Серия «Математика»
2023. Т. 45. С. 104—120

Онлайн-доступ к журналу:
http://mathizv.isu.ru

ИЗВЕСТИЯ
Иркутского
государственного
университета

Research article

УДК УДК 519.63:681.51
MSC 65N30, 35Q79
DOI https://doi.org/10.26516/1997-7670.2023.45.104

Finite Element Modeling of Nonstationary Problems of Heat Conduction under Complex Heat Transfer

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Abstract. The article presents a numerical simulation of nonstationary heat conduction problems under complex heat transfer, which includes such heat transfer mechanisms as heat conduction, convection, and radiation. The Stefan-Boltzmann law describes the resulting heat transfer by radiation between two bodies, where the heat transfer coefficient is a function of the body surface temperature. An algorithm and software for solving the heat conduction problem using the finite element method were developed, and the influence of external impacts on the temperature field distribution in the vicinity of an insulated circular hole in the center of the body was studied. The temperature fields were investigated for various boundary conditions in the hole of the plate and the corresponding isotherms were given.

Keywords: heat transfer, nonstationary process, thermal conductivity, convection, radiation, isotherms, hole, algorithm, FEM

For citation: Ikramov A. M. Polatov A. M. Finite Element Modeling of Nonstationary Problems of Heat Conduction under Complex Heat Transfer. *The Bulletin of Irkutsk State University. Series Mathematics*, 2023, vol. 45, pp. 104–120.
https://doi.org/10.26516/1997-7670.2023.45.104

Научная статья

Конечно-элементное моделирование нестационарных задач теплопроводности при сложном теплообмене

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Аннотация. Представлено численное моделирование нестационарных задач теплопроводности при сложном теплообмене, включающем такие механизмы теплообмена, как теплопроводность, конвекция и излучение. Закон Стефана – Больцмана описывает результирующий перенос тепла излучением между двумя телами, где коэффициент теплопередачи является функцией температуры поверхности тела. Разработан алгоритм и программное обеспечение для решения задачи теплопроводности методом конечных элементов, исследовано влияние внешних воздействий на распределение температурного поля в окрестности изолированного круглого отверстия в центре корпуса. Исследованы температурные поля при различных граничных условиях в отверстии пластины и приведены соответствующие изотермы.

Ключевые слова: теплообмен, нестационарный процесс, теплопроводность, конвекция, излучение, изотермы, отверстие, алгоритм, МКЭ

Ссылка для цитирования: Ikramov A. M. Polatov A. M. Finite Element Modeling of Nonstationary Problems of Heat Conduction under Complex Heat Transfer // Известия Иркутского государственного университета. Серия Математика. 2023. Т. 45. С. 104–120.
https://doi.org/10.26516/1997-7670.2023.45.104

1. Introduction

The study of heat transfer processes has always played an important role in the development of engineering and natural sciences. At first, the studies were initiated mainly by the needs of the thermal power industry. Further, the development of aviation, nuclear energy, and rocket and space technology put forward new statements of heat transfer problems and, at the same time, new, more stringent requirements for the completeness and reliability of the predictive capabilities of the theory.

At present, the scope of intensive research and application of heat transfer phenomena has expanded enormously; it includes the leading areas of technology and basic natural sciences. The theoretical study of heat transfer processes is based on their numerical simulation. This became possible due to significant progress in the development of computational methods for solving problems for partial differential equations and an increase in the power of modern computers. Numerical modeling of heat transfer processes is currently becoming increasingly important due to the fact that modern science and technology require a reliable prediction of such processes, the experimental study of which is difficult and expensive.

The main mechanisms of heat transfer are conduction, convection, and radiation. In practice, it often happens that the thermal conductivity inside the body and near its boundaries is different. This difference is due to a change in the conditions for the occurrence of heat transfer processes and a change in the structure of the substance. External factors, such as irradiation, can have a significant effect on thermal conductivity. In semi-transparent media, thermal conductivity is accompanied by radiative heat transfer. The effective thermal conductivity observed in such media is the sum of thermal conductivity and radiative heat transfer.

The study in [2] presents a finite-volume method to calculate transient radiative transfer in two-dimensional irregularly shaped enclosures. The fully implicit scheme was used to discretize the transient term. The step spatial differencing schemes and CLAM were used in this study. The ability of the present formulation in modeling absorbing, emitting and anisotropically scattering media was examined using wall heat fluxes and incident radiation. In [18], an integral equation for transient radiative transfer in a 3D absorbing and anisotropically scattering medium was formulated. The method developed was applied to transient radiative transfer in 1D planar and 2D cylindrical linearly anisotropically scattering media under pulse radiation.

The study in [3] describes the simulation of heat transfer under stationary and non-stationary conditions. In the case of non-stationary heat transfer, the influence of masonry joints on the thermal resistance of the enclosing structure was considered. Thermal resistance was characterized by the range of temperatures and the estimated damping of the range of temperatures. A global solution to the two-dimensional model is constructed in [6] by improving the relative moisture diffusion equation on the basis of experimental results. To prove the existence of a global solution, the extension method of local solutions was used.

The study in [17] investigates the features of the distribution of a nonstationary temperature field over the thickness of a multilayer hollow cylinder under convective heat exchange on its surfaces, taking into account the presence of internal (distributed) heat sources. The study in [5] proposes an iterative method for constructing an approximate solution at a given time interval, based on the boundary element approach. An exact solution of parabolic type is obtained and investigated; it is reduced to integrating the Cauchy problem for an ordinary differential equation.

The study presented in [14] offers the development of a model that includes all standard transport equations and transient conditions to predict the behavior of the process and materials upon heating to high temperature in an enclosed vessel made of composite material. In [13], the authors provide a software application designed to study heat transfer problems. The application is used to calculate and visualize the temperature distribution in a flat plate body when it is heated or cooled. In [11], non-stationary heat transfer in agarose gels of various concentrations was studied to make a breakthrough in the technology of 3-D additive bioprinting.

The authors of [4] investigated special boundary value problems for the nonlinear parabolic heat-transfer equation. In the case of a power-law dependence of the thermal conductivity coefficient on temperature, the equation is used to describe the processes of radiative heat conductivity, polytropic gas filtration in porous soil, migration of biological populations, etc. Natural convection heat transfer in a partitioned square cavity is studied in [10] utilizing nanofluids. According to the results, Rayleigh's number and location of the partition are important factors that extremely affect the streamlines and isotherms.

Mathematical modeling of the heat transfer during the pyrolysis process used for the treatment of end-of-life tires is presented in [19]. To simplify the modeling, a hierarchy of 2D models for the temperature, which describes the non-stationary heat transfer in such a pyrolysis station, was created. The study in [8] presents the results of computer simulation of non-stationary temperature fields arising in polar dielectrics irradiated with medium-energy focused electron beams when studied using scanning electron microscopy techniques. The mathematical model is based on the solution of the multidimensional non-stationary equation of heat conduction by the numerical finite element method. In [12], a finite element model, based on the discrete ordinates method and least-squares variational principle is developed to simulate the transient radiative transfer in absorbing and scattering media. The numerical formulations and detailed steps are given.

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This article provides a numerical simulation of nonstationary heat conduction problems under complex heat transfer, which includes such heat transfer mechanisms as heat conduction, convection, and radiation. The resulting heat transfer by radiation between two bodies is described by the Stefan-Boltzmann law, where the heat transfer coefficient is a function of the body surface temperature.

The scientific novelty of the article is the development of a numerical model of a non-stationary process of complex heat transfer and a finite element mesh of multiply connected domains.

The purpose of this article is to develop a computational algorithm for a non-stationary process of complex heat transfer under mixed boundary conditions and to study the temperature field distribution based on a finite element model of a multiply connected body.

2. Formulation of the problem

The process of heat conduction in a homogeneous rigid body is considered in the article. The equation of the two-dimensional boundary value problem of nonstationary heat conduction in a rectangular Cartesian coordinate system has the following form [20], [16]:

$$K_{xx}\frac{\partial^2 T}{\partial x^2}+K_{yy}\frac{\partial^2 T}{\partial y^2}+w=\rho c\frac{\partial T}{\partial t}, \tag{2.1}$$

where

$T=T(x,y,t)$ is the temperature field in domain Ω;

K_{xx}, K_{yy} are the coefficients of thermal conductivity in directions *Ox, Oy*, respectively;

$w=w\,(x,y,t)-$ is the energy of heat sources inside the body;

ρ is the density of the body material;

c is the specific heat capacity of the body material.

To solve problem (2.1), it is necessary to specify the initial and boundary conditions.

The initial condition is the body temperature at some initial time t_0:

$$T_0=T(x,y,t_0). \tag{2.2}$$

The boundary condition of the first kind is temperature T_{s_1} set on a part of surface S_1:

$$T_{s_1}=T(x,y,t),\quad x,y\in S_1. \tag{2.3}$$

The boundary condition of the second kind is the heat flux of density q set on a part of surface S_2:

$$K_{xx}\frac{\partial T}{\partial x}\ell_x+K_{yy}\frac{\partial T}{\partial y}\ell_y=-q, \tag{2.4}$$

where ℓ_x, ℓ_y – the direction cosines of the outer normal to surface S_2; the heat flux is positive if heat is dissipated from the body. If surface S_2 is insulated, then $q=0$.

The boundary condition of the third kind is when the convective heat transfer occurs on a part of surface S_3:

$$K_{xx}\frac{\partial T}{\partial x}\ell_x+K_{yy}\frac{\partial T}{\partial y}\ell_y=-h\left(T_s-T_\infty\right), \tag{2.5}$$

where

T_s is the temperature on surface S_3;

T_∞ is the ambient temperature;

h is the heat transfer coefficient;

$q_h = h\left(T_s - T_\infty\right)$ is the density of the heat flux dissipated from the body surface due to convection.

Radiative heat exchange occurs on a part of surface S_4:

$$K_{xx}\frac{\partial T}{\partial x}\ell_x + K_{yy}\frac{\partial T}{\partial y}\ell_y = -\varepsilon\cdot\sigma_0\left(T_s^4 - T_\infty^4\right), \tag{2.6}$$

where T_s is the temperature on surface S_4, ε is the absorption coefficient of the surface, σ_0 is the Stefan-Boltzmann constant, $q_r = \varepsilon\cdot\sigma_0\left(T_s^4 - T_\infty^4\right)$ is the density of the heat flux dissipated from the surface of the body due to radiation.

To linearize the boundary condition, the radiative component q_r, is presented in a form similar to the convective component $q_r = \alpha_r\left(T_s - T_\infty\right)$, where the radiative heat transfer coefficient is $\alpha_r = \epsilon\cdot\sigma_0\left(T_s^3 + T_s^2 T_\infty + T_\infty^2 T_s + T_\infty^3\right)$ [15]. The value of α_r is calculated from the temperature values at the previous time iteration.

Problem (2.1)–(2.6) has a variational formulation: solving equation (2.1) with boundary conditions (2.2)–(2.6) is equivalent to finding the minimum of the functional at a fixed time point [9]:

$$\begin{aligned}\chi &= \int\limits_V \tfrac{1}{2}\left[K_{xx}\left(\tfrac{\partial T}{\partial x}\right)^2 + K_{yy}\left(\tfrac{\partial T}{\partial y}\right)^2 - 2\left(w - \rho c\tfrac{\partial T}{\partial t}\right)T\right]dV + \\ &+ \int\limits_{S_2} qT\,dS + \int\limits_{S_3}\tfrac{h}{2}\left(T - T_\infty\right)^2 dS + \int\limits_{S_4}\tfrac{\varepsilon\cdot\sigma_0}{2}\left(T^4 - T_\infty^4\right)^2 dS\end{aligned} \tag{2.7}$$

Functional (2.7) is used to substantiate the FEM in relation to two-dimensional problems of heat conduction.

3. Solution method

The finite element solution to the problem consists of two stages: construction of a discrete model of the structure and solution of the nonstationary problem of heat conduction.

3.1. Method for constructing a finite element mesh

The method for constructing a finite element mesh is based on the connection of elementary subdomains. An elementary subdomain is a domain for which there is an algorithm for constructing a finite element mesh. It is assumed that the topology of the model of a complex 2D domain is represented by a system of surfaces interconnected along the boundary lines and intersected at nodal points; the boundary lines may have a number of intermediate nodes; surfaces can be connected by multiple lines. Two lines must be connected so that one of them intersects the other at the endpoint.

The finite element mesh of the domain is described by the following discrete set

$$\Omega = \{n, m, K, M\},$$

where n is the number of mesh nodes; m is the number of finite elements; K is the set of coordinates of nodes; M is the set of node numbers by elements.

The following relationship is the condition for the coincidence of boundary nodes of two sets Ω_1 and Ω_2

$$\left|x_i^1 - x_j^1\right| < \varepsilon \quad \& \quad \left|x_i^2 - x_j^2\right| < \varepsilon,$$

where $(x_i^1, x_i^2) \in K_1$ is the set of coordinates of the nodes of set $\Omega_1 = \{n_1, m_1, K_1, M_1\}$, $i = 1, 2, \ldots, n_1$, $(x_j^1, x_j^2) \in K_2$ is the set of coordinates of the nodes of set, and $\Omega_2 = \{n_2, m_2, K_2, M_2\}$, $j = 1, 2, \ldots, n_2$, $\varepsilon > 0$ is a sufficiently small number.

When the topology conditions of the model of complex domain of connection of two subsets Ω_1 and Ω_2 are satisfied, the following resulting set is formed $\Omega = \Omega_1 \cup \Omega_2$, where $n = n_1 + n_2 - q$; $m = m_1 + m_2$; $K = K_1 \cup K_2'$; $M = M_1 \cup M_2'$; $q = K_1 \cap K_2$ is the number of nodes located on the boundary of connection of subdomains; K_2' is the set of coordinates of the nodes of subset Ω_2, not considering the nodes with the same coordinates; M_2' is the set of renumbered nodes of subset Ω_2.

To number the local node numbers of sets Ω_1 and Ω_2, the following sets of natural numbers are introduced, respectively:

$$N_1 = \{\, i \,|i \le n_1 \} \in N \quad \text{and} \quad N_2 = \{\, j \,|j \le n_2 \,\},\ j \in N.$$

Then the following sets are introduced: A and B $A = B = q$. The elements of these sets are local numbers of coinciding boundary nodes from the sets of nodes N_1 and N_2, respectively, satisfying the relation of the Cartesian product of sets:

$$A \times B = \left\{ (i, j)|i \in N_1 \ \& \ \exists j \in N_2 : \sum_{k=1}^{l} \left| x_i^k - x_j^k \right| < l\varepsilon \right\}, \text{where } l = 2.$$

Algorithm for constructing set M:

1) elements of set M_1 are assigned to the initial elements $k = 1, 2, \ldots, m_1$ of set M, i.e. $M_1 \subset M$;

2) subsequent elements $k = m_1 + 1, m_1 + 2, \ldots, m_1 + m_2$ of set M are formed by replacing the local node numbers of set M_2, with global

numbers, i.e. $M = M_1 \cup M_2'$, where M_2' is a set consisting of global node numbers. The process of calculating global node numbers is conducted as follows. If the local node number i of the set of natural numbers N_2 $(i = 1, 2, \ldots, n_2)$ belongs to set B, i.e. $i \in B$, then it is assigned the corresponding local node number from set A. Otherwise, its value is calculated on the basis of relation $i = i + n_1 - z$. The value of variable z is defined as the number of elements of set B, the value of which is less than the value of the current number i:

$$z = |\,Q(i; B)|, \text{ where } Q(i; B) = \{j \in B : j < i\}.$$

Algorithm for constructing set K:

1) elements of set K_1 are assigned to the initial elements $k = 1, 2, \ldots, n_1$ of set K, i.e. $K_1 \subset K$;

2) subsequent elements $k = n_1 + 1, n_1 + 2, \ldots, n_1 + n_2 - q$ of set K are formed from the elements of subset K_2, without considering the coordinates of the node numbers located in set B, i.e. $K = K_1 \cup K_2'$, where $K_2' \subset K_2$ and $K_2' = n_2 - q$.

Thus, set Ω is formed – a finite element representation of a multiply connected domain.

3.2. Method for solving the non-stationary heat conduction problem with complex heat transfer

The method for solving the problem is based on the variational grid generation of the heat conduction process, which satisfies the integral identity for the generalized solution to the problem on the set of piecewise polynomial grid functions. A triangular finite element with three nodes at the vertices was considered as an example (Fig. 1). On the side face jk, the load is a heat flux of density q, on the side face ij the load is a convective heat transfer q_h, and on the side face ik the load is a radiative heat transfer q_r; heat sources of energy w act inside the body. The thickness of the element is assumed unit. The temperature values at the nodes of the finite element e form vector $\{T\}_e^T = \{T_i, T_j, T_k\}_e$, and the nodal thermal loads form vector $\{Q\}_e^T = \{Q_i, Q_j, Q_k\}_e$.

The temperature inside the element is approximated by a linear polynomial:

$$T\,(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y. \tag{3.1}$$

Substituting the values of nodal temperature and the corresponding coordinates of the nodes in relation (3.1), we obtain three equations, from which expressions for coefficients α_1, α_2, α_3 are determined. Substituting the found values into (3.1), we obtain the following expression [16]:

$$\{T\,(x, y)\} = [N\,(x, y)]_e\,\{T\}_e\,. \tag{3.2}$$

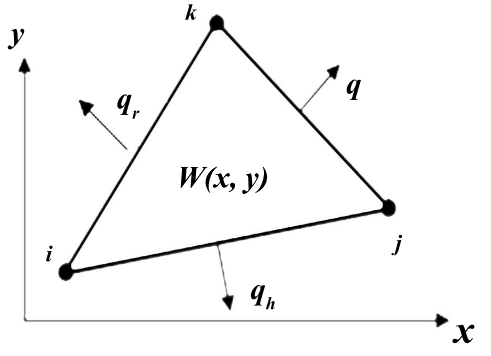

Figure 1. Triangular finite element

Differentiating expression (3.2) with respect to time, for element e, we have:

$$\frac{\partial T}{\partial t} = [N\,(x,y)]_e\,\frac{\partial}{\partial t}\,\{T\}_e\,. \tag{3.3}$$

Taking into account expressions (3.2) – (3.3), we write integral (2.7) over all finite elements:

$$\chi = \sum_{e=1}^{m} \chi_e = \sum_{e=1}^{m} \Bigg[\int\limits_{V_e} \frac{1}{2}\{T\}^T[B]_e^T[D]_e[B]_e\{T\}dV + \\ + \int\limits_{V_e} \rho c[N]_e\{T\}[N]_e\frac{\partial}{\partial t}\{T\}dV - \int\limits_{V_e} w[N]_e\{T\}dV \Bigg] + \\ + \sum_{e=1}^{m} \Bigg[\int\limits_{S_{2e}} q[N]_e\{T\}dS + \int\limits_{S_{3e}} \frac{h}{2}\{T\}^T[N]_e^T\,[N]_e\,\{T\}\,dS - \\ - \int\limits_{S_{3e}} hT_\infty[N]_e\{T\}dS + \int\limits_{S_{3e}} \frac{h}{2}T_\infty^2 dS + \int\limits_{S_{4e}} \frac{\alpha_r}{2}\,\{T\}^T\,[N]_e^T\,[N]_e\,\{T\}\,dS - \\ - \int\limits_{S_{4e}} \alpha_r T_\infty[N]_e\{T\}dS + \int\limits_{S_{4e}} \frac{\alpha_r}{2}T_\infty^2 dS \Bigg]. \tag{3.4}$$

Minimizing the functional (3.4) on the set of nodal values of the temperature field $\{T\}$, we obtain the following system of equations:

$$\frac{\partial \chi}{\partial\{T\}} = \frac{\partial}{\partial\{T\}}\sum_{e=1}^{m}\chi_e = \sum_{e=1}^{m}\frac{\partial \chi_e}{\partial\{T\}} = 0. \tag{3.5}$$

The contribution of the individual finite element e to the total sum (3.5) can be represented as a matrix differential relation:

$$\frac{\partial \chi_e}{\partial \{T\}} = [C]_e \frac{\partial}{\partial T}\{T\}_e + [K]_e\{T\}_e - \{Q\}_e^q - \{Q\}_e^g - \{Q\}_e^h - \{Q\}_e^r, \quad (3.6)$$

where the thermal conductivity matrix is:

$$[K]_e = \int\limits_{V_e} [B]_e^T [D]_e [B]_e dV + \int\limits_{S_{3e}} h[N]_e^T [N]_e dS + \int\limits_{S_{4e}} \alpha_r [N]_e^T [N]_e dS, \quad (3.7)$$

heat capacity matrix is:

$$[C]_e = \int\limits_{V_e} \rho c [N]_e^T [N]_e dV, \quad (3.8)$$

and vectors of nodal heat fluxes equivalent to the heat flux of density q, generation of heat sources w, convective heat transfer and radiative heat fluxes, respectively:

$$\{Q\}_e^q = -\int\limits_{S_{2e}} q[N]_e^T dS; \quad \{Q\}_e^g = -\int\limits_{V_e} w[N]_e^T dV;$$

$$\{Q\}_e^h = -\int\limits_{S_{3e}} hT_\infty [N]_e^T dS; \quad \{Q\}_e^r = -\int\limits_{S_{4e}} \alpha_r T_\infty [N]_e^T dS.$$

Summing up in (3.5) the contributions of all finite elements, a general system of differential equations is formed:

$$[C]\frac{\partial}{\partial t}\{T\} + [K]\{T\} = \{Q\}^q + \{Q\}^g + \{Q\}^h + \{Q\}^r, \quad (3.9)$$

where $[K]$, $[C]$ are the global matrices of thermal conductivity and heat capacity; $\{Q\}^q$, $\{Q\}^g$, $\{Q\}^h$, $\{Q\}^r$ are the global vectors of nodal heat loads equivalent to given surface heat fluxes, given generation of internal heat sources, convective heat and radiative heat flows, respectively. All thermal loads can be nonstationary.

The components of the matrices and vectors entering equation (3.9) are determined by summing over all the corresponding components with the same indices.

Next, we consider the solution to the matrix differential equation (3.9) by the finite-difference method, using the central-difference scheme. Equation (3.9) is written as:

$$[C]\,\frac{\partial}{\partial t}\{T\} + [K]\{T\} = \{Q\}, \quad (3.10)$$

where $[Q] = \{Q\}^q + \{Q\}^g + \{Q\}^h + \{Q\}^r$.

For the midpoint of the time interval $\Delta t = t_1 - t_0$, the derivative of the global vector $\{T\}$ is approximately represented by the following expression:

$$\frac{\partial}{\partial t}\{T\} = \frac{1}{\Delta t}\left(\{T\}_1 - \{T\}_0\right). \tag{3.11}$$

At the same midpoint of the time interval, the global vectors of nodal temperatures and thermal nodal loads are approximately calculated as:

$$\{T\} = \frac{1}{2}\left(\{T\}_1 - \{T\}_0\right) \tag{3.12}$$

$$\{Q\} = \frac{1}{2}\left(\{Q\}_1 - \{Q\}_0\right). \tag{3.13}$$

Substituting expressions (3.11) – (3.12) into differential equation (3.10), the following recursive relation is formed:

$$\left([K] + \frac{2}{\Delta t}[C]\right)\{T\}_1 = \left(\frac{2}{\Delta t}[C] - [K]\right)\{T\}_0 + 2\{Q\}. \tag{3.14}$$

Knowing the nodal temperatures at the beginning of the time interval, the nodal temperatures at the end of the time interval are determined by formula (3.14). Matrices $[K], [C], \{Q\}$ are calculated until equation (3.14) is solved, when the thermophysical properties (thermal conductivity coefficient, specific heat capacity, heat transfer coefficient under convection) do not depend on temperature. If the thermophysical properties depend on temperature, then equation (3.14) becomes nonlinear and must be solved by iterative methods.

If the load depends on temperature, then formula (3.14) becomes implicit and the solution is also obtained by iterations. The boundary conditions of the first kind, that is, the nodal temperatures specified on the surface of the model, are taken into account in equation (3.14). The boundary conditions of the second kind (2.4) – given heat fluxes and boundary conditions of the third kind (2.5) – convective flows are taken into account through the corresponding terms of the functional (3.4). In the nonstationary problem of heat conduction, the initial condition (2.2) is taken into account at the first step of calculations, as the values of the nodal temperatures at the beginning of the first time interval.

4. Algorithm and software for solving the problem

A computational algorithm for solving non-stationary problems of heat conduction with complex heat transfer has been developed based on the finite element method; it consists of the following parts:

1. Construction of a discrete model of a two-dimensional domain of complex configuration.

2. Calculation of the coefficients of the thermal conductivity matrix and the vector of thermal loads.

3. Summation over finite elements and construction of a resolving system of equations.

4. Accounting for the given boundary conditions of the first kind.

5. Solving the system of equations by the method of square roots.

6. Visualization of calculation results.

Based on the algorithm for solving the problem in the algorithmic language C++, a software package was developed that allows automation of the process of calculating the temperature field of a two-dimensional domain of a complex configuration and visualizing the calculation results.

5. Computational experiment

Consider as a test problem the process of heat transfer in a plate, on the two boundaries of which heat exchange with the external medium occurs due to radiation and convection (Fig. 2).

$$\rho c \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad \left| \frac{0 < x < L}{0 < y < H,} \right.$$

The initial and boundary conditions are written as:

$$t = 0: \quad T = T_0, \quad 0 \le x \le L, \quad 0 \le y \le H;$$

$$x = 0: \quad \lambda \frac{\partial T}{\partial x} = k_1 (T^{e1} - T) + \varepsilon \sigma ((T^{e1})^4 - T^4), \quad t > 0,\ k_1 > 0;$$

$$x = L: \quad \frac{\partial T}{\partial x} = 0, \quad t > 0;$$

$$y = 0: \quad \frac{\partial T}{\partial y} = 0, \quad t > 0;$$

$$y = H: \quad \lambda \frac{\partial T}{\partial y} = k_2 (T^{e2} - T) + \varepsilon \sigma \left((T^{e2})^4 - T^4 \right), \quad t > 0,\ k_2 > 0.$$

Plate dimensions are $L = H = 0.3\,m$. Plate material is hard rubber ($\lambda = 0.16\,W/(m \cdot {}^\circ C)$, $\rho = 1190\,kg/m^3$, $c = 1900\,J/(kg \cdot {}^\circ C)$). The initial temperature of the solution domain is $30^\circ C$, $k_1 = 50\,W/(m^2 \cdot {}^\circ C)$, $T^{e1} = 20^\circ C$, $\varepsilon = 0.8$, $k_2 = 35\,W/(m^2 \cdot {}^\circ C)$, $T^{e2} = 35^\circ C$.

To check the reliability of the developed computational algorithm and program, the test problem described above was solved. For comparison,

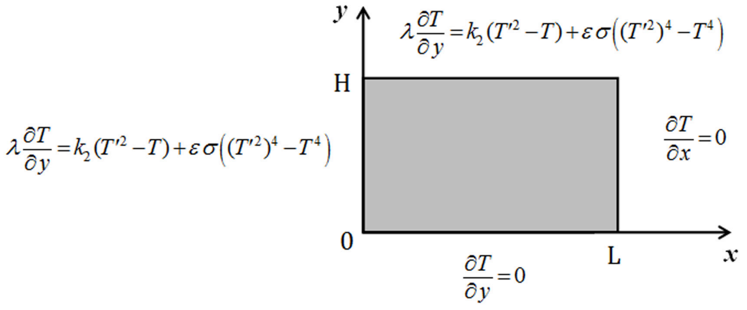

Figure 2. Domain of problem solution

Table 1 presents the results of calculations at the characteristic points of the domain, in case of the presence of a homogeneous plate at the boundaries: simultaneous heat transfer and radiation, and radiation and heat transfer.

Table 1

Temperature values at characteristic points of the domain

$t_n = 10\,h,\ \Delta t = 100\,s$	(0.0, 0.1552)	(0.1552, 0.3)	(0.1552, 0.1552)
heat transfer and radiation [7]	20.3645	34.7254	29.8328
heat transfer and radiation	20.3660	34.7302	29.9221
radiation	29.9896	30.0109	30.0000
heat exchange	20.3660	34.7301	29.9221

The presented results correspond to the boundary conditions of the problem and confirm the fact that at low temperatures of the body the effect of radiation on the temperature field is insignificant. Figure 3 shows a graphical representation of the temperature field.

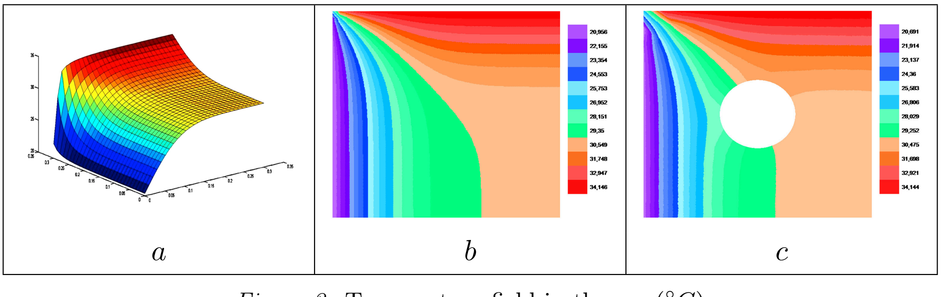

Figure 3. Temperature field isotherms ($^{\circ}C$)

To study the nonstationary heat conduction process under complex heat transfer ($t_n = 10\,h, \Delta t = 100\,s$), the above problem is solved for a plate with a round hole in the center.

The boundary conditions are:

I – the boundary of the hole is insulated (Fig. 3 c),

II – constant temperature of $20°C$ is set at the boundary of the hole (Fig. 4 *a*),

III – the heat flux $q = 10\,W/m^2$ is set on the right side of the plate (Fig. 4 *b*),

IV – a heat source $Q = 5\,W/m^3$ is set inside the plate (Fig. 4 *c*).

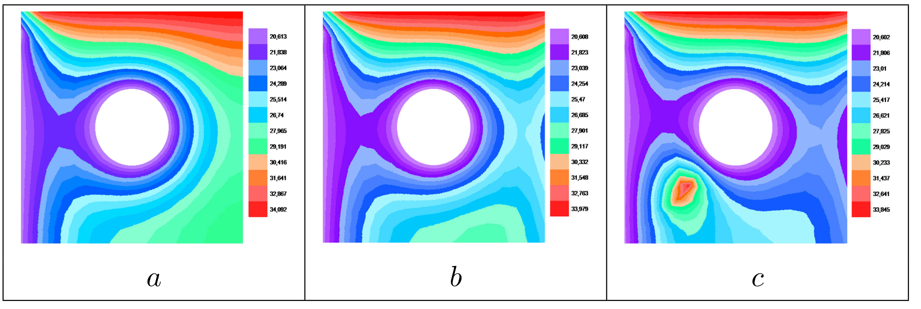

Figure 4. Temperature field isotherms ($°C$)

The presence of a round hole in the center of the plate, the boundaries of which are insulated, causes a slight change in the temperature around the hole. A constant temperature of $20°C$ at the hole boundary sharply lowers the temperature around the hole, and setting a heat flux of density $q = 10\,W/m^2$ on the right side of the plate leads to a further expansion of the domain with low temperatures. It should also be noted that the presence of a point heat source $Q = 5\,W/m^3$ inside the plate increases the temperature in the vicinity of its location.

To study the temperature field during the extension of finite time t_n, Table 2 shows the temperature values at the characteristic points of the plate (the location of the heat source).

In the plate, during the extension of finite time, a smooth cooling of the structure occurs, and in the presence of a constant temperature at the boundary of the hole and heat flow on the right side of the plate, the dynamics of the process is higher, while in the case of a solid plate and the presence of insulation of the boundary of the hole, the dynamics of the process is lower.

6. Conclusion

By conducting a computational experiment, it was established that:

Table 2

Temperature values at characteristic points of the domain

t_n, s	Homogeneous	Hole			
		I	II	III	IV
6000	29.98	29.88	29.89	29.88	31.41
12000	29.64	29.14	28.49	28.49	31.34
18000	29.12	28.40	27.17	27.17	31.30
24000	28.60	27.77	26.09	26.09	31.19
30000	28.13	27.24	25.21	25.21	31.09
36000	27.71	26.79	24.49	24.49	31.01

1. Verification of calculation results is numerically shown by the test problem.

2. Given the initial parameters: the presence of a constant temperature at the boundary of the hole sharply lowers the temperature in the vicinity of the hole, and setting the heat flux on the right side of the plate leads to a further expansion of the area with low temperatures.

3. In the process of increasing the finite amount of calculation time, the structure is smoothly cooled, and in the presence of a constant temperature at the hole boundary and heat flow on the right side of the plate, the dynamics of the process is higher, while with a solid plate and the presence of insulation of the hole boundary, it is lower.

4. A computational algorithm for constructing a finite element model of a multiply connected domain was developed.

5. A computational algorithm for a non-stationary process of complex heat transfer under mixed boundary conditions was developed and the temperature field distribution was studied on the basis of a finite element model of a multiply connected body.

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Поступила в редакцию / Received 29.12.2022
Поступила после рецензирования / Revised 24.04.2023
Принята к публикации / Accepted 04.05.2023