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## Boundary Value Problem of Magnetically Insulated Diode: Existence of Solutions and Complex Bifurcation

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**Abstract:** The paper focuses on the stationary self-consistent problem of magnetic insulation for a vacuum diode with space-charge limitation, described by a singularly perturbed Vlasov-Maxwell system of dimension 1.5. The case of insulated diode when the electrons are deflected back towards the cathode at the point  $x^*$  is considered. First, the initial VM system is reduced to the nonlinear singular limit system of ODEs for the potentials of electric and magnetic fields. The second step deals with the limit system's reduction to the new nonlinear singular ODE equation for effective potential  $\theta(x)$ . The existence of non-negative solutions is proved for the last equation on the interval  $[0, x^*)$  where  $\theta(x) > 0$ . The most interesting and unexplored case takes place if  $\theta(x) < 0$  on the interval  $(x^*, 1]$  and such case corresponds to the case of an insulated diode. For the first time, a numerical analysis of complex bifurcation of solutions in insulated diode is considered for  $\theta(x) < 0$  depending on parameters and boundary conditions. Bifurcation diagrams of the dependence of solution  $\theta(x)$  on a free point (free boundary)  $x^*$  were constructed. Insulated diode spacing is found. These results could lead to more efficient and effective magnetically insulated diodes in future power conversion systems.

**Keywords:** relativistic Vlasov-Maxwell system, magnetic insulation, effective potential, insulated diode, initial value problem, singular boundary value problem, contractive mapping, fixed point theorem, complex numerical bifurcation

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Научная статья

## Краевая задача для диода с магнитной изоляцией: существование решений и комплексная бифуркация

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**Аннотация:** Исследуется стационарная самосогласованная задача о магнитной изоляции вакуумного диода с пространственным ограничением заряда, описываемой сингулярно возмущенной системой Власова – Максвелла размерности 1,5. Рассматривается случай изолированного диода, когда электроны отклоняются назад к катоду в точке  $x^*$ . Сначала исходная система ВМ сводится к нелинейной сингулярной предельной системе ОДУ для потенциалов электрического и магнитного полей. На втором этапе происходит сведение предельной системы к новому нелинейному сингулярному уравнению ОДУ для эффективного потенциала  $\theta(x)$ . Для последнего уравнения доказано существование неотрицательных решений на интервале  $[0, x^*)$ , где  $\theta(x) > 0$ . Наиболее интересным и неисследованным является случай, когда  $\theta(x) < 0$  на интервале  $(x^*, 1]$  и соответствует случаю изолированного диода. Впервые проведен численный анализ бифуркаций решений в изолированном диоде для  $\theta(x) < 0$  в зависимости от параметров и граничных условий. Построены бифуркационные диаграммы зависимости решения  $\theta(x)$  от свободной точки (свободной границы)  $x^*$ . Найден изолированный интервал диода. Полученные результаты могут способствовать разработке более эффективных диодов с магнитной изоляцией для будущих систем преобразования энергии.

**Ключевые слова:** релятивистская система Власова – Максвелла, магнитная изоляция, эффективный потенциал, изолированный диод, задача с начальным значением, сингулярная краевая задача, сжимающие отображения, теорема о неподвижной точке, комплексная численная бифуркация

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## 1. Motivation

Power electronics is a cornerstone of contemporary power systems, bringing theoretical physics and practical engineering to enable efficient energy conversion and transmission. Among its critical challenges is ensuring the stability and reliability of high-voltage devices, such as those employed in high-voltage direct current (HVDC) systems. A fascinating intersection of applied mathematics and plasma physics arises in the study of magnetic insulation in vacuum diodes – a phenomenon where magnetic fields surpass electron backflow, allowing these devices to operate efficiently at extreme voltages. The seminal work of Langmuir and Compton [3] provides the fundamental theory to model electron behavior. For the comprehensive review the advancement of diode physics since the pioneering works of Child and Langmuir readers may refer to work [11].

The Vlasov-Maxwell theory provides a rigorous kinetic framework to model the collective behavior of charged particles (electrons) and electromagnetic fields. Here readers may refer to part 3 of book [6], chapter 5 in [7] and references therein including works [2; 3; 8]. The generic theory of the parametric families of small branching (bifurcation) solutions of nonlinear differential equations is proposed in [1; 5] and applied to magnetic insulation problem. The reduction of the boundary value problem for noninsulated magnetic regime in a vacuum diode to a singular system of nonlinear Fredholm equations is fulfilled in [4]. In [10] an equation for the space-charge limited current of a crossed-field diode is derived and it can be solved numerically.

The nonlinear dynamics of these models often leads to bifurcations – sudden qualitative shifts in system behavior caused by small parameter changes. Bifurcations analysis is crucial for predicting instabilities, optimizing insulation thresholds, and preventing catastrophic failures in devices like vacuum diodes. It is to be noted that electron transport in high energy devices such as vacuum diodes exhibits many nonlinear phenomena due to the extremely high applied voltages. One of these effects is the saturation of the current due to the self-consistent electric and magnetic field.

The effect of magnetic insulation consists in that the electrons emitted from cathode cannot reach the anode due to the extremely high applied electric and magnetic field; they are reflected by the magnetic forces back to the cathode. Here two basic regimes are possible: the first, when electrons reach the anode (“noninsulated” diode) and the second one, when electrons rotate back to the cathode (“insulated” diode).

## 2. Setting of the problem

We consider a plane diode consisting of two perfectly conducting electrodes, a cathode ( $X = 0$ ) and anode ( $X = L$ ). The system is described by the 1.5 dimensional VM model:

$$\begin{aligned} V_X \frac{\partial F}{\partial X} + e \left( \frac{d\Phi}{dX} - V_Y \frac{dA}{dX} \right) \frac{\partial F}{\partial P_X} + e V_X \frac{dA}{dX} \frac{\partial F}{\partial P_Y} &= 0, \\ \frac{d^2 \Phi}{dX^2} &= \frac{e}{\epsilon_0} N(X), \quad X \in (0, L), \\ \frac{d^2 A}{dX^2} &= -\mu_0 J_Y(X), \quad X \in (0, L). \end{aligned}$$

Here,  $F(X, P_X, P_Y)$  is electron distribution function,  $\Phi, A$  are potentials of electromagnetic field,  $\epsilon_0, \mu_0$  are the vacuum permittivity and permeability.

After appropriate scaling and taking the limit  $\epsilon \rightarrow 0$ , we obtain (see [2] and [7]) the following limit system

$$\begin{aligned} \frac{d^2 \varphi}{dx^2} &= j_x \frac{1 + \varphi(x)}{\sqrt{(1 + \varphi(x))^2 - 1 - (a(x))^2}}, \quad \varphi(0) = 0, \quad \varphi(1) = \varphi_L, \\ \frac{d^2 a}{dx^2} &= j_x \frac{a(x)}{\sqrt{(1 + \varphi(x))^2 - 1 - (a(x))^2}}, \quad a(0) = 0, \quad a(1) = a_L, \end{aligned} \quad (2.1)$$

where  $j_x > 0$ ,  $\alpha \in [0, 1]$ ,  $\varphi$  is a potential of electric field and the potential of magnetic fields is  $a$ .

Let us now define the effective potential by  $\theta(x) = (1 + \varphi(x))^2 - 1 - a^2(x)$ .

## 3. Main Results

We introduce the substitution  $u = 1 + \varphi$ ,  $a = v$  and reduce problem to the following form:

$$\begin{aligned} u'' &= j_x \frac{u}{\sqrt{u^2 - 1 - v^2}}, \quad u(0) = 1, \quad u(1) = \varphi_L + 1 = \alpha, \quad u'(0) = 0, \\ v'' &= j_x \frac{v}{\sqrt{u^2 - 1 - v^2}}, \quad v(0) = 0, \quad v(1) = a_L, \quad v'(0) = \beta > 0. \end{aligned} \quad (3.1)$$

**Definition 1.** A function  $(u, v) = (u(x), v(x))$  is a solution of the initial value problem (IVP) on  $[0, \varepsilon)$  if:

- $u, v \in C^1[0, \varepsilon) \cap C^2(0, \varepsilon)$ ,
- $\theta(u(x), v(x)) > 0$  for  $x \in (0, \varepsilon)$ ,
- $(u, v)$  satisfies the differential equations (3.1),
- the initial conditions hold.

**Proposition 1.** *Let  $(u, v)$  be a solution of the IVP on  $[0, \varepsilon)$  and define  $\theta = u^2 - 1 - v^2$ . Then:*

- $\theta(x) \in C^1[0, \varepsilon) \cap C^2(0, \varepsilon)$ ,
- $\theta(0) = \theta'(0) = 0$  and  $\theta(x) > 0$  on  $(0, \varepsilon)$ ,
- $\theta$  satisfies the differential equation:

$$\theta'' = j_x \left( 6\sqrt{\theta} + \frac{2}{\sqrt{\theta}} - 4\gamma \right), \quad \gamma = -\frac{1}{2j_x} \beta^2.$$

**Theorem 1.** *For every  $\gamma \in \mathbb{R}$  there exists a unique solution on  $[0, \infty)$  of the initial value problem:*

$$D'' = j_x \left( 6\sqrt{D} + \frac{2}{\sqrt{D}} - 4\gamma \right), \quad D(0) = D'(0) = 0. \quad (3.2)$$

*The solution has different properties depending on the value of  $\gamma$ :*

**Case 1:**  $\gamma < 2$ :  $D \in C^1[0, \infty) \cap C^2(0, \infty)$ ,  $D(\infty) = \infty$ .

**Case 2:**  $\gamma = 2$ :  $D \in C^1[0, \infty) \cap C^2(0, \infty)$ ,  $D(\infty) = 1$ .

**Case 3:**  $\gamma > 2$ :  $D \in C^1[0, a] \cap C^2(0, a]$ , can be extended periodically.

Employing Banach's fixed-point theorem, the existence of a unique solution to (3.2) is proved.

**Proposition 2** (Child-Langmuir Law). *Let  $0 < c \leq j_x \leq j_x^{max}$ ,  $a = 0$ . Then equation*

$$\varphi'' = j_x \frac{1 + \varphi}{\sqrt{\varphi(2 + \varphi)}}, \quad \varphi(0) = 0, \quad \varphi(1) = \varphi_L$$

*has a lower positive solution  $u_0 = \delta^2 x^{4/3}$  if  $4\delta^3 \geq 9j_x^{max}(1 + \delta^2)/\sqrt{2 + \delta^2}$  and an upper positive solution  $u^0 = \alpha + \beta x$  with  $\varphi_L \geq \delta^2$ .*

#### INSULATED CASE

For the study of the magnetically insulated diode (MID),  $\theta < 0$  we want to analyze the conditions under which this phenomenon occurs.

Considering the system (2.1) and the effective potential  $\theta$  defined in Proposition 1, we get the following first-order ODE after some simple algebraic and calculus-based manipulation:

$$(\theta')^2 = k\theta + 8j_x\theta^{\frac{3}{2}} + 8j_x\theta^{\frac{1}{2}} + 4\beta^2, \quad \theta(0) = 0,$$

where  $k$  is an integration constant that we got during the order reduction process. As we want to know when the electrons start to deflect, we will

use this expression to find where the  $\theta$  values decrease. We will call this point  $x_d$ . Then

$$(\theta(x_d'))^2 = \theta'_d = k\theta_d + 8j_x\theta_d^{\frac{3}{2}} + 8j_x\theta_d^{\frac{1}{2}} + 4\beta^2 = 0,$$

**Proposition 3** ( $\theta$  and  $u$  solutions relationship). *Let  $\hat{k} = \frac{k}{8j_x}$  and  $\hat{\beta} = \frac{\beta^2}{2j_x}$ , and consider the equations:*

$$u^3 + \hat{k}u^2 + u + \hat{\beta}^2 = 0, \quad \theta^{\frac{3}{2}} + \hat{k}\theta + \theta^{\frac{1}{2}} + \hat{\beta}^2 = 0.$$

*Then, any solution  $u$  of the first equation with  $\text{Re}(u) > 0$  induces a solution  $\theta = u^2$  of the second equation.*

As solving the equation described in terms of  $u$  induces the desired solutions for  $\theta_d$ , the efforts can be focused on solving the cubic expression, which is more tractable than the corresponding radical expression.

**Proposition 4.** *Let  $\Delta_u = 18\hat{k}\hat{\beta} + \hat{k}^2 - 4 - 4\hat{k}^3\hat{\beta} - 27\hat{\beta}^2$ , then the cubic equation  $u^3 + \hat{k}u^2 + u + \hat{\beta}^2 = 0$  has the following solutions in  $\mathbb{C}$ :*

–  $\Delta_u < 0$ :

$$\begin{aligned} u_1 &= -\frac{\hat{k}}{3} + \frac{\sqrt[3]{4}}{18} \left( \sqrt[3]{A_1 + A_2} + \sqrt[3]{A_1 - A_2} \right) \\ u_2 &= \left[ -\frac{\hat{k}}{3} - \frac{\sqrt[3]{4}}{18} \left( \sqrt[3]{A_1 + A_2} + \sqrt[3]{A_1 - A_2} \right) \right] + \\ &\quad + \left[ \frac{\sqrt{3}\sqrt[3]{4}}{36} \left( \sqrt[3]{A_1 + A_2} - \sqrt[3]{A_1 - A_2} \right) \right] i \\ u_3 &= \left[ -\frac{\hat{k}}{3} - \frac{\sqrt[3]{4}}{18} \left( \sqrt[3]{A_1 + A_2} + \sqrt[3]{A_1 - A_2} \right) \right] + \\ &\quad + \left[ -\frac{\sqrt{3}\sqrt[3]{4}}{36} \left( \sqrt[3]{A_1 + A_2} - \sqrt[3]{A_1 - A_2} \right) \right] i, \end{aligned}$$

where  $A_1 = -54\hat{k}^3 + 243\hat{k} - 729\hat{\beta}$  and  $A_2 = \sqrt{A_1^2 + 2916(3 - \hat{k}^2)^3}$ ;

–  $\Delta_u = 0$ ,  $\hat{k} = \pm\frac{\sqrt{3}}{9}$ ,  $\hat{\beta} = \pm\sqrt{3}$ :

$$u_1 = u_2 = u_3 = \mp \frac{\sqrt{3}}{3};$$

–  $\Delta_u = 0$ ,  $\hat{k} \neq \pm\frac{\sqrt{3}}{9}$ ,  $\hat{\beta} \neq \pm\sqrt{3}$ :

$$u_1 = \frac{\hat{k}^3 - 4\hat{k} + 9\hat{\beta}}{3 - \hat{k}^2}, u_2 = u_3 = \frac{-\hat{k} + 9\hat{\beta}}{2\hat{k}^2 - 6};$$

$$\begin{aligned}
& -\Delta_u > 0, \hat{k} \neq \pm \frac{\sqrt{3}}{9}, \hat{\beta} \neq \pm \sqrt{3}: \\
& u_1 = A_3 \cos\left(\frac{1}{3} \arccos(A_4)\right) - \frac{\hat{k}}{3}, u_2 = A_3 \cos\left(\frac{1}{3} \arccos(A_4) + \frac{2\pi}{3}\right) - \frac{\hat{k}}{3}, \\
& u_3 = A_3 \cos\left(\frac{1}{3} \arccos(A_4) + \frac{4\pi}{3}\right) - \frac{\hat{k}}{3}, \text{ where } A_3 = \frac{2}{3} \sqrt{\hat{k}^2 - 3} \text{ and } A_4 = \\
& \frac{4\hat{k}^3 - 9\hat{k} + 27\hat{\beta}}{6 - 2\hat{k}^2} (\hat{k}^2 - 3)^{-1/2}.
\end{aligned}$$

With the analytical description of the solutions of the cubic equation in Proposition 3 and the method to induce the solutions for  $\theta_d$ , the behavior of the solutions can be described by understanding and manipulating the parameters  $\hat{k}$  and  $\hat{\beta}$ .

#### 4. Bifurcation diagrams

In order to understand how the parameters  $\hat{k}$  and  $\hat{\beta}$  influence the existence or non-existence of the solutions required for the magnetic insulated diode, the different bifurcation diagrams over the solutions in the  $\theta$  and  $u$  space can be employed.

##### CONSTRUCTION OF BIFURCATION DIAGRAMS VIA ANALYTICAL SOLUTIONS

The bifurcation diagrams presented are constructed by analytically solving the steady-state condition of the system, specifically the differential equation  $\theta' = 0$ . This approach focuses on identifying equilibrium points of the system as a function of a control parameter, such as magnetic field strength.

##### *Methodology for Constructing the Bifurcation Diagram*

- 1) Analytical determination of equilibrium points. The condition  $\theta' = 0$  implies that the system is at equilibrium. By setting the derivative to zero, the resulting algebraic equation is solved analytically to find expressions for  $\theta$  in terms of the control parameter. This process yields the equilibrium solutions of the system.
- 2) Parameter variation and solution evaluation. The analytical expressions for  $\theta$  are evaluated across a range of values for the control parameter. This step involves substituting different parameter values into the analytical solutions to compute the corresponding equilibrium points.
- 3) Plotting the bifurcation diagram. The computed equilibrium points are plotted against the varying control parameter using Python's `matplotlib` library. Each point on the diagram represents a steady-state solution for a specific parameter value. Connecting these points reveals

the structure of the solution branches and highlights bifurcation points where qualitative changes in the system's behavior occur.

This analytical approach to constructing bifurcation diagrams provides a clear and precise understanding of the system's steady-state behavior, facilitating the identification of critical parameter values that lead to qualitative changes in dynamics. The Python code [9] used to generate the bifurcation diagrams.

## 2D DIAGRAMS FOR $u$ AND $\theta$

In this first set of diagrams, one parameter was fixed at a specific value, while the other was varied over the interval  $[-5, 5]$ . The fixed values were chosen to allow the existence of a real solution to the cubic equation with multiplicity three.

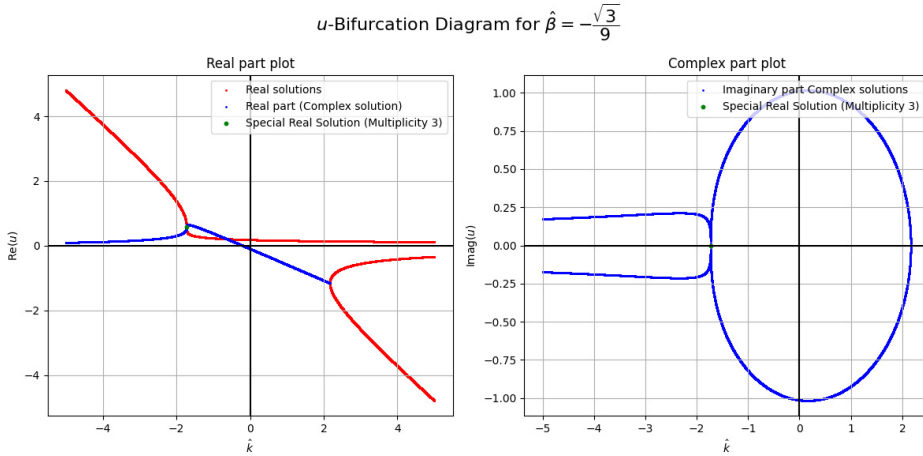


Figure 1. Bifurcation diagram of the solutions of the cubic equation related to  $u$  centered on the 3-multiplicity real solution

In Figure 1, we observe the behavior of the solutions in the  $u$ -space. As predicted by theory, the number of solutions in the complex plane ranges from one to three. However, since complex solutions occur in conjugate pairs, the plot reveals loop-like structures, which are of particular interest for further analysis.

In Figure 2, we observe the behavior of the solutions in the  $\theta$ -space. In this new setting, some solutions are lost compared to the  $u$ -space due to the real part constraint defined in Proposition 3. Nevertheless, the loop-like behavior persists, which gives hope for its potential application to the magnetically insulated scenario.



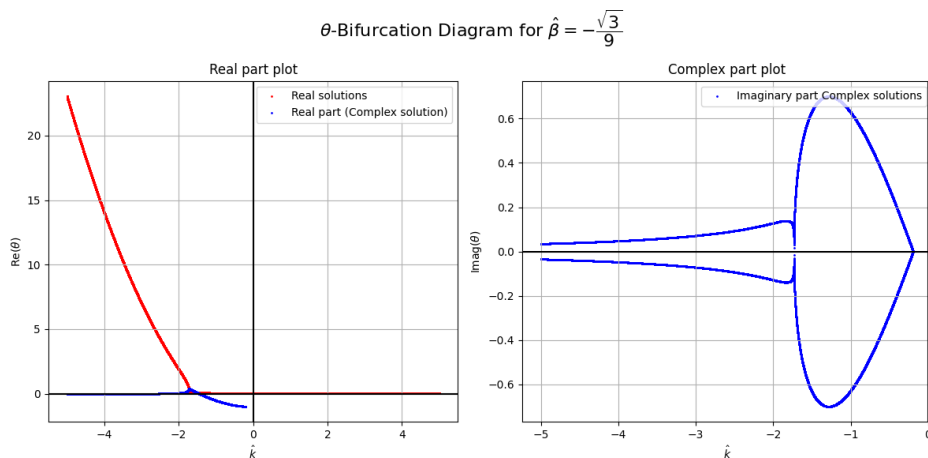


Figure 2. Bifurcation diagram of the solutions of the cubic equation related to  $\theta$  centered on the 3-multiplicity real solution

### 3D SURFACE DIAGRAMS FOR $u$ AND $\theta$

The bifurcation diagrams in the 2D space revealed several interesting features. However, to obtain a more comprehensive understanding of the phenomenon, we now examine the behavior of the solutions in a 3D space. In this setting, both parameters are allowed to vary freely within the interval  $[-5, 5]$ , and the  $z$ -axis represents either the real or imaginary part of the solutions.

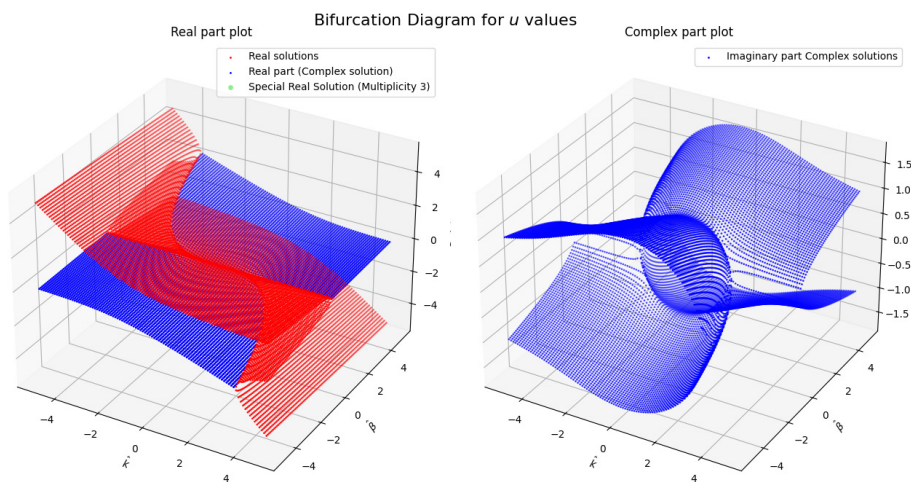


Figure 3. Surface gotten from the parameters  $\hat{k}$  and  $\hat{\beta}$  for the solutions of the cubic equation

In Figure 3, we analyze the behavior of the solutions in the  $u$ -space. As a first observation, we note that for every pair of values  $(\hat{k}, \hat{\beta})$ , there is at least one solution represented on the surface. Moreover, the loop-like behavior previously observed in the 2D case begins to manifest in a more structured and continuous form in the 3D surface, suggesting an underlying pattern that warrants further investigation.

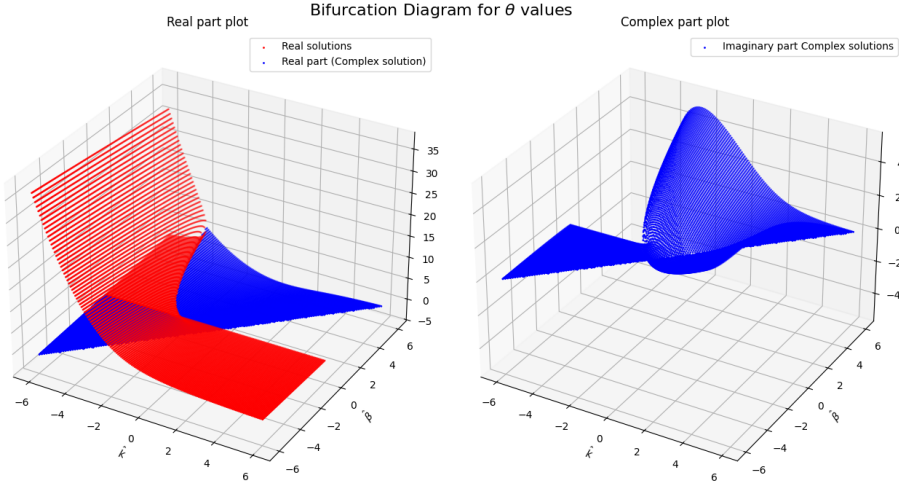


Figure 4. Surface gotten from the parameters  $\hat{k}$  and  $\hat{\beta}$  for the solutions of the stability of the  $\theta$ -based ODE

In Figure 4, we analyze the behavior of the solutions in the  $\theta$ -space. Notably, we observe that the solutions with a negative real component are trimmed due to the constraint defined in Proposition 3. Additionally, there are empty regions that exhibit a likely linear pattern, which could be an interesting subject for further investigation.

## 5. Conclusion

This study yielded critical insights into the boundary problem governing MID through a rigorous analysis of the singularly perturbed Vlasov-Maxwell system. By reducing the system to a nonlinear ordinary differential equation framework, the existence of physically admissible solutions under well-defined parametric constraints is proved. The bifurcation analysis underscored the important role of the free boundary point in shaping the solution topology, revealing distinct regimes dependent on system parameters. Numerical simulations further elucidated the mechanistic interplay between magnetic insulation and electron dynamics, particularly the redirection of electron trajectories toward the cathode under varying insulation

thresholds. These findings emphasize the need for continued exploration of parameter-driven bifurcation phenomena, which may advance both theoretical and applied understanding of MID in high-energy physics and related fields. The bifurcation analysis enables robust design strategies to avoid unstable regimes that could compromise device longevity and efficiency.

This study offers significant contributions to the design of high-performance power converters through the systematic optimization of MID operational parameters. Specifically, the analysis provides three key engineering advantages:

- 1) **Enhanced Operational Stability:** By identifying the optimal working point of the magnetically insulated diode, the proposed methodology ensures robust converter stability under dynamic load conditions. This equilibrium point minimizes oscillatory behavior and mitigates instability risks inherent in high-power systems.
- 2) **Improved Energy Density:** The approach enables the accurate calibration of the diode's rated power capacity, directly enhancing energy density. This optimization aligns the converter's output with application-specific requirements, thereby maximizing efficiency while avoiding overdesign.
- 3) **Cost-Effective Hardware Implementation:** Traditional designs often rely on conservative maximum capacity estimates, leading to oversized components and inflated costs. In contrast, the derived optimal power threshold allows for tailored diode specifications, reducing material expenditures without compromising performance or reliability.

Collectively, these insights advance the practical deployment of magnetically insulated diodes in next-generation converters.

## References

1. Ben Abdallah N., Degond P., Mehats F. Mathematical Models of Magnetic Insulation. *Rapport interne N 97.20*. 1997, MIP, Universite Paul Sabatier, Toulouse, France.
2. Degond P., Raviart P.-A. An asymptotic analysis of the one-dimensional Vlasov-Poisson system: the Child-Langmuir law. *Asymptotic Anal.*, 1991, vol. 4, no. 3, pp. 187–214.
3. Langmuir I., Compton K.T. Electrical discharges in gases. Part II. Fundamental phenomena in electrical discharges. *Rev. Mod. Phys.*, 1931, vol. 3, no. 2, pp. 191–257.
4. Rojas E.M., Sidorov N.A., Sinitsyn A.V. A boundary value problem for non-insulated magnetic regime in a vacuum diode. *Symmetry*, 2020, vol. 12, no. 4, 617.

5. Sidorov N.A., Sidorov D.N. Small solutions of nonlinear differential equations near branching points. *Russ Math.*, 2011, vol. 55, pp. 43–50. <https://doi.org/10.3103/S1066369X11050070>
6. Sidorov N., Sidorov D., Sinitsyn A. Toward General Theory of Differential-Operator and Kinetic Models, Book Series: *Chua L. (ed.) World Scientific Series on Nonlinear Science Series A*, vol. 97, World Scientific, Singapore, London, 2020, 496 p.
7. Sidorov D., Rojas E., Sinitsyn A., Sidorov N. Approximation and Regularisation Methods for Operator-Functional Equations / Book Series: *Bellomo N., Eftimie R. (eds.) Advances in Mathematics for Applied Sciences*, vol. 95, World Scientific, Singapore, London, 2025, 248 p.
8. Sinitsyn A.V. Positive solutions of a nonlinear singular boundary value problem of magnetic insulation. *Mathematical modelling*, 2001, vol. 13, no. 5, pp. 37–52.
9. *Github Repository* (2025).  
Available at: [https://github.com/omardtl24/Mag\\_Isol\\_Numerical](https://github.com/omardtl24/Mag_Isol_Numerical) (accessed 5 May 2025).
10. Wright J. K., Sree Harsha N. R., Garner A. L. A generalized equation for the critical current for a one-dimensional crossed-field gap in an orthogonal coordinate system. *arXiv e-prints*, 2025, no. 2504.13435.
11. Zhang P., Valfells Á., Ang L.K., Luginsland J.W., Lau Y.Y. 100 years of the physics of diodes. *Applied Physics Reviews*, 2017, vol. 4, no. 1, p. 011304.

### Список источников

1. Ben Abdallah N., Degond P., Mehats F. Mathematical Models of Magnetic Insulation // Rapport interne N 97.20. 1997, MIP, Universite Paul Sabatier, Toulouse, France.
2. Degond P., Raviart P.-A. An asymptotic analysis of the one-dimensional Vlasov-Poisson system: the Child-Langmuir law // *Asymptotic Anal.* 1991. Vol. 4, N 3. P. 187–214.
3. Langmuir I., Compton K.T. Electrical discharges in gases Part II. Fundamental phenomena in electrical discharges // *Rev. Mod. Phys.* 1931. Vol. 3, N 2. P. 191–257.
4. Rojas E. M., Sidorov N. A., Sinitsyn A. V. A boundary value problem for noninsulated magnetic regime in a vacuum diode // *Symmetry*. 2020. Vol. 12, N 4. Art. N 617.
5. Сидоров Н. А., Сидоров Д. Н. О малых решениях нелинейных дифференциальных уравнений в окрестности точек ветвления // *Известия вузов. Математика*. 2011. № 5. P. 53–61.
6. Sidorov N., Sidorov D., Sinitsyn A. Toward General Theory of Differential-Operator and Kinetic Models / ed. L. Chua ; World Scientific. Singapore, London, 2020. 496 p. (World Scientific Series on Nonlinear Science Series A ; vol. 97).
7. Approximation and Regularisation Methods for Operator-Functional Equations / D. Sidorov, E. Rojas, A. Sinitsyn, N. Sidorov / eds. N. Bellomo, R. Eftimie ; World Scientific. Singapore, London, 2025. 248 p. (Advances in Mathematics for Applied Sciences ; vol. 95).
8. Sinitsyn A.V. Positive solutions of a nonlinear singular boundary value problem of magnetic insulation // *Mathematical modelling*. 2001. Vol. 13, N 5. P. 37–52.
9. Github Repository. URL: [https://github.com/omardtl24/Mag\\_Isol\\_Numerical](https://github.com/omardtl24/Mag_Isol_Numerical) (дата обращения: 05.05.2025).

10. Wright J. K., Sree Harsha N. R., Garner A. L. A generalized equation for the critical current for a one-dimensional crossed-field gap in an orthogonal coordinate system // arXiv e-prints. 2025. № 2504.13435.
11. 100 years of the physics of diodes / P. Zhang, A. Valfells, L. K. Ang, J. W. Luginsland, Y. Y Lau // Applied Physics Reviews. 2017. Vol. 4, N 1. P. 011304.

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