set $\{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\}$. At the suggestion of the 3 this set is a subset of the set

$$\{J_1; L_2(q), q > 3, q = 3, 5 \pmod{8}; U_3(2^n); Sz(2^{2n+1}); Re(3^{2n+1})\}.$$

From the proposal 5 it follows that G isomorphic in one of the following groups:

$$\{J_1, L_2(P), Re(W), U_3(F), Sz(Q) \mid W, F, Q, P \text{ are locally-finite fields}\}.$$

Lemma 4. $G \not\cong J_1$.

Proof. Suppose Inverse Demand. In this case, the saturation condition implies that $J_1 \cong \{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\}$. Since the Sylow 2 subgroup J_1 is an elementary abelian group of order 8, what we have here is unique opportunity: $J_1 \simeq L_2(8)$, that it is impossible ([3, p. 6, p. 36]).

The Lemma is proved. □

Lemma 5. $G \not\cong L_2(P)$ if P is locally-finite field of an odd characteristic.

Proof. Suppose Inverse Demand: $G \simeq L_2(P)$. If the W field contains 3 elements, then the group G does not contain finite simple non-abelian subgroups. Contradict with the theorem conditions. If the P field contains 5 items, (since $L_2(5) \simeq L_2(2^2)$) $G \simeq L_2(4)$. Contrary to the fact that G is a counter-example. Let the P field contain more than 5 items. Let S be Sylow 2 subgroup of group G . If S contains more than 4 items, then

for any finite subgroup K from S in the latter there is a finite subgroup X , consisting of over 4 elements and containing K . the subgroup X is embeddable in some finite subgroup Y of G group such that $Y \simeq L_2(P_1)$ for a suitable finite subfield P_1 of P field. Therefore, X is a dihedral group of order greater than 4, and as a result, contains involutions that do not lie at its center (defenition 1). On the other hand, by the saturation condition and Lemma 3 $X < V < G$, where

$$V \tilde{\in} \{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\},$$

by defenition 3 all involutions from X must lie at its center. The resulting contradiction indicates that the order of S is 4, therefore, S – is an elementary abelian group of order 4. In this case, all Sylow 2 subgroups of G are conjugate (definition 4) with S group . But then for any group $M \in \mathfrak{M}(1)$, Sylow 2 subgroup of M elementary abelian of order 4. Therefore, $M \simeq L_2(2^2)$ (Lemma 3). Due to randomness of choice M , as an element of the set $\mathfrak{M}(1)$, we get that $\mathfrak{M}(1) = \{L_2(2^2)\}$, and any finite subgroup of G has an order of at most 60, which is not so. Contradiction.

The Lemma is proved. □

Lemma 6. $G \not\simeq Re(W)$.

Proof. Assume the opposite: $G \simeq Re(W)$. Let S is Sylow 2 subgroup of the group G . If S contain more than 8 elements, then for any finite subgroup K from S in the latter there is a finite subgroup X , consisting of more than 8 elements and containing K . The subgroup X is embeddable in some finite subgroup Y of G group such that $Y \simeq Re(W_1)$ for a suitable finite subfield W_1 of W field. Therefore the group X contains no more than 8 elements. Contradiction. So, S is a group of order at most 8, and since the group G contains a subgroup of Y , then S is an elementary abelian group of order 8.

In this case, all Sylow 2 subgroups of G are conjugate (theorem 4) with S group. On the other hand, by the saturation condition and Lemma 3 $Y < V < G$, where

$$V \tilde{\in} \{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\}.$$

By the above, the Sylow 2 subgroup of V group is elementary abelian of order 8. Therefore $V \simeq L_2(2^3)$ (lemma 3). But then $L_2(2^3)$ contains a subgroup H , isomorphic to the Y group . The latter is impossible due to the fact that $Y \simeq Re(3^{2k+1})$ for a suitable natural k ([3, p. 6]).

The lemma is proved. □

We complete the proof of the theorem. From definition 5 and the Lemmas 4, 5, 6 proved above it follows that G is isomorphic to one of the group of the following set

$$\{L_2(P), U_3(F), Sz(Q) \mid F, Q, P \text{ are locally finite fields of characteristic } 2\}.$$

This contradicts the fact that G is a counterexample.

The theorem is proved. \square

Theorem 2. *Let G be Shunkov group with a strongly embedded subgroup saturated with finite simple non-abelian groups. Then G has a periodic part $T(G)$, which is isomorphic to one of the groups of the following set:*

$$\{L_2(P), Sz(Q), U_3(F)\},$$

where P, Q, F are locally finite fields of characteristic two.

Proof. Let G be a counterexample to the theorem, let R be a strongly embedded subgroup of G , and \mathfrak{M} is the set of all finite simple non-abelian groups. By a direct verification we verify that the Lemmas 1, 2, 3 occurs for G group. Therefore, as a saturation set for the group G you can take

$$\{L_2(2^n), Sz(2^{2n+1}), U_3(2^n) \mid n = 1, 2, \dots\}$$

set. By the definition 3 this set is a subset of the set

$$\{J_1; L_2(q), q > 3, q = 3, 5 \pmod{8}; U_3(2^n); Sz(2^{2n+1}); Re(3^{2n+1})\}.$$

From the definition 6 follows that the group G has a periodic part $T(G)$, and $T(G)$ isomorphic to one of the groups of the following set:

$$\{J_1, L_2(W), Re(F), U_3(Q), Sz(P)\},$$

where W, F, Q, P are suitable locally finite fields. It is easy to see that the periodic group $T(G)$ has a strongly embedded subgroup $R_1 = T(G) \cap R$ and is saturated with finite simple non-abelian groups. In this case, for $T(G)$ all the conditions of Theorem 1 are satisfied, therefore, for $T(G)$ the conclusion of this theorem holds: $T(G)$ is isomorphic to one of the groups of the following set:

$$\{L_2(W), Sz(P), U_3(Q)\}.$$

The theorem is proved. \square

4. Conclusion

The concept of a strongly embedded subgroup is widely used in the theory of finite simple non-abelian groups to establish the internal structure of the group under study. Transferring classical results on groups containing a strongly embedded subgroup from the class of finite groups to the class of infinite groups (both with and without finiteness conditions) seems to be a

meaningful task, since it will allow us to study the structure of the infinite group, which the present work demonstrates.

Theorems 1, 2 proved above allow us to hope for a generalization of Bender's result formulated above to wider classes of groups and saturating sets themselves. For example,

Hypothesis A. Let G be a periodic group with a strongly embedded subgroup saturated with finite unsolvable groups. Then

$$O^2(G/O(G)) \cong \{L_2(P), Sz(Q), U_3(F)\}$$

where P, Q, F are locally finite fields of characteristic two.

Hypothesis B. Let G be Shunkov group with a strongly embedded subgroup saturated with finite unsolvable groups. Then

$$O^2(G/O(G)) \cong \{L_2(P), Sz(Q), U_3(F)\}$$

where P, Q, f are locally finite fields of characteristic two.

It is natural to consider these hypotheses at first for locally finite groups, periodic Shunkov groups, and then in general form.

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Группы с сильно вложенной подгруппой, насыщенные конечными простыми неабелевыми группами

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Аннотация. Важным понятием в теории конечных групп является понятие сильно вложенной подгруппы. Принципиальный результат о строении конечных групп с сильно вложенной подгруппой принадлежит М. Сузуки. Полная классификация конечных групп с сильно вложенной подгруппой получена Г. Бендером. Бесконечные периодические группы с сильно вложенной подгруппой впервые были исследованы В. П. Шунковым и А. Н. Измайловым при некоторых ограничениях на рассматриваемые группы. В работе установлено строение периодической группы с сильно вложенной подгруппой, насыщенной конечными простыми неабелевыми группами. Понятия сильно вложенной подгруппы и группы, насыщенной заданным множеством групп, не предполагают периодичности исходной группы. В связи с чем возникает вопрос о расположении элементов конечного порядка как в группах с сильно вложенной подгруппой, так и в группах, насыщенных некоторым множеством групп. Одним из интересных классов смешанных групп (т. е. групп, содержащих как элементы конечного порядка, так и элементы бесконечного порядка) является класс групп Шункова. Доказано, что группа Шункова с сильно вложенной подгруппой, насыщенная конечными простыми неабелевыми группами, обладает периодической частью.

Ключевые слова: периодическая группа, группа Шункова, группы, насыщенные заданным множеством групп, сильно вложенная подгруппа, теорема Бендера.

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