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An equilibrium model with mixed federal structures *

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Abstract. This paper examines the problem of meeting an inelastic demand for public goods of club type in an economy with a finite number of agents, who exhibit different preferences regarding the choice of public projects. The choice problem is assumed to be multidimensional as there are several dimensions of a societal decision.

From the formal point of view, the problem can be summarized as follows. There are n players, identified by points in a multidimensional space, who should be partitioned into a finite number of groups under the requirement that there exists no nonempty subset S of players, each member of which strictly prefers (in terms of utilities) group S to the group he was initially allocated.

Utilities which are inversely related to costs consist of two parts: monetary part (inversely proportional to the group's size), and the transportation part (distance from the location of a player to the point minimizing aggregate transportation cost within his group).

One cannot hope for a general result of existence of stable coalition structure even in a uni-dimensional setting. However, by allowing formation of several coalition structures, each pursuing a different facet of public decision, we obtain a very general existence result. Formally, this means that for each coalition there exists a *balanced* system of weights assigned to each of the dimensions of the public project.

Keywords: equilibrium, regions, federal structures, monetary contribution, equal share.

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1. Introduction

In this note we consider a model of country formation with a «world» consisting of multiple regions that may either form a unified entity or to be partitioned into several countries. Each country chooses a (possibly multidimensional) public policy whose cost is shared among country's regions. However, since regions have heterogenous preferences over public policies, some of them may find centrally chosen policies sufficiently distant from their ideal choices and may pose a threat of secession from the country to which they belong. A natural question is whether there are *stable partitions* of the world that do not admit a group of regions each benefiting by breaking away from the status quo. It turns out that, in general, the stability cannot be guaranteed. In particular, in the case where all regions within a country equally share the cost of public policies, even uni-dimensionality of the policy space and single-peakedness of regions' preferences do not guarantee the existence of a stable world structure [3].

The paradigm of the centralized decision making, however, has been recently revised on both theoretical and empirical grounds. As Alesina et al. [2] point out: "Historically, the nation state concentrated most of the authority in every policy domain. In recent decades, however, a more complex structure has begun to emerge, characterized by a demand for more autonomy (if not secession) at a sub-national level of country unions which assume certain policy prerogatives." Indeed, in the context of the federal structures and multiple public goods, the local governments assume an increasing responsibility for providing local public goods while macroeconomic and redistribution policies with the federal government. Regional projects that tackle the trade, environmental and migration issues play an ever increasing role across the globe. These observations indicate that from the theoretical point of view, the players (regions) may belong to several unions, each assigned to a certain aspect of the public policy and we therefore permit an option of the multiple union membership for every region. Note that a membership in several unions is a wide-spread phenomena as some European countries may sustain a simultaneous membership in the European Union, European Monetary Union, NATO, United Nations or WTO.

Thus, the regions could be members in several unions and we even allow for an opt-out option [8] where some regions forego their participation in the provision of some public goods. Every formed union of regions S is assigned a certain public project and a participation weight $\Lambda(S)$, where for every region r the sum of the participation weights over the unions r belongs to, must be equal to one. A set of unions with the corresponding participation weights will be called a *federal structure*.

Given a federal structure, some of the regions may reject the proposed arrangement and pose a threat of secession. Our secession requirement is very mild: a group of regions S poses a threat of secession if it can guarantee to every region in S a higher payoff than at least in one of the unions r belongs to. The main result of this paper yields the existence of stable federal structures. In order to prove our result we use the framework of cooperative game without transferable utility and rely on the Danilov's (1999) variant [5] of the celebrated Scarf (1967) theorem on nonemptiness of the core[9].

The paper is organized as follows. In the next section we present the model and definitions and indicate the non-existence of a stable partition. In Section 3 we introduce the notion of federal structures and show the existence of a stable federal structure under the same assumptions that yield the nonexistence result in Section 2. Section 4 concludes.

2. The Model

Consider a model with a finite set $N = \{1, \ldots, n\}$ of regions, which can either constitute one (unified) country or be partitioned into several countries. Each country chooses a public policy, and we assume that the set of feasible policies is given by a multi-dimensional Euclidean space $P = \mathcal{R}^k$, where $k \geq 1$.

If a country $S \subset N$ forms it must choose a policy p in P. The policy implementation incurs monetary costs, denoted by $g(S)$. We naturally assume that the costs are positive for every S and are weakly increasing with respect to inclusion:

A.1 – Cost Monotonicity: If the set of regions S is contained in a larger set S' then $g(S') \geq g(S)$.

Every region $r \in N$ has an ideal point $p^r \in P$ and the choice of any other policy $p \in P$ would generate a disutility for r, represented by the Euclidean distance $||p^r - p||$ between its ideal policy p^r and the policy p.

We assume that every region $r \in N$ has an initial endowment $y_r > 0$, a part of which is spent on the implementation of the public policy chosen in the country of region r. That is, if country S chooses a policy $p \in P$ then every region $r \in S$ is assigned the monetary contribution t_r. We assume the total contributions of the regions cover the cost of public projects they participate in:

A.2 – Budget-balancedness: $\displaystyle\sum_{r \in S} t_r = g(S)$.

To simplify the matters, we assume separability and linearity of the regions' preferences:

A.3 – Utilities: The utility of region r assigned to a monetary contribution t_r in the country with a public policy p, is given by

$$u(y_r, t_r, p^r, p) = y_r - t_r - ||p^r - p||.$$

In order to proceed with our results we make further assumptions on policy choices and allocation of their costs across regions. We assume that the policy choice in the country is determined through the majority voting mechanism. If the policy set P is unidimensional, for every country $S \subset N$ consider the set of its *median locations*. It is easy to see that every median location minimizes the aggregate cost of regions in S, and is, in fact, a solution to the following minimization problem:

$$\min_{p \in P} \sum_{r \in S} ||p^r - p||. \tag{2.1}$$

We denote the set of solutions to (2.1) by $M(S)$. If the policy set P is multidimensional and the set of ideal points p^r is not located along a straight line, a solution to (2.1), denoted by $m(S)$, is unique.[1] In the uni-dimensional case, $M(S)$ could be an interval, and in this case $m(S)$ will stand for the middle point of $M(S)$. We impose the efficiency requirement:

A.4 – Efficiency: Every country S chooses its public policy at $m(S)$.

Following Alesina and Spolaore [1], Casella [4], Jéhiel and Scotchmer [7], Haimanko et al. [6], Bogomolnaia et al. [3] we assume, for simplicity, that all regions of the same country make an equal contribution towards the policy cost. The regions are hold responsible for their preferences, and are not compensated for their disutility of location of the chosen policy in the policy space P:[2]

A.5 – Equal Share: If the country S is created, every region in S makes the same monetary contribution: $t_r = \dfrac{g(S)}{|S|}$ for every $r \in S$.

The assumptions we impose allow us to reduce a country formation problem described above to determination of countries' composition; once formed, their policy actions and cost contributions are prescribed as above. We will denote by $v(r, S)$ the indirect utility (or payoff) of the region $r \in S$ when country S forms: $v(r, S) = y_r - \dfrac{g(S)}{|S|} - ||p^r - m(S)||$.

We will examine partitions which are stable under secession threats. In other words, no group of regions could reduce their costs (or equivalently, increase their payoffs) by forming a new country. Formally,

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1 In the mathematical programming literature, the value of the problem (2.1) is called Minimal Aggregate Transportation Cost of the set S.

2 We could have assumed that each region contributes proportionally to its population, without altering the paper's main result.

Definition 1: A collection $\pi = \{S_1, \ldots, S_K\}$ of pairwise disjoint subsets of N is called a *partition* if $\bigcup_{k=1}^{K} S_k = N$. The set of all partitions of N is denoted by Π.

Consider a partition π of N and denote by $S^r(\pi) \in \pi$ the country in π that contains r. Then, the utility of the region r is given by $v(r, \pi) = v(r, S^r(\pi))$.

We now offer the standard definition of (core) stability:

Definition 2: A partition $\pi = \{S_1, \ldots, S_K\}$ of N is called *stable* if there is no group of regions $S \subset N$ such that $v(r, S) > v(r, \pi)$ for every $r \in S$.

A stable partition may however fail to exist:

Proposition 1 [3]: There exists a set of regions N, satisfying assumptions A1 – A5, which does not admit a stable partition.

3. Federal Structures

Note that the definition of partitions introduced in the previous section rules out situations where different facets of public policy are carried out under different group structures. For example, defense or foreign policy can be implemented by the grand coalition of regions, while education or health fall into jurisdiction of local authorities. A natural framework to incorporate this possibility would be allowing the regions to enter several unions, each responsible for a certain facet of public policy. A union does not necessarily include all regions, as one could easily imagine the case where only a group of regions is keen on developing of a public policy on, say, environment or migration, while other (possibly distant) regions may have a limited interest in those issues. In line with this comment we allow unions of regions to pursue different aspects of public policy. A degrees of participation intensity may vary across unions but is the same for all members of the same "rigid" (in terminology of Alesina [2]) union. Some "disinterested" regions may even decide to forego their participation in certain parts of the public project, and, following Makarov [8], we allow for an opt-out option. Formally, we consider a notion of *federal structure*, which consists of unions of regions formed to pursue different facets of public projects. Each union is assigned a participation weight and we only require that the sum of the participation weights over the unions a region belongs to, is equal to one for all regions:

Definition 3: A *federal structure* is a function $\Lambda : 2^N \setminus \emptyset \to [0, 1]$ that assigns every nonempty subset S of N a nonnegative value such that the equality $\sum_{S \in \mathcal{S}^r} \Lambda(S) = 1$ holds for all $r \in N$, where $\mathcal{S}^r$ is the collection of subsets of N that contain r.[3]

Note that every partition $\pi \in \Pi$ induces a federal structure Λ_π by assigning the value of one to every S from π and zero to all other subsets.

We will now introduce a secession requirement. First, for every federal structure Λ define the set of *essential* unions S with a positive degree of participation: $\mathcal{S}_\Lambda = \{S \subset N | \Lambda(S) > 0\}$.

Then the utility level derived by region r, given Λ would be defined as follows:

$$v_m(r, \Lambda) = \min_{S \in \mathcal{S}_\Lambda \cap \mathcal{S}^r} v(r, S). \tag{3.1}$$

Now a *secession threat* by a group of regions S would simply require that S can guarantee to every region $r \in S$ a payoff which is higher than at least in one of effective unions r belongs to. Formally,

Definition 4: A group of regions S poses a threat of secession to the federal structure Λ if $v(r, S) > v_m(r, \Lambda)$ for every $r \in S$. A federal structure Λ is called stable if no group of regions poses a threat of secession to Λ.

Now, we state our main result.

Proposition 2: Under Assumptions A.1 –A.5, there exists a stable federal structure Λ.

Proof of Proposition 2: To prove this result we will use Danilov [5]. For every nonempty $S \subset N$ denote by $\Re^S$ the projection of the set $\Re^n$ on coordinates in S. For every vector $y = (y_1, \ldots, y_n) \in \Re^n$ let $y^S \in \Re^S$ be a natural projection of y, that is, $y_r^S = y_r$ for every $r \in S$. A *non-cooperative NTU-game* V is a correspondence that assigns to each S a subset $V(S)$ of $\Re^S$. Danilov [5] proves the following variant of the Scarf's result:

D-Theorem: Consider a non-cooperative NTU-game V, where for every $S \subset N$ the set $V(S)$ is closed, bounded from above and satisfies the free disposal condition. Then there exists a vector $y \in \Re^n$, and a balanced collection (federal structure) Λ, satisfying two requirements:

(i) There is no $S \subset N$ such that $y^S \in int\{V(S)\}$, where $int\{V(S)\}$ stands for the interior of $V(S)$;

(ii) If $\Lambda_S > 0$ then $y^S \in V(S)$.

[3] Note that in the cooperative game theory, a collection $\{\Lambda_S\}_{S \subset N}$ satisfying this equality is called *balanced*.

Now, for every nonempty $S \subset N$ define the set $\hat{V}(S) \subset \Re^S$ as follows:

$$\hat{V}(S) = \{y^S \in \Re^S : \ v(r, S) \geq y_r^S \ \forall r \in S\}. \tag{3.2}$$

Clearly, the game $\hat{V}$ satisfies the conditions of the D-theorem. Thus, there exist vector $y \in \Re^n$ and a balanced collection Λ which satisfy the requirements of D-Theorem. We claim that Λ is a stable mixed federal structure.

Indeed, suppose a group of regions S poses a threat of secession to Λ. That is, for each region $r \in S$ there exists $T(r) \in \mathcal{S}_\Lambda \cap \mathcal{S}^r$ such that $v(r, S) > v(r, T(r))$.

Since $\Lambda_{T(r)} > 0$, the assertion (ii) of the D-theorem implies that $y^{T(r)} \in \hat{V}(T(r))$, and, therefore, $v(r, T(r)) \geq y_r^{T(r)} = y_r$ for every $r \in S$. The last two inequalities guarantee that $v(r, S) > y_r$ for all $r \in S$. But then $y^S \in int\{\hat{V}(S)\}$, a contradiction to assertion (i) of the D-theorem. □

4. Conclusions

In this paper we consider a model of the "world" with multiple regions. The world is partitioned into countries, each consisting of one or several regions. The regions have distinct preferences over public policies chosen in their country and finance the cost of public projects through the equal share mechanism. Bogomolnaia et al. [3] have shown that stable partitions, that are immune against threats of secession by groups of regions, do not necessarily exist. In order to rectify this problem and to examine a more flexible distribution of power and responsibility within countries, we allow every regions to belong to several unions, assigned to different facets of the public good project. This so-called *federal structure* consists of unions of regions, where each union is assigned a participation weight so that for every region the sum of the participation weights over the unions a region belongs to is equal to one. By using the Danilov variant (1999) of the Scarf theorem (1967) on nonemptiness of the core of a balanced game without side payments, we show that under our assumptions there always exists a stable federal structure.

There are two natural questions related to the result of this paper that remain open and are left for future research. What are conditions on the distribution of regions' preferences that guarantee the existence of stable partitions? Could one characterize the set of stable federal structures, especially in the environments that do not admit stable partitions?

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Равновесие в модели формирования дробных коалиционных структур

Аннотация. В статье ставится и решается задача об удовлетворении неэластичного спроса на общественное благо клубного типа внутри экономической системы, состоящей из конечного числа действующих лиц (агентов). Как и в других исследованиях данного направления, предполагается, что существует конфликт интересов относительно выбора конкретных параметров блага. Этот конфликт, однако, в настоящей работе предполагается *многомерным*, то есть включающим произвольное число параметров разногласия.

Математически, задача может быть сформулирована следующим образом. Дано конечное число точек-игроков $x_1, \ldots, x_n$ в многомерном евклидовом пространстве. Нужно их разбить на конечное (но не фиксированное!) число групп $S_1, \ldots, S_k$ с соблюдением следующего свойства: не существует подмножества игроков S, любой участник которого получает в группе S больший выигрыш, чем в той группе S_j, к которой был исходно приписан.

Входящие в выигрыш со знаком минус издержки суммируются из монетарной составляющей, обратно пропорциональной размеру группы, к которой прибавляется расстояние до центра группы (то есть до точки, минимизирующей суммарную транспортировку внутри группы).

В этих условиях нельзя рассчитывать на общую теорему существования коалиционно устойчивого решения задачи, как показывает ряд примеров даже для одномерных постановок. Однако если допустить формирование *дробных* групповых структур, то теорему о существовании коалиционно устойчивого решения задачи можно установить в самой большой степени общности. Под дробной структурой понимается здесь набор интенсивностей λ_S функционирования для всевозможных непустых подмножеств множества игроков — набор, удовлетворяющий формализуемым в работе условиям сбалансированности.

Ключевые слова: равновесие, регионы, федерализм, функция выигрыша, принцип равнодолевого участия.

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