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Algebras of Binary Isolating Formulas for Theories of Root Products of Graphs*

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Abstract. Algebras of distributions of binary isolating and semi-isolating formulas are derived objects for given theory and reflect binary formula relations between realizations of 1-types. These algebras are associated with the following natural classification questions: 1) for a given class of theories, determine which algebras correspond to the theories from this class and classify these algebras; 2) to classify theories from a given class depending on the algebras defined by these theories of isolating and semi-isolating formulas. Here the description of a finite algebra of binary isolating formulas unambiguously entails a description of the algebra of binary semi-isolating formulas, which makes it possible to track the behavior of all binary formula relations of a given theory. The paper describes algebras of binary formulae for root products. The Cayley tables are given for the obtained algebras. Based on these tables, theorems describing all algebras of binary formulae distributions for the root multiplication theory of regular polygons on an edge are formulated. It is shown that they are completely described by two algebras.

Keywords: algebra of binary isolating formulas, root product of graphs.

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Algebras of distributions of binary isolating and semi-isolating formulas are derived objects for this theory and reflect the binary formula relations between 1-type realizations. These objects play an important role both in the classification of these theories and in the study of general issues of reconstruction of information recovery from derived objects. Along with the developed general theory of binary formula algebras [10; 11; 13], a series of classification results is obtained on ordered and countably categorical theories [3; 4; 14], theories of abelian groups [?], theories of unars [2], polygonometric theories [5], including theories of regular polyhedrons, simplex theories [6], etc.

This paper continues the study of algebras of distributions of binary isolation formulas: such algebras are described for theories of root products of graphs.

1. Distribution algebras of binary isolating formulas for graph root product theories

Initial concepts and notations used in this paper can be found in [1; 2; 10–13].

Definition 1. [7–9] The *root product* $G \circ H$ of graph G and of root graph H is defined as follows: take $|V(G)|$ disjoint copies of graph H and for each vertex v_i of graph G , identify v_i with the root vertex of i -th copy H . Thus $G \circ H$ is the union of G with copies of H whose roots are identified with correspondent vertices of G .

It is known that the diameter of the graph for the product $G \circ H$ equals $m + 2l$, where m is the diameter of the graph G and l is the diameter of the graph H .

If $H_1 = H_2 = \dots = H_k = H$ then the root product $H \circ H \circ \dots \circ H$ is called *k th root degree* of the graph H and it is denoted by H^k .

Definition 2. We denote by \mathfrak{H}^2 the algebra $\langle \mathcal{P}(\{0, 1, 2, 3\}); * \rangle$ of binary isolating formulas for the graph H^2 , where H is a graph for an edge, given by the following table:

·	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{1}	{0, 2}	{1, 3}	{0, 2}
2	{2}	{1, 3}	{0, 2}	{1, 3}
3	{3}	{0, 2}	{1, 3}	{0, 2}

Definition 3. We denote by \mathfrak{H}^k the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, \dots, n\}); * \rangle$ of binary isolating formulas for the graph H^k , where H is a graph for an edge, $n = 2 * k - 1$, given by the following table:

·	0	1	2	3	4	...	n
0	{0}	{1}	{2}	{3}	{4}	...	{ n }
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	...	Foe($n + 1$)
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	...	Foe($n + 2$)
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	Odd(n)	...	Foe($n + 3$)
4	{4}	{1, 3, 5}	{0, 2, 4, 6}	Odd(n)	Ev(n)	...	Foe($n + 4$)
...
n	{ n }	Foe($n + 1$)	Foe($n + 2$)	Foe($n + 3$)	Foe($n + 4$)	...	Ev(n)

where Ev(n) is the set of all even numbers up to n , Odd(n) is the set of all odd numbers up to n , Foe(n) takes the value Ev(n) if n is even, and Odd(n) otherwise.

Definition 4. For the product $Q \circ H$ of a square Q and an edge H we denote by $\mathfrak{Q}\mathfrak{H}$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4\}); * \rangle$ given by the following table:

·	0	1	2	3	4
0	{0}	{1}	{2}	{3}	{4}
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3}
2	{2}	{1, 3}	{0, 2, 4}	{1, 3}	{0, 2, 4}
3	{3}	{0, 2, 4}	{1, 3}	{0, 2, 4}	{1, 3}
4	{4}	{1, 3}	{0, 2, 4}	{1, 3}	{0, 2, 4}

Definition 5. For the product $Q \circ H^2$ of a square Q and an edge H we denote by $\mathfrak{Q}\mathfrak{H}^2$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, 5, 6\}); * \rangle$ given by the following table:

·	0	1	2	3	4	5	6
0	{0}	{1}	{2}	{3}	{4}	{5}	{6}
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}
4	{4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}
5	{5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}
6	{6}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5}	{0, 2, 4, 6}

Definition 6. For the product $Q \circ H^k$ of a square Q and an edge H we denote by $\mathfrak{Q}\mathfrak{H}^k$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, \dots, n\}); * \rangle$, where $n = 2 * k + 1$, given by the following table:

·	0	1	2	3	4	...	n
0	{0}	{1}	{2}	{3}	{4}	...	{ n }
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	...	Foe($n + 1$)
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	...	Foe($n + 2$)
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	Odd(n)	...	Foe($n + 3$)
4	{4}	{1, 3, 5}	{0, 2, 4, 6}	Odd(n)	Ev(n)	...	Foe($n + 4$)
...
n	{ n }	Foe($n + 1$)	Foe($n + 2$)	Foe($n + 3$)	Foe($n + 4$)	...	Ev(n)

where Ev(n) is the set of all even numbers up to n , Odd(n) is the set of all odd numbers up to n , Foe(n) takes the value Ev(n) if n is even, and Odd(n) otherwise.

Remark 1. Note that the algebra \mathfrak{H}^\natural absorbs the algebra $\mathfrak{Q}\mathfrak{H}^\natural$.

Definition 7. For the product $T \circ H$ of a triangle T and an edge H we denote by $\mathfrak{T}\mathfrak{H}$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3\}); * \rangle$ given by the following table:

·	0	1	2	3
0	{0}	{1}	{2}	{3}
1	{1}	{0, 1, 2}	{0, 1, 2, 3}	{0, 1, 2, 3}
2	{2}	{0, 1, 2, 3}	{0, 1, 2, 3}	{0, 1, 2, 3}
3	{3}	{0, 1, 2, 3}	{0, 1, 2, 3}	{0, 1, 2, 3}

Definition 8. For the product $T \circ H^2$ of a triangle T and the graph H^2 we denote by $\mathfrak{T}\mathfrak{H}^2$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, 5\}); * \rangle$, given by the following table:

·	0	1	2	3	4	5
0	{0}	{1}	{2}	{3}	{4}	{5}
1	{1}	{0, 1, 2}	{0, 1, 2, 3}	{0, 1, 2, 3, 4}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}
2	{2}	{0, 1, 2, 3}	{0, 1, 2, 3, 4}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}
3	{3}	{0, 1, 2, 3, 4}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}
4	{4}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}
5	{5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}

Definition 9. For the product $T \circ H^k$ of a triangle T and the graph H^k we denote by $\mathfrak{T}\mathfrak{H}^k$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, \dots, n\}); * \rangle$, where $n = 2 * k$, given by the following table:

Definition 12. For the product $P \circ H^k$ of a pentagon P and the graph H^k we denote by \mathfrak{H}^k the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, \dots, n\}); * \rangle$, where $n = 2 * k + 1$, given by the following table:

·	0	1	2	3	4	...	n
0	{0}	{1}	{2}	{3}	{4}	...	{ n }
1	{1}	{0, 2}	{1, 2, 3}	{0, 1, 2, 3, 4}	{0, 1, 2, 3, 4, 5}	...	{0, 1, 2, 3, ..., n }
2	{2}	{1, 2, 3}	{0, 1, 2, 3, 4}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5, 6}	...	{0, 1, 2, 3, ..., n }
3	{3}	{0, 1, 2, 3, 4}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6, 7}	...	{0, 1, 2, 3, ..., n }
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	{ n }	{0, 1, ..., n }	{0, 1, ..., n }	{0, 1, ..., n }	{0, 1, ..., n }	...	{0, 1, ..., n }

Definition 13. For the product $D \circ H$ of a hexagon D and an edge H we denote by \mathfrak{H} the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, 5\}); * \rangle$ given by the following table:

·	0	1	2	3	4	5
0	{0}	{1}	{2}	{3}	{4}	{5}
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4}
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4}	{1, 3, 5}
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4}
4	{4}	{1, 3, 5}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4}	{1, 3, 5}
5	{5}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4}

Definition 14. For the product $D \circ H^2$ of a hexagon D and the graph H^2 we denote by \mathfrak{H}^2 the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, 5, 6, 7\}); * \rangle$ given by the following table:

·	0	1	2	3	4	5	6	7
0	{0}	{1}	{2}	{3}	{4}	{5}	{6}	{7}
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}
4	{4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}
5	{5}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}
6	{6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}
7	{7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}

Definition 15. For the product $D \circ H^k$ of a hexagon D and the graph H^k we denote by \mathfrak{H}^k the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, \dots, n\}); * \rangle$, where $n = 2 * k + 2$, given by the following table:

·	0	1	2	3	4	...	n
0	{0}	{1}	{2}	{3}	{4}	...	{ n }
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	...	{Odd(n)}
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	...	{Ev(n)}
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5, 7}	...	{Odd(n)}
4	{4}	{1, 3, 5}	{0, 2, 4, 6}	{1, 3, 5, 7}	{0, 2, 4, 6}	...	{Ev(n)}
...
n	{ n }	{Odd(n)}	{Ev(n)}	{Odd(n)}	{Ev(n)}	...	{Ev(n)}

where $Ev(n)$ is the set of all even numbers up to n , $Odd(n)$ is the set of all odd numbers up to n .

Definition 16. For the product $G \circ H$ of a heptagon G and an edge H we denote by $\mathfrak{G}\mathfrak{H}$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, 5\}); * \rangle$ given by the following table:

·	0	1	2	3	4	5
0	{0}	{1}	{2}	{3}	{4}	{5}
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 2, 3, 4, 5}	{1, 2, 3, 4, 5}
2	{2}	{1, 3}	{0, 2, 4}	{1, 2, 3, 4, 5}	{1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}
3	{3}	{0, 2, 4}	{1, 2, 3, 4, 5}	{1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}
4	{4}	{1, 2, 3, 4, 5}	{1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}
5	{5}	{1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}	{0, 1, 2, 3, 4, 5}

Definition 17. For the product $N \circ H$ of a nonagon N and an edge H we denote by $\mathfrak{N}\mathfrak{H}$ the algebra $\langle \mathcal{P}(\{0, 1, 2, 3, 4, 5, 6\}); * \rangle$ given by the following table:

·	0	1	2	3	4	5	6
0	{0}	{1}	{2}	{3}	{4}	{5}	{6}
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 1, 2, 3, 4, 5, 6}	{1, 2, 3, 4, 5, 6}
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 1, 2, 3, 4, 5, 6}	{1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 1, 2, 3, 4, 5, 6}	{1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}
4	{4}	{1, 3, 5}	{0, 1, 2, 3, 4, 5, 6}	{1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}
5	{5}	{0, 1, 2, 3, 4, 5, 6}	{1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}
6	{6}	{1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}	{0, 1, 2, 3, 4, 5, 6}

Definition 18. The algebra of binary isolating root product formulas of general form $A \circ H$ for regular polygons A with an odd number of vertices with edge H is denoted by $\mathfrak{A}\mathfrak{S}^{\mathfrak{t}}$ and it is represented as $\langle \mathcal{P}(\{0, 1, 2, 3, 4, \dots, n\}); * \rangle$ with the following table:

\cdot	0	1	2	3	4	...	n
0	{0}	{1}	{2}	{3}	{4}	...	{ n }
1	{1}	{0, 2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	...	Fr($n + 1$)
2	{2}	{1, 3}	{0, 2, 4}	{1, 3, 5}	{0, 1, 2, 3, 4, 5, 6}	...	Fr($n + 1$)
3	{3}	{0, 2, 4}	{1, 3, 5}	{0, 1, 2, 3, 4, 5, 6}	{1, 2, 3, 4, 5, 6}	...	Fr($n + 1$)
...
n	{ n }	Fr($n + 1$)	Fr($n + 1$)	Fr($n + 1$)	Fr($n + 1$)	...	{0, 1, 2, 3, 4, ..., n }

where for the number m of vertices of a regular polygon, Fr(k) takes value $\{0, 1, 2, 3, 4, \dots, k\}$ for even k , and for odd k there are two cases: first, if $k \geq m$ then Fr(k) = $\{0, 1, 2, 3, 4, \dots, k\}$; second, if $k < m$ then Fr(k) = $\{1, 2, 3, 4, \dots, k\}$.

On the basis of the obtained description of the Cayley tables for the algebras of binary isolating formulas of root product theories we have the following theorems.

Theorem 1. *If the root product of binary isolating formulas for n -gons results in at least one simplex, then the algebra for the result is isomorphic to the algebra of simplexes.*

Theorem 2. *If T is a theory of root product of graphs per edge, \mathfrak{B} is an algebra of binary isolating formulas of the theory T , then the algebra \mathfrak{B} is given by exactly one of the following algebras: the algebra $\mathfrak{S}^{\mathfrak{t}}$ and the algebra $\mathfrak{A}\mathfrak{S}^{\mathfrak{t}}$.*

2. Conclusion

In the paper, we obtained Cayley tables for the root product of regular polygons by an edge and by an edge degree. Based on these tables, theorems are shown describing all algebras of binary formula distributions for a theory of root product of regular polygons by an edge. It is produced that they are completely described by two forms of algebras.

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Алгебры бинарных изолирующих формул для теорий корневых произведений графов

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Аннотация. Алгебры распределений бинарных изолирующих и полуизолирующих формул являются производными объектами для данной теории и отражают бинарные формульные связи между реализациями 1-типов. Эти алгебры связаны со следующими естественными классификационными вопросами: 1) по данному классу теорий определить, какие алгебры соответствуют теориям из этого класса, и классифицировать эти алгебры; 2) классифицировать теории из класса в зависимости от определяемых этими теориями алгебр изолирующих и полуизолирующих формул. При этом описание конечной алгебры бинарных изолирующих формул однозначно влечет и описание алгебры бинарных полуизолирующих формул, что позволяет отслеживать поведение всех бинарных формульных связей данной теории.

В статье описаны алгебры бинарных формул для корневых произведений. Для полученных алгебр приведены таблицы Кэли. На основании этих таблиц сформулированы теоремы, описывающие все алгебры распределений бинарных формул для теории корневого умножения правильных многоугольников на ребро. Показано, что они полностью описываются двумя алгебрами.

Ключевые слова: алгебра распределений бинарных формул, корневое произведение графов.

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