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Satisfiability Problem in Interval FP-logic

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Abstract. The article investigates the interval modal logic, in which an action of the modal operator $\Diamond$ is limited by the boundaries of an interval. In addition, the language of modal logic is extended by the operator $D(\alpha, \beta)$, the truth of which is determined qualitatively: it is true only if the number of points on the interval $[c_i; c_{i+1}]$ where the formula α is true is strictly less than the number of points in this segment where the formula β is true. The problem of satisfiability of formulas is solved, and as a consequence, the decidability of logic.

Keywords: modal logic, frame and model Kripke, satisfiability problem

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Научная статья

Проблема выполнимости в интервальной FP-логике

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Аннотация. Исследуется интервальная модальная логика, в которой действие модального оператора $\Diamond$ ограничено границами интервала. Кроме того, язык модальной логики расширен оператором $D(\alpha, \beta)$, истинность которого определяется качественно: он истиннен, только если число точек на отрезке $[c_i, c_{i+1}]$, в которых истинна формула α, строго меньше числа точек этого отрезка, в которых истинна формула β. Решается проблема выполнимости формул, и как следствие, разрешимость логики.

Ключевые слова: модальная логика, фрейм и модель Крипке, выполнимость формул, проблема выполнимости в логике, разрешимость логики

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1. Introduction

Nowadays Symbolic Logic is an active tool in research concerning Information Sciences and Artificial Intelligence. Several domains are in active use, for example these are temporal logic [4;5;12], multi-modal logics, multi-agent logics [1–3; 6–9]. For example linear temporal logic LTL after its invention by Pnually [12] got to be very popular (cf. [6; 7; 15; 16; 18–20]. Multi-agent logics used in areas of analysis of information on reliability and safety by many authors (cf. [21; 22]).

Therefore, instruments of temporal logic are rather popular in such kind of research and usually to be combined with elements of multi-agency, parallel computing and multi-agent logics (in a sense of a multi-modal logics). It seems the first substantive example of a two-modal logic is Arthur Prior's tense logic, with two modalities, F and P, corresponding to "sometime in the future" and "sometime in the past". A logic with infinitely many modalities is dynamic logic, introduced by Vaughan Pratt in 1976, it has a separate modal operator for every regular expression.

In multi-agents' logic modalities are interpreted often as agent's temporal accessibility operations, or the ones oriented to model checking, they were used widely for study interaction and autonomy, effects of cooperation (cf. e.g. Babenuschev and Rybakov [1–3], Woldridge and Lomuscio [21], Woldridge [22], Lomuscio et al [9], Rybakov [16; 18]).

Working with knowledge representation also often deals with analysis of information by logical instruments (e.g. description logics) close to temporal and modal logics (cf. Horrocks, Satler et al [11; 13], Baader et al [6–8]). Representation of agents interaction (as a dual of common knowledge) was suggested also using it as a base of agents' knowledge (S5-like) modalities. Knowledge, as a concept itself, came from multi-agency, since individual knowledge may be received only from interaction of agents, learning.

In this paper we wish to touch rather novel approach to information concerning reasoning about implicit knowledge on truth of statements, which might be expressed by modal like logical information. This considers a comparison which statements are looking more close to be true regarding them to each other. We use a language of modal logic extended by a new logical operation $D(\alpha, \beta)$ which compare the truth of formulas α and β on time intervals in terms the possible amount of states where they may be true. Mathematically we work with ways to invent algorithms recognizing satisfiability of such modal formulas and find a deciding algorithm.

2. Denotation, Preliminary Facts

We assume our reader to be familiar with the algebraic and Kripke semantics for modal logics and to have basic knowledge concerning inference rules and their admissibility (though we briefly recall all necessary facts below). Following modern trends by a *logic* we understand the set of all theorems provable in a given axiomatic system, or the set of valid formulas for a certain class of Kripke frames. In particular, a normal modal logic λ is a set of modal formulas which is closed under substitution, modus ponens $\{\alpha,\ \alpha \to \beta/\beta\}$ and necessitation rule $\{\alpha\ /\ \Box\alpha\}$, and contains all theorems of the minimal propositional modal logic K. In what follows by a modal logic we understand an algebraic propositional logic extending $S4$.

The language of modal logics consists of a countable set of propositional variables $p_1, \dots; p_n \dots$, logical connectives of classical logic $\neg, \wedge, \vee, \to$ and the unary modal operator $\Diamond$. A normal modal logic is a set of modal formulas L that contains all propositional tautologies, an axiom scheme $\Box(\alpha \to \beta) \to (\Box\alpha \to \Box\beta)$, and closed with respect to substitutions, the modus ponens $\{\alpha,\ \alpha \to \beta/\beta\}$ and necessitation rule $\{\alpha\ /\ \Box\alpha\}$. The minimal modal logic is denoted as the logic K. If L is a normal modal logic, then for formulas $\alpha \in L$ we write $\vdash_L \alpha$ or $L \vdash_L \alpha$ (i.e. there is a theorem of logic - it is deduced from the axioms using the postulated rules of inference). If the logic L is fixed or clear from the context, then we denote $\vdash \alpha$ for simplicity.

A frame $\mathcal{F} := \langle F, R\rangle$ is a pair, where F is a non-empty set and R is a binary relation on F. The frame and its underlying set are often denoted by the same letter for simplicity. Further, we consider only frames where R is a transitive and reflexive relation.

A model is a triple $\mathcal{M} = \langle W,\ R,\ V\rangle$, where $\mathcal{F} := \langle F, R\rangle$ is a frame and V is a valuation of a set of propositional letters P in the frame $\mathcal{F}$ that is $V : P \to 2^W$. $Dom(V) = P$ is called the domain of V.

A frame $\mathcal{F} = \langle F, R\rangle$ is called an open subframe of frame $\mathcal{G} = \langle G, R\rangle$ (denoted $\mathcal{F} \sqsubseteq \mathcal{G}$) if $F \subseteq G$ and $\forall a \in F \forall b \in G\,(aRb \Longrightarrow b \in F)$ holds. If $\mathcal{M}_1 = \langle W_1,\ R_1,\ V_1\rangle$, $\mathcal{M}_2 = \langle W_2,\ R_2,\ V_2\rangle$ are models then we call $\mathcal{M}_1$ an open submodel of $\mathcal{M}_2$ (denoted $\mathcal{M}_1 \sqsubseteq \mathcal{M}_2$) if : 1) $\langle W_1,\ R_1\rangle$ is open subframe of $\langle W_2,\ R_2\rangle$; 2) $Dom(V_1) = Dom(V_2)$ and $\forall p \in Dom(V_1)\ V_1(p) = V_2(p) \cap W_1$.

A mapping $f : \langle F, R\rangle \to \langle G, S\rangle$ is called a p-morphism if (1) $aRb \Longrightarrow f(a)Sf(b)$; (2) $f(x)Sz \Longrightarrow \exists y \in F : f(y) = z \,\&\, xRy$.

We say a mapping $f : \mathcal{M}_1 = \langle W_1,\ R_1,\ V_1\rangle \to \mathcal{M}_2 = \langle W_2,\ R_2,\ V_2\rangle$ is a p-morphism from the model $\mathcal{M}_1$ into the $\mathcal{M}_2$ if 1) f is a p-morphism of the frame $\mathcal{F}_1 = \langle W_1,\ R_1\rangle$ into the frame $\mathcal{F}_2 = \langle W_2,\ R_2\rangle$; 2) the valuations $V_1,\ V_2$ are defined on the same set of propositional letters; 3) $\forall\, p \in Dom(V_1), \forall a \in W_1 (a \models_{V_1} p \iff f(a) \models_{V_2} p)$.

We say a frame F is an λ–frame for a logic λ if all theorems of λ are valid at F, and $\lambda(F)$ – the set of all formulas valid in F – is the logic generated by F.

A logic λ satisfies the finite model property (FMP) if for any $\alpha \notin \lambda$ there exists a finite λ-model on which α is not valid.

We fix an interval partitioning In of the set of natural numbers

$$N = \bigcup_{i \in In} [c_i, c_{i+1}], c_i \ < \ c_{i+1}.$$

The interval linear FP-frame represents a special structure

$$\mathcal{F}^{FP} = \langle \bigcup_{i \in In} [c_i, c_{i+1}]$$

where $\leq$ is the standard linear order relation on natural numbers. The language of our logic is the standard language of modal propositional logic extended by a binary logical operation $D(x, y)$, which may be applied to any (arbitrary) formulas - cf. - $D(\alpha, \beta)$.

The valuations V of a set $Prop$ of propositional variables in frames $\mathcal{F}^{FP}$ from models $\mathcal{M}^{FP}$, here $\forall p \in Prop\ V(p) \subseteq \bigcup_i [c_i, c_{i+1}]$. The basis of any such model is $\bigcup_i [c_i, c_{i+1}]$ - the set of natural numbers N. If $a \in \mathcal{M}^{FP}$, and $a \in V(p)$ then write $(\mathcal{M}^{FP}, a) \models_V p$ and say that p is true on element a when V is valued.

Valuations of variables $Prop$ to be expanded to formulas as follows:

- Truth of Boolean connectives $\neg, \wedge, \vee, \rightarrow$ is defined in the standard way;
- $(\mathcal{M}^{FP}, x) \models_V \Box\alpha \iff \exists i \in N : x \in [c_i, c_{i+1}] \Longrightarrow \forall\ y \in [c_i, c_{i+1}]\ (x \leq y \leq c_{i+1} \Longrightarrow y \models_V \alpha)$;
- $(\mathcal{M}^{FP}, x) \models_V \Diamond\alpha \iff \exists i \in N : x \in [c_i, c_{i+1}] \Longrightarrow (\exists y \in [c_i, c_{i+1}]\ (x \leq y \leq c_{i+1}\ \&\ (\mathcal{M}^{FP}, y) \models_V \alpha))$;

– $(\mathcal{M}^{FP}, x) \models_V D(\alpha,\beta) \iff \exists i \in N : x \in [c_i, c_{i+1}]$ *and* the number of states on the segment $[c_i, c_{i+1}]$ in which the formula α is true, is strictly less then the number of states of this segment at which the formula β is true.

So, we define the modal operator $\Diamond$ almost as usual, but we limit his action by the right end of the segment – "Focus Point" of available information. The binary logical operation $D(\alpha,\beta)$ can be interpreted as an expert assessments at this stage interval; that is a sort of comparision statements in an implicite situation – when precise amount of states where the statements are true is not known in precise numerical value.

The introduced constraint on the $\Diamond$ modal operator causes, leads, for example, to the implementation on the introduced $\mathcal{M}^{FP}$ models of the following properties':

(1) $\Box p \wedge \Diamond\Diamond\neg p$, and as consequence $\Box p \wedge \Diamond^k \neg p$, $k > 1$.
(2) $\Box\neg p \wedge \Diamond\Box p$.
(3) $D(\alpha,\beta) \wedge \Diamond\Box\neg D(\alpha,\beta)$.

In what follows we will consider only formulas where the operation $D(\alpha,\beta)$ may occur only once. The reason for that is a technical one which we will comment after all main proofs.

Definition 1. *The logic L^{FP} is the set of all such formulas that are true on all models $\mathcal{M}^{FP}$ defined above.*

3. Satisfiability problem.

Recall some definitions. Let some class of models K and some formula be given. A formula is said to be satisfiable (in a given class K) if it is true at some element of some model from this class. In our case for logics L^{FP} the class of models is given by the set of models of the form $\mathcal{M}^{FP}$. Thus, a formula ϕ is satisfiable in logic L^{FP} if there exists a specific model $\mathcal{M}^{FP}$ with some valuation V, such that for some element $x \in \mathcal{M}^{FP}$ of this model $(\mathcal{M}^{FP}, x) \models_V \phi$ holds. Satisfiability is directly related to decidability: the logic is decidable if there is an algorithm that checks for any given formula ϕ whether $\phi \in L^{FP}$ is true. It is clear that $\phi \in L^{FP}$ if and only if the formula $\neg\phi$ is not satisfiable. And vice versa, ϕ is satisfiable if and only if $\neg\phi \notin L^{FP}$.

Thus, it is necessary to find an algorithm that checks the satisfiability of formulas at such finite models, where the size of which is limited by the value of some computable function from the length of formulas ϕ.

Theorem 1. *A formula f is satisfiable in a model $\mathcal{M}^{FP} = \langle \bigcup_{i \in In}[c_i, c_{i+1}]\rangle$ iff it is satisfiable in a model based at a frame of kind $\mathcal{F}^{FP}$ in the interval $[c_1, c_2]$ size of which is computable from the size of f.*

Proof. To start proof, let a modal formula f is given and it has propositional variables $Prop(f) = \{p_1, p_2, \dots, p_k\}$. Assume that this formula is satisfiable at some element x of the model $\mathcal{M}^{FP}$, e.g $(\mathcal{M}^{FP}, x) \models_V f$. Let us show that in this case the formula f also holds on some finite model $\mathcal{M}_c^{\mathcal{M}}$, whose structure and size of this model will be determined later in the course of the proof. Without loss of the generality, we can assume that $x \in [1, c_1]$.

We denote by $Sub(f)$ the set of all subformulas of the formula f, so $Sub(f)$ is closed w.r.t subformulas. Let $SR := \{A_1, A_2, \dots A_k\} \subseteq Sub(f)\}$ be the set of all sets of all subformulas of the formula f, which are true at some elements of the interval $[1, c_1]$.

That is, for each subset A_i, all formulas of A_i are true on some element x_i of $[1, c_1]$, and for any subformula of f if it is true at this element x_i then it belongs to A_i. For each such subset A_i, choose and fix the $\leq$–maximal element x_i with this property from the given segment $[1; c_1]$. Let's arrange all these elements x_i in $\leq$-increasing order: $t_{i_0} < t_{i_1} < \dots < t_{i_k}$. It is clear that the number of such elements is not greater than $2^{|Sub(f)|}$. Let M_1 be the set of all shown states $t_{i_0} < t_{i_1} < \dots < t_{i_k}$.

Lemma 1. *For any submodel M_2 of our model $M = \mathcal{M}^{FP}$ containing M_1 and for all formulas g from $Sub(f)$ without subformulas of sort $D(\alpha, \beta)$ the truth values of g at states from M_2 and the original model M are the same.*

Proof. It is an immediate induction of the length of formulas. Indeed, for Boolean formulas inductive steps are obviously the case (we keep the valuation when thinning).

Consider the case $(M, t) \models_V \Diamond\phi$. By definition of truth for the modal operator $\Diamond$ this means:

$$\exists b \in [1, c_1] : t \leq b \leq c_1 \ \& \ b \models_V \phi.$$

If $\exists j : b = t_j \in M_1$ then everything is clear, by the inductive hypothesis $(M_2, b) \models_V \phi$ and $(M_2, t) \models_V \Diamond\phi$, the truth of the formula is preserved.

Consider now the case (*) when no such b, and for all such existing b, where $b \models_V \phi$, $b \neq t_j$ for all $t_j \in M_1$.

Consider any b where $t \leq b$ and $b \models_V \phi$. It is clear that for some subset $A_b \in SR$ all formulas of A_b are true at b, and for any subformula of f if it is true at b then it belongs to A_s. But then, due to the choice of elements t_i, $\leq$-maximal element for this A_b was chosen, and the case (*) is impossible. So the statement $(M, t) \models_V \Diamond\phi$ give us $(M_2, t) \models_V \Diamond\phi$ is proven.

Now let $(M_2, t) \models_V \Diamond\phi$. Then

$$\exists b \in [1, c_1] : t \leq b \leq c_1 \ \&\ (M_2, b) \models_V \phi.$$

By the inductive hypothesis we get $(M, b) \models_V \phi$ and hence $(M, t) \models_V \Diamond\phi$. This concludes the proof of Lemma 1. □

Lemma 2. *Let M_3 be any submodel of $M := \mathcal{M}^{FP}$ containing model on M_1 (from previous lemma). Then we may effectively extend M_1 to a model M_3, which has the following property: for any subformula β of f, for any state $x \in M_3$, the truth of β at x in M_3 and M is the same.*

Proof. At the beginning consider itself the model M_1 from the previous lemma. Then all formulas from the set $Sub(f)$ of all subformulas of formula f without subformulas of sort $D(\alpha, \beta)$ have the same truth values at any state of M_1 and M by Lemma 1. For all subformulas of f without subformulas of type $D(\alpha_0, \beta_0)$ the statement of our current lemma holds.

Consider now a possible subformula $D(\alpha_0, \beta_0)$ of the formula f (notice that then formulas α_0 and β_0 do not have occurrences of subformulas of kind $D(\alpha_1, \beta_1)$). In this stage we possibly will extend the model.

If the formula $D(\alpha_0, \beta_0)$ is true at a state from $[1, c_1]$ in M and it is true in the model M_1, we do not add to the model M_1 anything.

If the formula $D(\alpha_0, \beta_0)$ is true at a state from $[1, c_1]$ in M but it is not the case for the current model M_1 then the following holds. The amount of states in $[1, c_1]$ where α is true in M_1 is not less then amount of states in $[1, c_1]$ where β is true in M_1.

This means that in the original model M there were some additional amount of states in $[1, c_1]$ where the formula β is true in M and this made the formula $D(\alpha_0, \beta_0)$ to be true at M. Then we may add these additional states to the model M_1, by Lemma 1 this does not change the truth values of subformulas of f with no formulas starting with D and this will make the formula $D(\alpha_0, \beta_0)$ true in the extended model.

Notice that we compare the amount of states in M_1, and those where α and (or) β_0 is true in M. And the amount of states where α is true in M_1 is finite and not bigger than the size of M_1 itself. Therefore we may need add at most such much states for β as big is M_1. After this extension of the formula $D(\alpha_0, \beta_0)$ will be true at the obtained model.

If the formula $D(\alpha_0, \beta_0)$ is not true at a state from $[1, c_1]$ in M our extension the model works similar as above (but so to say in an opposite site). This concludes the proof of our lemma 2 and theorem from left to right.

The opposite is obviously true: if the formula f is satisfiable on a finite model based at the frame of kind $\mathcal{F}_c^{FP}$ with the base set $[1, c_1]$ for some valuation V, then this formula is also satisfiable on some infinite model based on frame of kind $\mathcal{M}^{FP}$.

The base set of this model $\mathcal{M}^{FP}$ is defined as

$$\mathcal{F}^{FP} = [1, c_1] \cup \bigcup_{2 \leq i} [c_i, c_{i+1}].$$

Valuation of V into segments $[1, c_1]$ is transferred from the model $\mathcal{M}_c^{FP}$, on the remaining segments $[c_i, c_{i+1}], 2 \leq i$ of arbitrary length, all variables of the formula f are considered true. The theorem 1 has been proven. □

Therefore as a consequence we have

Theorem 2. *The problem of satisfiability for formulas in the logic L^{FP} is decidable.*

4. Conclusion

In this short paper we investigate the interval modal logic, in which the action of the modal operator $\Diamond$ is limited by the boundaries of the interval. In addition, the language of modal logic is extended by the operator $D(x; y)$, the truth of which is determined implicitly and in a sense qualitatively: it is true only if the number of points on the interval $[c_i; c_{i+1}]$ where the formula x is true is strictly less than the number of points in this segment where the formula y is true. The problem of satisfiability for formulas is solved, and as a consequence, the decidability of logic. But we have to confess that we do not allow nested or repeated occurrences of the operation $D(x; y)$ in formulas. Following our scheme of proof this would allow appearance of infinite loop, and it would break our proof. So, the problem with nested occurrences is yet open.

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