Серия «Математика»
2025. Т. 51. С. 141—150

Онлайн-доступ к журналу:
http://mathizv.isu.ru

И З В Е С Т И Я
Иркутского государственного университета

Research article

УДК 510.665, 510.643
MSC 03B45, 03H05
DOI https://doi.org/10.26516/1997-7670.2025.51.141

Intransitive Temporal Multi-agent Logic with Agents' Multi-valuations. Decidability

Konstantin V. Grekovich[1], **Vladimir V. Rybakov**[1✉], **Vitaliy V. Rimatskiy**[1✉]

[1] Siberian Federal University, Krasnoyarsk, Russian Federation
✉ Rybakov@mail.ru
✉ Gemmeny@rambler.ru

Abstract. We study intransitive temporal multi-agent logic with agents' multi-valuations for letters and formulas. In previous wide accepted research the time and knowledge primarily were modeled by Kripke models with structure looking as simply a single time cluster with multi-relations for agents' accessibility relations. Here we develop this approach and use Kripke models with linear intransitive time and states represented by arbitrary time clusters for agents accessibility multi-relations.

This logic is defined in a semantic way, as a set of formulas, which are true at linear models with multi-valued variables by agents' and clusters of states. We propose a background for such approach and a technique for computation truth values of formulas. Main result concerns decidability problem. We prove that the resulting logic is decidable and obtain a sort of finite model property.

Keywords: modal logic, frame and model Kripke, multi-agent logics, decidability problem

Acknowledgements: The research was financially supported by the Russian Science Foundation, Krasnoyarsk Regional Fund of Science (Project No. 23-21-00213 and Project No. 25-21-20011 https: //rscf.ru/project/25-21-20011/)

For citation: Grekovich K. V., Rybakiov V. V., Rimatskiy V.V. Intransitive Temporal Multi-agent Logic with Agents' Multi-valuations. Decidability.. *The Bulletin of Irkutsk State University. Series Mathematics*, 2025, vol. 51, pp. 141–150.
https://doi.org/10.26516/1997-7670.2025.51.141

Научная статья

Нетранзитивная временная многоагентная логика с мультиозначиваниями агентов. Разрешимость

К. В. Грекович[1], **В. В. Рыбаков**[1✉], **В. В. Римацкий**[1✉]

[1] Сибирский федеральный университет, Красноярск, Российская Федерация
✉ Rybakov@mail.ru
✉ Gemmeny@rambler.ru

Аннотация. Изучается нетранзитивная временная мультиагентная логика с мультиозначиваниями агентов для переменных и формул. Ранее время и знания моделировались с помощью моделей Крипке, структура которых выглядела как простой единый временной кластер с множеством отношений достижимости для агентов. Здесь мы развиваем этот подход и используем модели Крипке, которые представляют собой линейное нетранзитивное время и состояния, представленные произвольными временными кластерами для мультиотношений достижимости агентов. Эта логика определяется семантически как множество формул, истинных на линейных моделях с мультиозначиваниями переменными и кластерами состояний. Мы предлагаем обоснование такого подхода и методику вычисления истинностных значений формул. Основные результаты касаются проблемы разрешимости. Мы доказываем, что полученная логика разрешима и финитно аппроксимируема.

Ключевые слова: модальная логика, фрейм и модель Крипке, мультиагентная логика, разрешимость логики

Благодарности: Работа выполнена при финансовой поддержке Российского научного фонда и Красноярского краевого фонда науки (проект № 23-21-00213 и проект № 25-21-20011 https: //rscf.ru/project/25-21-20011/).

Ссылка для цитирования: Grekovich K. B., Rybakiov V. V., Rimatskiy V.V. Intransitive Temporal Multi-agent Logic with Agents' Multi-valuations. Decidability. // Известия Иркутского государственного университета. Серия Математика. 2025. Т. 51. С. 141–150.
https://doi.org/10.26516/1997-7670.2025.51.141

1. Introduction

Nowadays mathematical logics widely applied in research concerning computer science, information sciences overall. The interaction here is of both side and tasks and problems in computer science generates new areas in mathematical logic and induces creation new technique and tools in mathematical logic itself. Conception of knowledge arise in the analysis of distributed systems, and leaded to development multi-agent and multi-valued logical systems. More details about this can be found in the works of Halpern, Vardy (Reasoning About Knowledge [11]), Rybakov [17].

It concern also the fixpoint approach, omniscience, monotonicity, justified knowledge, etc (cf. for example Artemov [1], Artemov [2], Artemov [3]). It also was implemented in research concerning uncertainty and plausibility (cf. V Rybakov [18]). Later some works were done towards consolidation such technique and to refine it towards elements of hybrid cooperation of the agents [4–6]. Also technique for formalization of knowledge was enriched by research in description logics (cf. Baader and Satler [7]), first-order logic was also implemented (cf. Belardinelli F., Lomuscio A. [8]). Various semantic technique was used (cf. Horrocks, Sattler [9]; Horrocks, Giese, Kharlamov, Waler [10]).

In resent time research concerning knowledge was combined with implementation of temporal logic (cf. Rybakov [13–16]). An automata-theoretic approach to multi-agent planning was evolved at Wooldridge [19].

In this our short paper we study intransitive temporal multi-agent logic with agents' multi-valuations for letters and formulas. Earlier common knowledge in [11] was modeled at Kripke models with structure looking as simply a single time cluster with multi-relations for agents'accessibility relations. This brought interesting strong results correlating well with observed examples and intuition. Here we wish to develop this approach towards modeling knowledge with Kripke frames which are linear time models with states represented by arbitrary clusters for agents accessibility relations. Here time is intransitive and it acts to only connect clusters. We propose a background for such approach and a technique for computation truth values of formulas. Main results concerns solvability problem, we prove that the resulting logic is decidable, prove existence of some deciding algorithm.

2. Denotation, Preliminary Facts

It is assumed that the reader is familiar with algebraic and Kripke semantics for modal logics, In accordance with the modern interpretation, by *logic* we mean the set of all theorems that can be proven in a given axiomatic system, or all formulas which are true at a chosen set of models. By propositional logic in his paper we mean algebraic propositional logic (see [12]), although the reader may consider λ to be just a modal logic, which is necessary for our purposes. Necessary information and all known facts used further, can be found in widely known literature (for example cf. [12], ch. 2.2-2.5; 4.1). To briefly recall definitions, a *frame* $\mathcal{F} := \langle F, R \rangle$ is a pair, where F is a non-empty set and R is a binary relation on F. The basic set and the frame itself will be further denoted by the same letter.

A non-empty set $C \subseteq F$ is called a *cluster* if: 1) for any x, y from C, xRy holds; 2) for any $x \in C$ and $y \in W$, $((xRy \& yRx) \Longrightarrow y \in C)$ holds. A cluster is called *proper* if $|C| > 1$; otherwise we say it is a *singleton or degenerate cluster*. For an element $a \in F$, let $C(a)$ will denote the cluster (i.e., the set of elements mutually comparable with respect to R with a given element a) generated by the element a. Frame is said to be intransitive if the accessibility relation R of this frame is intransitive.

A sequence of elements (or clusters) $\{a_0; a_1; \ldots; a_n\}$ of a frame is called a chain of length $n+1$ if, for all $i < n$, element a_{i+1} is R-accessible from element a_i (that is a_iRa_{i+1}) and there are no other frame elements between them.

The depth of an element x of a model (frame) F is the maximal number of clusters in chains of clusters starting from the cluster $C(x)$ containing x. The set of all elements (worlds) in a frame (model) F of depth not more than n will be denoted by $S_{\leq n}(F)$, and the set of elements of depth n we denote by $S_n(F)$.

Recall some more definitions. Let some class of models K and some formula be given. A formula is said to be satisfiable (in a given class K) if it is true at some element of some model from this class.

A logic L is said to be decidable if there is an algorithm which for any formula may compute if this formula belongs to L. A logic L satisfies the finite model property (FMP) if for any $\alpha \notin L$ there exists a finite L-model at which α is not valid.

3. Logic NML.

Now we introduce models and technique for our logic. NML-frame is a tuple

$$\mathcal{F}_{NML} = \langle W, Next, R_{\sim}, R_1, \dots R_n \rangle,$$

where

$$W = \bigcup_{i \in J} C_i, J = [0, 1, \dots m] \vee J = N,$$

and any C_i is the $R_{\sim}$-cluster, that is $R_{\sim}$ is the equivalence relation such that : $xR_{\sim}y \iff \exists i \in J : (x \in C_i \ \&\ y \in C_i)$; and all $R_j, 1 \leq j \leq n$ are arbitrary relations on any C_i (in particular equivalence relations on C_i).

Temporal binary relation $Next$ is defined as follows:

$(x\ Next\ y) \iff \exists i \in J : (x \in C_i\ \&\ y \in C_{i+1})$ for some single chosen unique y.

In a case when $J = [0, \dots, m]$ we assume $\forall x \in C_m(x\ Next\ y)$ for some single chosen unique y from C_m.

Definition 1. *NML-model is a tuple* $\mathcal{M}_{NML} = \langle \mathcal{F}, V_1, V_2, \dots V_n \rangle$ *, where* $\mathcal{F}$ *is NML-frame,* $V_i \subseteq 2^W$ *is a valuation of i-th agent.*

If $a \in \mathcal{M}_{NML}$, and $a \in V_j(p)$ then write $(\mathcal{M}_{NML}, a) \models_{V_j} p$ and say that p is true at element a w.r.t V_j valuation of j-th agent.

The language of our NML logic consists of:

1. countable set of propositional variables $P := \{p_1; p_2; \dots\}$;

2. Boolean operations $\{\neg, \vee, \wedge, \rightarrow\}$;

3. a set of unary modal operations $\Box_{\sim}, \Box_1, \dots \Box_n$
and temporal operator $Next$;

4. auxiliary symbols: brackets.

A formula in the NML-language is defined in a standard way, that are terms constructed out of letters by chosen logical operations.

Any sets of valuations of agents – V_j, of a set $Prop$ of propositional variables in frames $\mathcal{F}$ from models $\mathcal{M}$ to be extended to formulas as follows:

- Truth of Boolean connectives $\neg, \wedge, \vee, \rightarrow$ is defined in the standard way;
- $(\mathcal{M}_{NML}, x) \models_{V_j} Next\,\alpha \iff$
 $\exists k \in N : x \in C_k \;\wedge \exists\, y \in C_{k+1}\; ((x\ Next\ y)\ \&\ y \models_{V_j} \alpha)$;
- $(\mathcal{M}_{NML}, x) \models_{V_j} \Box_i \alpha \iff \exists k \in N : x \in C_k\ \&(\forall y \in C_k\ (xR_i y \implies (\mathcal{M}_{NML}, y) \models_{V_j} \alpha))$;
- $(\mathcal{M}_{NML}, x) \models_{V_j} \Box_{\sim} \alpha \iff \exists k \in N : x \in C_k\ \&\ \forall y \in C_k (y \models_{V_j} \alpha)$.

Each element (world) of the NML-frame can be understood as an information point. The time relation Next links such points into a linear and discrete time stream. For two points w and z, the expression $w\ Next\ z$ means that z will be available at the next time relative to w. Although time is usually thought of as continuous, in view of computational steps in a computer, it can also be thought of as discrete. In this context, the discreteness of the time flow is understood as follows: between two time moments C_i and C_{i+1} there are no other time points. The time chain of states (worlds) has a beginning (the computation starting point).

Also important for us is the assumption of the linearity of the time flow, that is, the absence of branching: after each time moment C_n there can only be one moment C_{i+1} such that $C_i Next C_{i+1}$. The relation $R_{\sim}$ connects all information points that are potentially available at the same moment. Thus, in the interpretation under consideration, $R_{\sim}$ defines the information potentially known at each information point of the current temporary state. The relation R_i connects with each other those points that are accessible to some agent i at the considered moment of time. Each point provides the agent with some information that is relevant at the moment in the current state.

Definition 2. *Logic NML is the set of all formulas which are true at each state of each introduced and described above model w.r.t. any agents' valuation.*

Recall the notion of temporal degree. Let α be a modal formula.

Definition 3. *The modal degree $md(\alpha)$ of the formula α is determined as follows: $md(p) = md(\top) = md(\bot) = 0$, $md(\alpha \wedge \beta) = md(\alpha \rightarrow \beta) = md(\alpha \vee \beta) = \max\{md(\alpha), md(\beta)\}$, $md(\neg\alpha) = md(\alpha)$, $md(\Box_i \alpha) =$*

$md(\Diamond_i\alpha) = md(\alpha)+1, md(\Box_\sim\alpha) = md(\alpha)+1$. *The temporal degree is defined analogously with a difference for not-boolean formulas:* $td(Next\alpha) = td(\alpha)+1$.

Now we are ready to go to proof of the following main theorem.

Theorem 1. *The logic NML is decidable (since it has a sort of FMP).*

Proof. Assume a formula α is given and that formula is not a logical law of our logic, so α is not valid at one model: $(\mathcal{M}_{NML}, x) \not\models_{V_j} \alpha$. Let α has temporal degree $td(\alpha) = k$.

Without loss of generality we can assume that we take given element x from first cluster C_0 of that model. First we define on each cluster of model $\mathcal{M}_{NML}$ relationship $\equiv$ as follows.

Let $Sub(\alpha)$ be the set of all subformulas of formula α. We define:

$$w \equiv z \iff \exists\, n : w, z \in C_n \;\&\; \forall A \in Sub(\alpha)\ [\forall\ V_j, j \leq n$$

$$[(\mathcal{M}_{NML}, w) \models_{V_j} A \iff (\mathcal{M}_{NML}, z) \models_{V_j} A]].$$

Then we define

$$\forall w \in \mathcal{M}_{NML},\quad [w] := \{t : w \equiv t\};,\ W^c = \{[w] : w \in W\} \text{ and}$$

$$[w]R_\sim[t] \iff wR_\sim t;$$

$$[w]R_i[t] \iff \forall i,\, j\forall\Box_i A \in Sub(\alpha)\&$$
$$\&((\mathcal{M}_{NML}, w) \models_{V_j} \Box_i A \Longrightarrow (\mathcal{M}_{NML}, t) \models_{V_j} A);$$

$$\forall\ p \in Sub(\alpha)\quad [V_i^c(p) := \{[w] : w \in V_i(p)\}].$$

Temporal binary relation $Next$ is defined as follows:

$$([x]\ Next\ [y]) \iff \exists iJ : ([x] \in [C_i]\ \&\ [y] \in [C_{i+1}])$$

for some single chosen unique $[y]$, for $x, y \in \mathcal{M}_{NML}$ such that $x\,Next\,y$. In a case when $J = [0, \ldots, m]$ we assume $\forall [x] \in [C_m]([x]\ Next\ [y])$ for the same single chosen unique $[y]$ from $[C_m]$. It's easy to check that the given definition of $Next$ is correct:
if $x_1\,Next\,y_1,\ x_2\,Next\,y_2\ \&\ x_1 \equiv x_2$, then must also be fulfilled $y_1 \equiv y_2$.

We denote obtained model by $M_\infty = \langle W^c, Next, R_1, \ldots R_n; V_1, \ldots R_n\rangle$. The size of the clusters in the resulting model does not exceed $n \cdot 2^{|Sud(\alpha)|}$, i.e. limited by some computable function.

The relation defined in this way has the following properties (often called filtering properties): A) $[a]R_i[b] \implies (\forall\Box_i A \in Sub(\alpha)\ (\mathcal{M}_{NML}, a) \models_{V_j} \Box_i A \Longrightarrow (\mathcal{M}_{NML}, b) \models_{V_j} A)$; B) $aR_ib \implies [a]R_i[b]$. They are performed by virtue of the definition of R_i and $Next$ on the model M_∞ of the given earlier.

Because with this relationship the elements inside one cluster are actually compressed together and the structure of the model does not change (a chain of clusters $[C_i]$ connected by a temporal relationship $Next$), the resulting model M_∞ is also a NML-model.

We may generalize standard well known filtration lemma for the case of multi-valuation (the proof is very similar to the original proof), and using it we obtain that the next statement hold:

$$\forall A \in Sub(\alpha) \quad [(\mathcal{M}_{NML}, x) \models_{V_j} A \iff (M_\infty, [x]) \models_{V_j} A].$$

Hence we get: for some $[x]$ from C_o^c,

Proposition 1. $(M_\infty, [x]) \not\models_{V_j} \alpha$.

Let's take NML-models

$$M_\infty = \langle \bigcup_N C_i^c, Next, R_\sim, R_1, R_2, \dots R_n; V_1, V_2 \dots V_n \rangle,$$

N – set of natural numbers or $[0, 1, \dots, k]$ for some arbitrary large k, and finite model

$$M_k = \langle [C_0, C_1, \dots, C_k, C_{k+1}], Next, R_\sim, R_1, R_2, \dots R_n; V_1, V_2 \dots V_n \rangle$$

and $\forall i \leq k+1$ the valuations on clusters C_i^c and C_i coincide on this models.

Theorem 2. *Let $td(\alpha) = k$. Then the truth of the formula α on any element $[x] \in C_0$ of model M_∞ is uniquely determined by the values of all propositional variables included in the formula on elements of clusters $C_0,\ C_1,\ \dots, C_{k+1}$ of frame M_∞.*

Proof. By induction on temporal degree of formula α. Lets show next: for any formula α such that $td(\alpha) = k$ holds:

$$\forall x \in C_0 (M_\infty, [x]) \models_{V_i} \alpha \iff (M_k, x) \models_{V_i} \alpha) \ \mathbf{(1)}$$

(I) If $td(\alpha) = 0$ then by induction on modal degree $md(\alpha)$ one can prove next statement:

Proposition 2. *If $td(\alpha) = 0$ then $\forall\, x \in C_i, i \in N$, holds:*

$$(M_\infty, [x]) \models_{V_i} \alpha \iff (M_k, x) \models_{V_i} \alpha)$$

Proof. If $md(\alpha) = 0$, then the formula consists only of propositional variables and standard Boolean operations. In this case, the truth of the formula on the element $[x] \in C_i$ is determined only by the valuation of the propositional variables included in the formula only on the element of

$[x] \in C_i$. Thus, according to the construction of model M_k, the statement is true.

Let us assume that for formulas such that $md(\alpha) \leq l$ the statement is true. Let us show that for formulas such that $md(\alpha) = l + 1$ the statement also holds.

Let's $\alpha = \Box_t \beta$ and $(M_\infty, [x]) \models_{V_j} \alpha$. Then $\forall z \in C_t \in M_\infty : xR_t z$ holds $(M_\infty, [z]) \models_{V_j} \beta$ and $md(\beta) \leq l$. Then by IH holds $\forall z \in C_i \in M_k$: $(xR_t z \Longrightarrow (z \models_{V_j} \beta))$ in model M_k. Hence we conclude $(M_k, x) \models_{V_j} \alpha$. The proof for formula $\alpha = \Box_\sim \beta$ is similar.

The proof in the opposite direction is similar. The proposition 2 is proved. $\Box$

Hence, in particular, it holds:

$$td(\alpha) = 0 \Longrightarrow \forall x \in C_i \in M_k((M_k, x) \models_{V_j} \alpha)$$

(II) Let us assume that for formulas such that $td(\alpha) \leq l < k$ (i.e. temporal degree not more then l) the statement is true and for all $i \leq l$ holds :

$$td(\alpha) = i \Longrightarrow \forall j \leq n \, \forall z \in C_{k-i}(((M_\infty, [z]) \models_{V_j} \alpha \Longleftrightarrow (M_k, z) \models_{V_j} \alpha)).(*)$$

Let us show that for formulas such that $td(\alpha) = l + 1$ the statement also holds. Let's take a formula $\alpha = Next\,\beta$ & $td(\beta) \leq l$. And assume $[x] \in C_{k-l-1} \in M_\infty$ & $(M_\infty, [x]) \models_{V_j} \alpha$. Then $\exists [z] \in C_{k-1} \in M_\infty$: $([z] \models_{V_j} \beta)$. Hence by **(*)** we get $\exists z \in C_{k-1} \in M_k : (z \models_{V_j} \beta)$. So we obtain $x \in C_{k-l-1} \in M_k$ & $(M_k, x) \models_{V_j} \alpha$

The proof in the opposite direction is similar. The Theorem 2 is proved. $\Box$

The collection of proven above statements implies the following result: the logic NML is decidable (since has a property related to FMP). Indeed, if a formula α belongs to the logic NML, then it is satisfied on the basic model $\mathcal{M}_{NML}$ for any valuations of the variables of the formula α. Consequently, this formula is true on the finite models M_k for any valuations of the variables. Conversely, if the formula α does not belong to the logic NML, then by what was proved above it is refuted on some finite model M_k with computable k and of computable size overall. Main Theorem 1 is proved. $\Box$

4. Conclusion

In this paper we introduce and study intransitive temporal multi-agents' logic with multi-valued variable letters. This logic is defined in a semantic way as a set of formulas that are true on linear models with time clusters and multi-agents' valuations. The main result states that the resulting logic is decidable (with a sort of FMP).

References

1. Artemov S. Evidence-Based Common Knowledge. *Technical Report TR-2004018 CUNY Ph.D.* Program in Computer Science, revised version), 2006.
2. Artemov S. Explicit Generic Common Knowledge. *Lecture Notes in CS LFCS: Logical Foundations of Computer Science*, 2013, pp 16–28.
3. Artemov S. Justification awareness. *Journal of Logic and Computation*, 2020, vol. 30, no. 8, pp. 1431–1446.
4. Babenyshev S., Rybakov V. Decidability of Hybrid Logic with Local Common Knowledge Based on Linear Temporal Logic LTL. *CiE 2008: Lecture Notes in Computer Science*, 2008, vol. 5028, pp. 32–41.
5. Babenyshev S., Rybakov V. Logic of Discovery and Knowledge: Decision Algorithm. *KES (2), Lecture Notes in Computer Science*, 2008, vol. 5178, pp. 711–718.
6. Babenyshev S., Rybakov V. Describing Evolutions of Multi-Agent Systems. *KES (1) Lecture Notes in Computer Science*, 2009, vol. 57, no. 1, pp. 38–45.
7. Baader F., Sattler U. Expressive Number Restrictions in Description Logics. *J. Log. Comput*, 1999, vol. 9, no. 3, pp. 319–350.
8. Belardinelli F., Lomuscio A. Interactions between Knowledge and Time in a First-Order Logic for Multi-Agent Systems, Completeness Results. *Journal of Artificial Intelligence Research*, 2012, vol. 45, pp. 1–45.
9. Horrocks I., Sattler U. A Description Logic with Transitive and Inverse Roles and Role Hierarchies. *Description Logics*, 1998.
10. Horrocks I., Giese M., Kharlamov E., Waaler A. Using Semantic Technology to Tame the Data Variety Challenge. *IEEE Internet Computing*, 2016, vol. 20, no. 6, pp. 62–66.
11. Fagin R., Halpern J. Y., Moses Y., Vardi M. MIT, 1995.
12. Rybakov V.V. *Admissibility of logical inference rules. Vol. 136 Studies in Logic and the Foundations of Mathematics.* Elsevier Sci. Publ., New-York, Amsterdam, 1997.
13. Rybakov V.V. Non-transitive linear temporal logic and logical knowledge operations, *J. Logic and Computation*, 2016, vol. 26, no. 3, pp. 945–958.
14. Rybakov V.V. Temporal logic with overlap temporal relations generated by time states themselves. *Siberian Mathematical Reports*, 2020, vol. 17, pp. 923–932.
15. Rybakov V.V. Multi-agent temporal nontransitive linear logics and the admissibility problem. *Algebra and Logic*, 2020, no. 59, pp. 87–100. https://doi.org/10.33048/alglog.2020.59.108
16. Rybakov V.V. Branching Time Logics with Multiagent Temporal Accessibility Relations. *Siberian Mathematical Journal*, 2021, vol. 62, no. 3, pp. 503–510. https://doi.org/10.33048/smzh.2021.62.313
17. Rybakov V. V. Refined common knowledge logics or logics of common information. *Archive for Mathematical Logic*, 2003, vol. 42, pp. 179–200.
18. Rybakov V. Temporal Multi-Agent's Logic, Knowledge, Uncertainty, and Plausibility. *Agents and Multi-Agent Systems: Technologies and Applications, LNCS*, 2021, pp. 2005–2014.

19. Wooldridge M. An Automata-theoretic approach to multi-agent planning. *Proceedings of the First European Workshop on Multi-agent Systems (EUMAS)*, December 2003, Oxford University.

Об авторах

Грекович Константин Викентьевич, Сибирский федеральный университет, Красноярск, 660041, Российская Федерация, propro879@gmail.com

Рыбаков Владимир Владимирович , д-р физ.-мат. наук, проф., Сибирский федеральный университет, Красноярск, 660041, Российская Федерация, Rybakov@mail.ru,

Римацкий Виталий Валентинович, канд. физ.-мат. наук, доц., Сибирский федеральный университет, Красноярск, 660041, Российская Федерация, Gemmeny@rambler.ru

About the authors

Konstantin V. Grekovich, Siberian Federal University, Krasnoyarsk, 660041, Russian Federation, propro879@gmail.com

Vladimir V. Rybakov, Dr. Sci. (Phys.-Math.), Prof., Siberian Federal University, Krasnoyarsk, 660041, Svobodniy Ave. 79, Russian Federation, Rybakov@mail.ru

Vitaliy V. Rimatskiy, Cand. Sci. (Phys.-Math), Assoc. Prof., Siberian Federal University, Krasnoyarsk, 660041, Russian Federation, Gemmeny@rambler.ru

Поступила в редакцию / Received 24.04.2024
Поступила после рецензирования / Revised 26.07.2024
Принята к публикации / Accepted 25.09.2024