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Kinds of Pregeometries of Cubic Theories

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Abstract. The description of the types of geometries is one of the main problems in the structural classification of algebraic systems. In addition to the well-known classical geometries, a deep study of the main types of pregeometries and geometries was carried out for classes of strongly minimal and ω-stable structures. These studies include, first of all, the works of B.I. Zilber and G. Cherlin, L. Harrington, A. Lachlan in the 1980s. Early 1980s B.I. Zilber formulated the well-known conjecture that the pregeometries of strongly minimal theories are exhausted by the cases of trivial, affine, and projective pregeometries. This hypothesis was refuted by E. Hrushovski, who proposed an original construction of a strongly minimal structure that is not locally modular and for which it is impossible to interpret an infinite group. The study of types of pregeometries continues to attract the attention of specialists in modern model theory, including the description of the geometries of various natural objects, in particular, Vamos matroids.

In this paper we consider pregeometries for cubic theories with algebraic closure operator. And we notice that for pregeometries $\langle S, \mathrm{acl}\rangle$ in cubic theories, the substitution property holds if and only if the models of the theory do not contain infinite cubes, in particular, when there are no finite cubes of unlimited dimension. By virtue of this remark, we introduce new concepts of c-dimension, c-pregeometry, c-triviality, c-modularity, c-projectivity and c-locally finiteness. And besides, we prove the dichotomy theorem for the types of c-pregeometries.

Keywords: pregeometry, cubic theory, c-pregeometry, c-triviality, c-modularity, c-projectivity, c-locally finiteness

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Научная статья

Виды предгеометрий кубических теорий

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Аннотация. Описание видов геометрий является одной из основных задач при структурной классификации алгебраических систем. Помимо известных классических геометрий глубокое исследование основных видов предгеометрий и геометрий проводилось для классов сильно минимальных и ω-стабильных структур. К этим исследованиям необходимо отнести прежде всего работы Б. И. Зильбера и Г. Черлина, Л. Харрингтона, А. Лахлана 1980-х гг. В начале 1980-х гг. Б. И. Зильбером была выдвинута известная гипотеза о том, что предгеометрии сильно минимальных теорий исчерпываются случаями тривиальных, аффинных и проективных предгеометрий. Эта гипотеза была опровергнута Э. Хрушовским, который предложил оригинальную конструкцию сильно минимальной структуры, не являющейся локально модулярной и для которой невозможно проинтерпретировать бесконечную группу. Исследование видов предгеометрий продолжает привлекать внимание специалистов по современной теории моделей, включая описание геометрий различных естественных объектов, в частности, матроидов Вамоса.

В настоящей работе дается классификация предгеометрий для кубических теорий с алгебраическим оператором замыкания. Устанавливается, что для предгеометрий $\langle S, \mathrm{acl}\rangle$ в кубических теориях выполняется свойство замены тогда и только тогда, когда модели теории не содержат бесконечных кубов, в частности, когда нет конечных кубов неограниченной размерности. В силу этого свойства мы вводим новые понятия c-размерности, c-предгеометрий, c-тривиальности, c-модулярности, c-проективности и c-локально конечности. Кроме того, доказываем теорему о дихотомии для типов c-предгеометрий.

Ключевые слова: предгеометрия, кубическая теория, c-предгеометрия, c-тривиальность, c-модулярность, c-проективность, c-локально конечность

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1. Introduction

The structural classification of algebraic systems involves the description of the types of their geometries. In addition to the well-known classical geometries, an in-depth study of the main types of pregeometries and geometries was carried out for classes of strongly minimal and ω-stable structures.These studies should include, first of all, the works of B.I. Zilber [14–17] and G. Cherlin, L. Harrington, A. Lachlan [4] of the 1980s. The classification of ω-categorical strictly minimal sets is a fundamental result that was established by G. Cherlin and B.I. Zilber. Such objects turned out to be either (infinite-dimensional) affine or projective spaces over finite fields, or (infinite) indistinguishable sets. In the 1970s, B.I. Zilber obtained a number of results and formulated hypotheses about uncountably categorical theories, among which the key hypothesis was the possibility of classifying such theories with accuracy up to b and interpretability. In the early 1980s, B.I. Zilber put forward a well-known hypothesis that the pregeometries of strongly minimal theories are exhausted by the cases of trivial, affine and projective pregeometries.This hypothesis was refuted by E. Hrushovski [7], who proposed an original construction of a strongly minimal structure that is not locally modular and for which it is impossible to interpret an infinite group. The study of the types of pregeometries continues to attract the attention of specialists in modern model theory, including the description of the geometries of various objects [1–3], in particular, Vamos matroids [9]. Natural questions arise about the classification of pregeometries and geometries for various significant classes of structures and their theories.

In this paper, we find conditions for the types of pregeometries of cubic theories with an algebraic closure operator. Definitions of pregeometry and its types are taken from the work, and [10] cubic theory from [11;12]. Based on them, we introduce new concepts of types of pregeometries, such as c-modular c-pregeometry, and give a description of types of pregeometries for cubic theories.

2. Kinds of pregeometries of cubic theories

From the works [4–6; 8; 10–12] and [13] we will take the definitions we need.

Definition 1. *A pregeometry is a set S together with a certain closure operation* $\mathrm{cl} : P(S) \to P(S)$ *satisfying the following conditions:*

1) for anyone $X \subseteq S$ in progress $X \subseteq \mathrm{cl}(X)$;

2) for anyone $X \subseteq S$ in progress $\mathrm{cl}(\mathrm{cl}(X)) = \mathrm{cl}(X)$;

3) for anyone $X \subseteq S$ and any $a, b \in S$ if $a \in \mathrm{cl}(X \cup \{b\}) - \mathrm{cl}(X)$ then $b \in \mathrm{cl}(X \cup \{a\})$;

4) for anyone $X \subseteq S$ if $a \in \mathrm{cl}(X)$ then $a \in \mathrm{cl}(Y)$ for some finite $Y \subseteq X$.

In the presence of pregeometry $\langle S, \mathrm{cl}\rangle$, each subset $X \subseteq S$ has a minimum set $X' \subseteq X$ such that $\mathrm{cl}(X) = \mathrm{cl}(X')$. This minimal set X' is called the basis of the set X. In this case, the power $|X'|$ does not depend on the choice of the basis in the X, and this power is called the dimension of the set X in the pregeometry $\langle S, \mathrm{cl}\rangle$, denoted by $\dim(X)$.

By definition, we have $\dim(X) = \dim(\mathrm{cl}(X))$, i.e. the dimension is preserved during the transition to the closure of the set X in the pregeometry $\langle S, \mathrm{cl}\rangle$.

If $\dim(X) \in \omega$ then set X called finite.

Definition 2. *A pregeometry $\langle S, \mathrm{cl}\rangle$ is called trivial or degenerate if for any $X \subseteq S$, $\mathrm{cl}(X) = \bigcup\{\mathrm{cl}(\{a\}) \mid a \in X\}$.*

A pregeometry $\langle S, \mathrm{cl}\rangle$ is called modular if for any closed sets $X_0, Y_0 \subseteq S$, X_0 independently of Y_0 with respect to $X_0 \cap Y_0$, i.e. for any finite-dimensional closed sets $X \subseteq X_0$, $Y \subseteq Y_0$ is true

$$\dim(X) + \dim(Y) - \dim(X \cap Y) = \dim(X \cup Y).$$

A pregeometry $\langle S, \mathrm{cl}\rangle$ is called a local modular if, for any $a \in S$, the pregeometry $\langle S, \mathrm{cl}_{\{a\}}\rangle$ is modular, where $\mathrm{cl}_{\{a\}}(X) = \mathrm{cl}(X \cup \{a\})$.

A pregeometry $\langle S, \mathrm{cl}\rangle$ is called projective if it is modular and not trivial, and locally projective if it is locally modular and not trivial.

A pregeometry $\langle S, \mathrm{cl}\rangle$ is called locally finite if, for any finite subset $A \subseteq S$, the set $\mathrm{cl}(A)$ is finite.

Definition 3. *Let S be a model of the theory of T. Then the algebraic closure operator for the model M is an operator $\mathrm{acl} : P(M) \to P(M)$ such that for any subset $X \subseteq S$, $\mathrm{acl}(X) = \{a \in S \mid S \models \exists^{<\omega} x\phi(x, \bar{b}) \wedge \phi(a, \bar{b})$ for some formula $\phi(x, \bar{y})$ and $\bar{b} \in X\}$.*

Below pregeometries of the form $\langle S, \mathrm{acl}\rangle$ will be considered.

Definition 4. *Let's call an n-dimensional cube, or n-cube, any graph isomorphic to a graph with a carrier $\{0, 1\}^n$ in which two vertices $(\delta_1, ..., \delta_n)$ and $(\delta'_1, ..., \delta'_n)$ are adjacent if and only if they differ by exactly one coordinate. In this case, the described graph Q_n with carrier $\{0, 1\}^n$ is called a canonical representative of the class of n-cubes.*

Definition 5. *Let λ be some infinite cardinal. Let's call an λ-dimensional cube, or an λ-cube, any graph isomorphic to a graph $Q = \langle X; R\rangle$ satisfying the following conditions:*

1) the carrier $X \subseteq \{0, 1\}^\lambda$ is generated from an arbitrary function $f \in X$ by the operator $\langle f\rangle$, which attaches to the set $\{f\}$ the result of replacing any finite set of values of $(f(i_1), \dots, f(i_m))$ by a set of values of $(1 - f(i_1), \dots, 1 - f(i_m))$;

2) the relation R consists of edges connecting functions that differ by exactly one coordinate.

Definition 6. *A cubic system is a graph $\Gamma = \langle X; R\rangle$, in which each component of connectivity is a cube. The theory T of a graph signature*

$\{R^{(2)}\}$ is called cubic if $T = \mathrm{Th}(M)$ for some cubic system M. In this case, the system M is called a cubic model of the theory T.

Definition 7. *An invariant of the cubic theory T is a function*

$$Inv_T : \omega \cup \{\infty\} \to \omega \cup \{\infty\},$$

satisfying the following conditions:

1) each natural number n is assigned a number $Inv_T(n)$ is the connectivity component of any model of the theory T, which are n-cubes if this number is finite, and the symbol ∞ if this number is infinite;

2) the symbol ∞ corresponds to the value 0 if there are no infinite-dimensional cubes in the models of the theory T (or, equivalently, the dimensions of the cubes are limited in aggregate), and the value 1 - otherwise.

Definition 8. *The diameter $d(T)$ of the cubic theory T is called the maximum distance between the elements of the models of the theory T, if these distances are limited, and it is assumed that $d(T) \rightleftharpoons \infty$ otherwise. The carrier (respectively, the ∞-carrier) $\mathrm{Supp}(T)$ ($\mathrm{Supp}_\infty(T)$) of the theory T is called the set $\{n \in \omega \cup \{\infty\} \mid Inv_T(n) \neq 0\}$ ($\{n \in \omega \cup \{\infty\}$ — $Inv_T(n) = \infty\}$).*

Remark 1. For systems $\langle S, \mathrm{acl}\rangle$ in cubic theories T, the substitution property holds if and only if the models S of the theory T do not contain infinite cubes, in particular, when there are no finite cubes of unlimited dimension.

Indeed, for finite cubes we have a degenerate pregeometry, for which replacing an element of an algebraic closure with any other element from this closure will mean either replacing an element from $\mathrm{acl}(\emptyset)$ with another element from this set, or replacing one element of a finite cube C with another with the capture of all elements from C. And if we consider three different elements a, b, c of some infinite cube C', for which $d(a, c) > 1$ and b belong to some shortest (a, c)-route, then $b \in \mathrm{acl}(\{c, a\}) \setminus \mathrm{acl}(\{c\})$, but $a \notin \mathrm{acl}(\{c, b\})$.

By virtue of the remark 1, for systems $\langle S, \mathrm{acl}\rangle$, the dimension of cubes should be considered as the dimension and we should talk about the c-modularity of pregeometries, i.e. about the connection of the dimensions of cubes, without relying on the substitution property. In this case, the systems $\langle S, \mathrm{cl}\rangle$ satisfying the conditions 1), 2), 4) of the definition of pregeometry will be called c-pregeometry.

Definition 9. *For cubic theory T, the value $\mu_A + \sum\limits_{C'} \nu_{A \cap C'}$ is considered as the c-dimension $\dim_c(A)$, where $A \subseteq M \models T$, where μ_A is the number* *of finite cubes $C \subseteq M$ with the condition $C \cap A \neq \emptyset$, and $\nu_{A\cap C'}$ is the dimension of the smallest bribes of K infinite cubes $C' \subseteq M$ with the condition $(A \cap C') \subseteq K$.*

Definition 10. *A c-pregeometry $\langle S, \mathrm{cl}\rangle$ is called c-modular if for any* acl-*closed sets $X_0, Y_0 \subseteq S$, X_0 independently from Y_0 with respect to $X_0 \cap Y_0$, i.e. for any finite-dimensional* acl-*closed sets $X \subseteq X_0$, $Y \subseteq Y_0$ are true:*

1) if there exists an infinite-dimensional cube C for which $X \cap Y \cap C = \emptyset$, $X \cap C \neq \emptyset$, $Y \cap C \neq \emptyset$, then the equality holds:

$$\begin{aligned}&\dim_c(X \cap C) + \dim_c(Y \cap C)+\\ &+\rho(X \cap C, Y \cap C) = \dim_c((X \cup Y) \cap C),\end{aligned} \tag{2.1}$$

where $\rho(X \cap C, Y \cap C)$ is shortest distance between vertices $x \in X \cap C$ and $y \in Y \cap C$;

2) in other cases, for the connectivity components C, the equality is fulfilled:

$$\dim_c(X\cap C)+\dim_c(Y\cap C)-\dim_c(X\cap Y\cap C) = \dim_c((X\cup Y)\cap C). \tag{2.2}$$

Remark 2. According to the definitions of c-dimension and c-modularities when summing the relations (2.1) and (2.2) over all connected components of C, some generalized analogue of the modularity formula in the pregeometry for c-pregeometry is obtained:

$$\begin{aligned}&\sum_C \dim_c(X \cap C) + \sum_C \dim_c(Y \cap C) - \sum_C \dim_c(X \cap Y \cap C)+\\ &+\sum_C \rho(X \cap C, Y \cap C) = \sum_C \dim_c((X \cup Y) \cap C).\end{aligned} \tag{2.3}$$

With the exception of the first case in the definition c-modularity equality is satisfied:

$$\dim_c(X) + \dim_c(Y) - \dim_c(X \cap Y) = \dim_c(X \cup Y). \tag{2.4}$$

Note that according to the definition of c-dimension, the formula (2.3) can be rewritten as follows:

$$\mu_X + \mu_Y - \mu_{X\cap Y} = \mu_{X\cup Y},$$

$$\sum_C \nu_{X\cap C}+\sum_C \nu_{Y\cap C}-\sum_C \nu_{X\cap Y\cap C}+\sum_C \rho(X\cap C, Y\cap C) = \sum_C \nu_{(X\cap C)\cup(Y\cap C)}.$$

Indeed, let be an infinite-dimensional cube C: $X \cap Y \cap C = \emptyset$, $X \cap C \neq \emptyset$, $Y \cap C \neq \emptyset$. In this case $\nu_{X\cap Y} = 0$, $\nu_{(X\cap C)\cup(Y\cap C)}$ equal to the dimension of two cubes $X \cap C$, $Y \cap C$ and the dimensions of the cube between them, that in turn coincides with the shortest distance between vertices $x \in X \cap C$ and $y \in Y \cap C$ equals $\rho(X, Y)$. If the sets X and Y intersect in an infinite-dimensional cube C, so $X \cap Y \cap C \neq \emptyset$, then $\rho(X, Y) = 0$ and the following equality is fulfilled:

$$\nu_{X\cap C} + \nu_{Y\cap C} - \nu_{X\cap Y\cap C} = \nu_{(X\cap C)\cup(Y\cap C)}.$$

Definition 11. *A c-pregeometry $\langle S, \mathrm{cl}\rangle$ is called c-projective if it is c-modular and nontrivial.*

Theorem 1. *Let T be a cubic theory. Then for any model $M = \langle S, R\rangle$ of the theory T one of the following two conditions holds:*

1) *all the connectivity components of the model M are finite and have limited cardinality, and the pregeometry $\langle S, \mathrm{acl}\rangle$ is degenerate;*

2) *The model M has an infinite connectivity component, which is a λ-cube for some cardinal λ, and the c-pregeometry $\langle S, \mathrm{acl}\rangle$ is c-modular.*

Proof. 1. We prove that the c-pregeometry satisfying the conditions of point 1 is degenerate.

When taking an algebraic closure, we will consider formulas

$$\exists^{<\omega} x\phi_n(x, \bar{b}),$$

where ϕ_n are formulas that do not depend on $\bar{b}$ and take the value of truth if vertex x is incident to n edges and there are finitely many such vertices, $n \in \mathbb{N}$, or tuple $\bar{b}$ consists of one element if there are infinitely many vertices of degree n. In the first case, the solutions of formulas $\phi_n(x, \bar{b})$ will be the set $\mathrm{acl}(\emptyset)$, and in the second case, the set of cubes containing elements b and constituting the closure $\mathrm{acl}(X)$ of the set $X \subseteq S$, the elements of which lie in finite-dimensional cubes.

Let there be an infinite number of cubes of the same finite dimension. The set of all their vertices is denoted S'. Thanks to the formulas $\exists^{<\omega} x\phi_n(x, \bar{b})$, the closure of any subset of S will contain $S'' = S \setminus S'$, that is, for any $X \subseteq S$, $S'' \subseteq \mathrm{acl}(X)$ is true, in particular, $S'' \subseteq \mathrm{acl}(\emptyset)$.

Note that for any set of vertices X from the cube $C \subseteq S'$, $\mathrm{acl}(X) = S'' \cup C$ is true. If the set X contains elements from several cubes $C_1, \ldots, C_m$ such that $C_1, \ldots, C_m \subseteq S'$, then $\mathrm{acl}(X) = S'' \cup C_1 \cup \ldots \cup C_m$ is true.

It turns out that for any set X, some elements of which are contained in S', that is, $X \cap S' \neq \emptyset$, $\bigcup\{\mathrm{acl}(\{a\}) |\ a \in X\} = S'' \cup C_1 \cup \ldots \cup C_1 \cup \ldots \cup C_m \cup \ldots \cup C_m = S'' \cup C_1 \cup \ldots \cup C_m = \mathrm{acl}(X)$ is true.

If the number of cubes, of any finite dimension, is finite, that is, $S' = \emptyset$, then the closure of any subset of the model, $X \subseteq S$, carrier will coincide with the entire S carrier. So the following equality $\mathrm{acl}(X) = S = S \cup S \ldots \cup S = \bigcup\{\mathrm{acl}(\{a\}) |\ a \in X\}$ is true.

It turns out for anyone $X \subseteq S$, $\mathrm{acl}(X) = \bigcup\{\mathrm{acl}(\{a\}) \mid a \in X\}$. So by the definition the pregeometry is degenerate.

Suppose that the model M contains an infinite connectivity component. We prove that in this case the pregeometry is non-degenerate. The closure of any set of two or more vertices of this connectivity component is the smallest in terms of the number of incident edges by vertices containing all the vertices from this set. And the closure of any singleton set $\{a\}$, for any $a \in S$, translates it into itself. Therefore, by the definition, the pregeometry is non-degenerate.

2. Due to the fact that any non-empty intersection of cubes in the model of cubic theory is again a cube, we note that, with the exception of the first case, in the definition of c-modularity, under the conditions of the second point of the theorem, the equality holds (2.4), specifying is a c-modularity.

By virtue of remark 2, the equalities are fulfilled:

$$\mu_X + \mu_Y - \mu_{X\cap Y} = \mu_{X\cup Y},$$

$$\nu_{X\cap C} + \nu_{Y\cap C} - \nu_{X\cap Y\cap C} + \rho(X\cap C, Y\cap C) = \nu_{(X\cap C)\cup(Y\cap C)}.$$

Thus, the c-pregeometry is c-modular. □

Corollary 1. *Let T be a cubic theory, and the $M = \langle S, R\rangle$ model of T theory has an infinite connectivity component, which is a λ-cube for some cardinal λ. Then the c-pregeometry of $\langle S, \mathrm{acl}\rangle$ is c-projective.*

Definition 12. *A c-pregeometry $\langle S, \mathrm{cl}\rangle$ is called c-locally finite if for any finite subset $A \subseteq S$, the set $\mathrm{cl}(A)$ is finite.*

Theorem 2. *Let T be a cubic theory. Then for any model $M = \langle S, R\rangle$ of the T theory, the c-pregeometry $\langle S, \mathrm{cl}\rangle$ is not locally finite if and only if there is an infinite number of natural numbers n for which $0 < Inv_T(n) < \infty$.*

Proof. First, consider the proof from right to left. Let there be an infinite number of natural numbers n for which $0 < Inv_T(n) < \infty$.

Then we can take the algebraic formula $\phi_n(x, \bar{b})$, independent of $\bar{b}$, and taking the value of truth when x is an element of an n-cube. For numbers n such that $0 < Inv_T(n) < \infty$, algebraic formulas $\phi_n(x, \bar{b})$ will have a finite number of solutions, which means that all vertices of such n-cubes will be contained in the closure of any set. Since the number of such n is infinite, then the closure of any set will be infinite. In particular, $|\mathrm{acl}(\emptyset)| \geq \omega$. Thus, if there are an infinite number of natural numbers n for which $0 < Inv_T(n) < \infty$, then the c-pregeometry of $\langle S, \mathrm{cl}\rangle$ is not c-locally finite.

Let's give the proof from left to right. Let c-pregeometry $\langle S, \mathrm{acl}\rangle$ not be c-locally finite, but there is no infinite number of natural numbers n for which $0 < Inv_T(n) < \infty$. Then we consider:

1) a finite number of natural numbers n for which $0 < Inv_T(n) < \infty$;
2) a n-cubes for which $Inv_T(n) = \infty$;
3) a λ-cube for some cardinal λ.

In the first case, the closure of a finite set will be finite. Secondly, any formula will give an infinite number of solutions, which means they will not be contained in the closure of any set. Hence, the presence or absence of such cubes does not affect the c-locally finiteness. In the third case, the closure of any finite set A will be the vertices of the smallest cube containing all the elements from A and lying in the λ-cube. It turns out that the closure will be of course. This means that the c-pregeometry $\langle S, \mathrm{cl}\rangle$ is not c-locally finite only in one case - there is an infinite number of natural numbers n for which $0 < Inv_T(n) < \infty$. $\square$

Corollary 2. *Let T be a cubic theory. Then one of the following conditions is fulfilled for some model $M = \langle S, R\rangle$ of theory T:*

1) a c-pregeometry $\langle S, \mathrm{acl}\rangle$ is c-locally finite;
2) the algebraic closure of any set $A \subseteq M$ is infinite.

3. Conclusion

The dependences of the types of c-pregeometries on the connectivity components of the cubic theory model and the algebraic closure operator are shown. In addition, a condition for fulfilling the substitution property from the definition of a pregeometry with an algebraic closure operator in cubic theories is given.

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