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The Satisfiability Problem in Linear Multi-agent Knowledge Logic Based on $\mathbb{N}$

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Abstract. In this paper we explore the linear logic of multi-agent knowledge using multivalued models. The logic of the language contains the unary operators K_j — j — the agent knows, ULK_G — unstable local knowledge, E_G — stable local knowledge in the group, and the binary logical operator AP_G - the majority opinion. We will show some examples that demonstrate the diversity of this language and its capabilities. Technically we prove decidability of satisfiability problem in the resulting models for our multi-agent logic, develop verification technique and provide some examples.

Keywords: modal logic, temporal logic, common knowledge, deciding algorithms, multi-agent logic

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Научная статья

Проблема выполнимости в линейной мультиагентной логике знаний, основанной на $\mathbb{N}$

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Аннотация. Исследуется линейная мультиагентная логика знаний с использованием моделей с мультиозначиванием. Язык логики содержит унарные операторы: K_j — j-й агент знает, ULK_G — нестабильные локальные знания, E_G — стабильные локальные знания в группе, бинарный логический оператор AP_G — мнение большинства. Показаны несколько примеров, демонстрирующих разнообразие этого языка и его возможности. Технически доказана разрешимость проблемы выполнимости в результирующих моделях для нашей многоагентной логики, разработана методика проверки и приведено несколько примеров.

Ключевые слова: модальная логика, темпоральная логика, общеизвестные знания, разрешающие алгоритмы, мультиагентная логика

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1. Introduction

Definition of knowledge and its representation has been and still is one of the areas of modern research in the field of computer science. Research has come a long way since the first publications in this area around the 1980s. Representation and modelling verification of information and knowledge very often uses so-called multi-agent logics (cf. [1; 2; 4; 8]). It involves often different variants of modal logic or temporal logic (cf. [13; 14; 17]).

On the one hand, knowledge arises in the simplest situations during interaction between people, but knowledge can also arise in the analysis of distributed systems, let say that agent 1 knows something about agent 2. More details about this can be found in the works of Moshe Vardi, Barwise and others [3; 6; 7; 16]. Previously, the apparatus of mathematical logic, namely techniques from the section of non-classical logics, had already been used in the field of computer science, for example, in the work of Amir Pnueli it is proposed to use linear temporal logic or simply LTL [11] for formal verification of programs [10]. As it turned out later, LTL can indeed be actively used in this area.

Nowadays, artificial intelligence is probably one of the most popular areas in mathematical logic applications. That is, the area of knowledge and modeling of agent interaction.

Also, logical methods in computer science are widely used to analyze the correctness, compatibility, and reliability of information. In the work [15] V.V. Rybakov explores multi-agent logic using temporal relational models with multivalued values. Then V.V. Rybakov together with M.A. Moore [9] investigated multi-valued multimodal logic and adapted the filtering method to some type of such logic. Further, in our recent joint article with V.V. Rimatsky, interval logic FP [12] was of a research nature, although the article considered logic without ambiguity, it considered an approach to evaluating implicit information. Some modern studies with novelty features in related areas [5] also are interesting.

In our present work, we want to explore multimodal logic with ambiguity, in which there will be a statement that, according to agents, could tell about the truth of this or that information at the current moment in time. We consider the operator AP_G – the opinion of the majority of agents from group G and ULK_G is unstable local knowledge. When assessing the reliability of information under conditions of uncertainty, sometimes it is necessary to appeal to the opinion of the majority, although this approach is quite controversial, since situations may arise when the majority of agents have erroneous or unreliable information or situations when, on the contrary, unstable local knowledge has become stable. The novelty part of our this paper is usage of multi-valuations for different agents and switching agent distinct modalities with arbitrary valuations. Technically we prove decidability of satisfiability problem in the resulting models for our multi-agent logic, develop verification technique and provide some examples.

2. Notation, Preliminary Facts, Example

2.1. Denotation

We now briefly recall some basic definitions regarding Kripke semantics. Very general definition is as follows.

Definition 1. *A Kripke frame $\langle W, R\rangle$ is a pair where W is a non-empty set and R binary relation on W.*

In our paper we will extend the structure of such models.

Definition 2. *For any $C \subseteq W$, C is a cluster if:*
- $\forall a, b \in C\ (aRb) \wedge (bRa)$
- $\forall a \in C\ \forall c \in W\ (aRc) \wedge (cRa) \rightarrow (c \in C)$

Definition 3. *A Kripke model $\langle W, R, V\rangle$ is a triple where $\langle W, R\rangle$ is a Kripke frame, and $V : Prop \mapsto 2^W$, where $Prop$ is a set of propositional variables.*

The definitions for single-modal cases are provided above. However, in the case of multi-agent systems, additional definitions are necessary, we provide it below.

First we define the language we will use. The language of our multi-agent logic consists of a countable set of propositional variables $p_1, ..., p_n, ...$, logical connectives of classical logic $\neg, \wedge, \vee, \rightarrow$, the temporal unary operator $\mathcal{N}$ (with the meaning next), unary agents' operations $\{K_i\}_{i=1}^n$, E_G, ULK_G and the binary logical operator AP_G. Below we will illustrate what these operators represent and why they are useful. Now we need to say how look like our formulas.

Definition 4. *The set $Form$ of all formulas in our language is defined in standard manner:*

- *All propositional letters are contained in the $Form$;*
- *$(\alpha \oplus \beta)$ where $\oplus \in \{\wedge, \vee, \rightarrow, \neg\}$ are contained in $Form$;*
- *$ULK_G(\alpha)$ are contained in $Form$;*
- *$AP_G(\alpha, \beta)$ are contained in $Form$;*
- *$\mathcal{N}(\alpha)$ are contained in $Form$;*
- *$K_j(\alpha)$,$E_G(\alpha)$ are contained in $Form$;*

Now we provide light modification of standard models for the case of multi-agent systems.

Definition 5. *A LMKL frame $\mathcal{F}^{LMKL} := \langle \mathbb{N}, \leq, Next\rangle$ is a triple, where $\mathbb{N}$ is a set of natural number and $\leq$ is the standard linear order relation on natural numbers. $Next$ is the binary relation on natural numbers. $a\ Next\ b$ iff $b = a + 1$.*

Definition 6. *A LMKL model is a set $\mathcal{M}^{LMKL} = \langle \mathcal{F}^{LMKL}, V_1, ..., V_n\rangle$, where any V_i is a valuation of a set propositional letters $Prop$ in the frame $\mathcal{F}^{LMKL}$ that is $V_i : Prop \mapsto 2^{\mathbb{N}}$.*

Such structures have the following interpretation - these are linear time models with some possibility of interaction between agents. We fix now the number of agents of our multi-agent system. Let $\mathcal{A} = \{1, ..., n\}$ be a set of names of all agents. I.e. 1 is the first agent and V_1 is its valuation for letters, etc. We are not always interested in considering the whole system as a whole, sometimes we want to observe the behavior of some group of agents $G \in 2^{\mathcal{A}}$. I.e., G is some "marked" or highlighted group of people, which can be any of the many possible associations of the entire group, except perhaps an empty set.

Below we will give some illustration, which show that this approach is really interesting and has a place to be. Such approach has been studied and applied in practice, cf. for example [7; 16], but it was used when there were many accessibility relations (corresponding to abilities of the agents for access) in the models. Computational rules for truth of formulas in our case are below.

Definition 7. *We extend valuations of letters $Prop$ on all formulas as follows:*

- *Truth of Boolean connectives $\neg, \wedge, \vee, \rightarrow$ is defined as always;*

– $(\mathcal{M}^{LMKL}, x) \models_{V_i} K_j\alpha$ *iff* $(\forall y)[(x \leq y) \to (\mathcal{M}^{LMKL}, y) \models_{V_j} \alpha]$
– $(\mathcal{M}^{LMKL}, x) \models_{V_i} \mathcal{N}\alpha$ *iff* $(\forall y \in \mathbb{N})(x\ Next\ y \to (\mathcal{M}^{LMKL}, y) \models_{V_i} \alpha)$
– $(\mathcal{M}^{LMKL}, x) \models_{V_i} E_G(\alpha)$ *iff* $(\forall j \in G)((\mathcal{M}^{LMKL}, x) \models_{V_i} K_j\alpha)$
– $(\mathcal{M}^{LMKL}, x) \models_{V_i} ULK_G(\alpha)$ *iff* $(\forall j \in G)((\mathcal{M}^{LMKL}, x) \models_{V_j} \alpha)$
– $(\mathcal{M}^{LMKL}, x) \models_{V_i} AP_G(\alpha, \beta)$ *iff*
$|\{j \in G : (\mathcal{M}^{LMKL}, x) \models_{V_j} \alpha\}| > |\{j \in G : (\mathcal{M}^{LMKL}, x) \models_{V_j} \beta\}|$

It is easy to see that the operator E_G can be expressed using the operators K_j and ULK: $\bigwedge_{j\in G} K_j(ULK_{\{j\}}(\alpha)) = E_G(\alpha)$.

The definition of operations K_j will allow us to evaluate the agent's opinion and analyze the reliability of information based on the opinion of the i-th agent. E_G is stable local knowledge for the group G. ULK_G is unstable local knowledge for the group G. AP_G is the opinion of the majority of agents from the group G.

Assume that the class $\mathcal{K}$ of all described models is given.

Definition 8. *A formula f is said to be satisfiable in class of models $\mathcal{K}$ if there is a model $\mathcal{M}^{LMKL} \in \mathcal{K}$ and a state $a \in \mathcal{M}^{LMKL}$ such that $(\mathcal{M}^{LMKL}, a) \models_{V_j} f$ for some j.*

The satisfiability problem for $\mathcal{K}$ is to resolve for any given formula if it is satisfiable in some model from $\mathcal{K}$. Assuming that $\mathcal{K}$ is chosen we may define the logic $\mathcal{L}(\mathcal{K})$ of this class, e.g., as follows:

$$\mathcal{L}(\mathcal{K}) := \{\varphi \in Form | \forall \mathcal{M} \in \mathcal{K}, \forall a \in \mathcal{M}, \forall V_j[(\mathcal{M}, a) \models_{V_j} \varphi]\}$$

The satisfiability problem for the logic $\mathcal{L}(\mathcal{K})$ generated by some $\mathcal{K}$, is the satisfiability problem for the class $\mathcal{K}$ itself. For brevity we will write $\mathcal{L}$ instead $\mathcal{L}(\mathcal{K})$ on assuming $\mathcal{K}$ fixed. By a model $\mathcal{M}$ (if not specified otherwise) we understand a model from $\mathcal{K}$.

2.2. Examples, Illustration

To begin with, here are some examples of formulas that can be compiled using this approach. Let V_0 be an agent who has reliable information. Then K_0 agents' knowledge is V_0. The majority opinion may be wrong:

$$(M, x) \models_{V_0} K_0(\alpha) \wedge (E_{\mathcal{A}}(AP_{\mathcal{A}}(\neg\alpha, \alpha))).$$

If most agents consider α to be true, then agent i considers α to be true:

$$(M, x) \models_{V_i} K_i(AP_{\mathcal{A}}(\alpha, \neg\alpha,)) \to K_i\alpha.$$

If agent (1) is in opposition to the opinion of the majority of agents then agent (i) will consider this to be true after tomorrow:

$$(M, x) \models_{V_i} K_1(AP_{\mathcal{A}}(\neg\alpha, \alpha)) \wedge \alpha \to \mathcal{N}(K_i(\alpha)).$$

Unstable local knowledge of the group G, when passing through the point x+1, turns into stable knowledge for a particular group, but cannot find support in the opinion of the majority.

$$(M, x) \models_{V_i} \mathcal{N}(ULK_G(\alpha) \to E_G(\alpha)) \wedge AP_{\mathcal{A}}(\neg\alpha, \alpha).$$

3. Satisfiability problem

We introduce now definitions which we will need for following theorems. For any formula f, $Sub(f)$ is the set of all subformulas of f. For any formula f, any model $\mathcal{M}$ and any state x from it:

Definition 9. $Theory_i^f(x) = \{\varphi | \varphi \in Sub(f) \ \ (\mathcal{M}, x) \models_{V_i} \varphi\}$

Definition 10. $Opport_i^f(x) = \{Theory_i^f(y) | x \leq y\}$

Lemma 1. *There is a countable set of states, in the basis set of the model $\mathcal{M}^{LMKL}$, where each state and all the following after it states have the same $Opport_i^f$. That is,* $(\exists x \in \mathbb{N})$ *such that* $(\forall i \in [1; n])$ $(\forall y \in \mathbb{N})$ $(y \geq x) \to (Opport_i^f(x) = Opport_i^f(y))$.

Proof. This lemma follows from the observation that the formula f has a finite length and the number of realizable sub-formulas of the formula at subsequent states cannot always increase. □

3.1. Model $\mathcal{M}_{line+circle}$

To continue exploring our logic system, we must reduce the power of the base set of models. Let us define models of a new type $\mathcal{M}_{line+circle}$. Below we describe this model and the algorithm for constructing the model.

Definition 11. *Interval* $[a, b]$ *has a stabilization property if* $(\forall y > b)$ $(\exists point \in [a, b])$*:* $(\forall i)(Theory_i^f(y) = Theory_i^f(point))$.

We use now Lemma 1 : we find three states $stable_0$,$stable_1$,$stable_2$ (we can take any number of states, more than two, but not less, otherwise we will not be able to move on to reduced models in the future) such that $stable_0 < stable_1 < stable_2$ and all these three points are stabilizers (that is have properties of x from formulation of Lemma 1: cf. $((\exists x \in \mathbb{N}))$ 1), and both $[stable_0, stable_1]$ and $[stable_1, stable_2]$ have the stabilization property.

We also additionally impose some more conditions on states $stable_0$ and $stable_2$ (which also hold by that lemma and its consequence formulated after end of its proof.):

$$(\forall i \in [1, n])[(Theory_i^f(Next(stable_0)) = Theory_i^f(Next(stable_2)))$$

and

$$(Theory_i^f(stable_0) = Theory_i^f(stable_2)).$$

Now we merge the sates $stable_0$ and $stable_2$ in one state and put that $(stable_2)Next(stable_0+1)$, and delete all states situated upper as $(stable_2)$.

Now, in obtained model, the sates situated between $stable_0$ and $stable_2$ form a cluster which we denote by C_{stable}.

Definition 12. *Definition of the model*

$$\mathcal{M}_{line+circle} : \mathcal{M}_{line+circle} = \langle [1, stable_0] \cup C_{stable}, \leq, Next, V_1, ..., V_n \rangle.$$

The binary relation $\leq$ is a linear order on a subset of $[1; stable_2] \subset \mathbb{N}$, and the relation $Next$ is simply translated from the original model, so, in particular, makes cycle C_{stable}.

3.2. Satisfiability by finite models

Notice that a formula f is satisfiable in a model $\mathcal{M}$ iff it is satisfiable at the initial state 0 of some model. Therefor we assume that $(\mathcal{M}, 0) \models_{V_i} f$.

Lemma 2. *$(\forall \alpha \in Sub(f))$ for all $x \in (\mathcal{M}_{line+circle})$ the following hold. For all valuations V_i,*

$$(\mathcal{M}_{line+circle}, x) \models_{V_i} \alpha \;\; iff \;\; (\mathcal{M}, x) \models_{V_i} \alpha.$$

If $(\exists x \in \mathbb{N})$ such that $[(\mathcal{M}, x) \models_{V_i} \alpha]$ then $(\exists y \in [1, stable_0] \cup C)$ such that $[(\mathcal{M}_{line+circle}, y) \models_{V_i} \alpha]$, or if α is satisfiable at $x \in \mathcal{M}$ then α is also satisfiable at some $y \in \mathcal{M}_{line+circle}$.

Proof. We will prove it by induction on the length of formula α. First assume $(\mathcal{M}, x) \models_{V_i} \alpha$. We need to show that $(\mathcal{M}_{line+circle}, x) \models_{V_i} \alpha$, that is to prove the implication below:

$$(\mathcal{M}, x) \models_{V_i} \alpha \Rightarrow (\mathcal{M}_{line+circle}, x) \models_{V_i} \alpha.$$

For propositional variables, it is true by assumption and definitions. Inductive step is obvious for logical operations of classical logic. Consider modal and temporal logical operations: K_j, $\mathcal{N}$, AP_G and ULK_G.

Proof for K_j: Let $(\mathcal{M}, x) \models_{V_i} K_j\alpha$. Then for all $y \geq x$, $(\mathcal{M}, y) \models_{V_j} \alpha$. In this case by inductive assumption for all $y \geq x$, where $y \in \mathcal{M}_{line+circle}$,

$$\mathcal{M}_{line+circle}, y \models_{V_j} \alpha.$$

Therefore $\mathcal{M}_{line+circle}, x \models_{V_i} K_j\alpha$.

Proof for $\mathcal{N}$: Let $(\mathcal{M}, x) \models_{V_i} \mathcal{N}\alpha$. Therefore $(\mathcal{M}, Next(x)) \models_{V_i} \alpha$. Then by inductive assumption and our choice to merge $stable_0$ and $stable_2$ while construction of $\mathcal{M}_{line+circle}$ and choice there that

$$(\forall i \in [1,n])[(Theory_i^f(Next(stable_0)) = Theory_i^f(Next(stable_2)))$$

and

$$(Theory_i^f(stable_0) = Theory_i^f(stable_2)).$$

we have that $(\mathcal{M}_{line+circle}, x) \models_{V_i} \mathcal{N}\alpha$ what we needed.

Proof for AP_G and other remaining modal operation: immediately follows from definitions and inductive assumptions. Opposite direction : we need to show that

$$(\mathcal{M}_{line+circle}, x) \models_{V_i} \alpha \Rightarrow (\mathcal{M}, x) \models_{V_i} \alpha.$$

The argumentation for this proof is exactly as the proof for opposite case above using the inductive assumptions. □

Lemma 3. *$(\forall \alpha \in Sub(f))$ if, for some x $(\mathcal{M}_{line+circle}, x) \models_{V_i} \alpha$, for arbitrary finite model with structure of $\mathcal{M}_{line+circle}$, then α is satisfiable in some usual infinite model M_1 on the set of all natural numbers.*

Proof. It is sufficient to use well known unraveling technique. That is to tear the model upper cluster to the infinite future by rolling the final cluster to the future. □

Theorem 1. *The logic $\mathcal{L}(\mathcal{K})$ has the finite model property w.r.t. satisfiability. For a formula α to be satisfiable it is sufficient to check the satisfiability of α in the models of structure $\mathcal{M}_{line+circle}$ which all are finite.*

Proof. It follows directly from the lemmas 2 and 3 □

Based on this theorem, we cannot yet to concluded that the problem of determining whether a given formula is satisfiable is solvable. That is because we have yet only unbounded finite model property. The size of satisfying models not showed to be computable from the size of formulas for test. We fix this problem in the following section.

4. The problem of decidability

Theorem 2. *The satisfiability problem for the logic $\mathcal{L}(\mathcal{K})$ is decidable. For verification that a formula f is satisfiable it is sufficient to check its satisfiability in the models with structure of $\mathcal{M}_{line+circle}$ of size with a bound computable from the size of f.*

Proof. First we do calculate the total number of possible theories at the states x of the possible model of sort $\mathcal{M}_{line+circle}$ based on the number of sub-formulas of the formula f.

Let $Theory^f(x) = \{Theory^f_i(x)\}_{i=1}^n = \{Theory^f_1(x), ..., Theory^f_n(x)\}$.

$|Theory^f_i(x)| \leq |Sub(f)|$ and the number of possible theories of a single state cannot exceed $2^{|Sub(f)|}$. Total number then is at most: $2^{|Sub(f)|} \cdot 2^{|Sub(f)|} \cdot ... \cdot 2^{|Sub(f)|} = 2^{|Sub(f)| \cdot n}$

To prove this theorem, we present the following algorithm which is a sort of rarefication algorithm (because filtration technique do not work here).

Recall that our given finite model $\mathcal{M}_{line+circle} = \langle [1, stable_0] \cup C_{stable}$ has the base set $[1, stable_0] \cup [stable_0, stable_1] \cup [stable_1, stable_2]$. Recall that we merged the sates $stable_0$ and $stable_2$ in one state and put that $(stable_2)Next(stable_0 + 1)$.

Let Th be a set of all theories $Theory^f(x)$ which are true at some states x of $\mathcal{M}_{line+circle}$.

1. Consider the interval $[1, stable_0]$ and mark all maximal (by $\leq$) in this interval states which have theories form the set Th (mark one maximal state for each possible theory). The set of all marked states is finite with upper bound of its size computable from f (as we saw upper).

2. We do similar marking inside of each interval [stable0, stable1] and [stable1, stable2].

Notice now that if we will consider only subformulas of f not containing the operation next – $\mathcal{N}$, – then the truth values of all these subformulas at states of any model obtained from $\mathcal{M}_{line+circle}$ by deleting all states of any chosen set of states not containing marked already states, and at the these stats of the original model, – are the same. This follows by trivial induction proof by the length of formulas. But with presence of the operation $\mathcal{N}$, the situation is different and we need to perform more work.

3. Now moving from first internal $[1, stable_0]$ to up we will insert some necessary states. We start from 1 and will do some similar inserting in the following proof. Actually, assume that we did all necessary insertions until a marked state s_j and s_{j+1} is the next up marked state (at the beginning, (first step) $s_j = 1$). Consider the interval $[s_j, s_{j+1}]$. If $s_j Next s_{j+1}$ we inset in this interval nothing and continue to work with s_{j+1}. If not, we consider the maximal state s in $[s_j, s_{j+1}]$ such that $s_j Next q$ and

$$Theory^f(q) = Theory^f(s).$$

Now we delete all states situated between s_j and s. And then we continue to work with s as we did now with s_j. These transformation will work until we will reach s_{j+1}. It is easy to see by induction on the length of formulas that this transformation does not change truth values of the formulas at the sates of the model itself and its reduced part.

The size of the resulted interval $[s_j, s_{j+1}]$ will be not bigger then the set of the set of possible theories plus 2, so has a computable bounded as we noted above. Such way, we work with each interval $[s_j, s_{j+1}]$ in the part $[1, stable_0]$. The set of such intervals again is computable size bounded from the amount of possible theories. Then we do the similar transformation with intervals $[stable_0, stable_1]$ and $[stable_1, stable_2]$. As the result, we obtain a finite model with the size computable from the size of f and satisfying f. $\square$

Theorem 3. *The logic $\mathcal{L}(\mathcal{K})$ is decidable.*

5. Conclusion

In this short paper, a poly-modal linear multi-agent time logic based on $\mathbb{N}$ was investigated. The satisfiability problem for the formulas of this logic has been solved. An algorithm has been proposed that allows the collapse of any infinite model of such logic into a finite model for verifying satisfiability.

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