КРАТКИЕ СООБЩЕНИЯ
SHORT PAPERS

Серия «Математика»
2020. Т. 33. С. 96—105

Онлайн-доступ к журналу:
http://mathizv.isu.ru

И З В Е С Т И Я
Иркутского государственного университета

УДК 519.71, 519.714.7
MSC 06E30, 94C10
DOI https://doi.org/10.26516/1997-7670.2020.33.96

On Length of Boolean Functions of a Small Number of Variables in the Class of Pseudo-Polynomials *

S. N. Selezneva[1], A. A. Lobanov[1]
[1] *Lomonosov Moscow State University, Moscow, Russian Federation*

Abstract. Minimization of Boolean functions in their various representations is required in logic design of digital devices. EXOR sums (polynomial forms) are considered among other representations. An EXOR sum (a polynomial form) is an expression that is an EXOR sum of products of factors in a certain form. We can accentuate some classes of EXOR sums (of polynomial forms) such that Fixed Polarized Reed-Muller forms, FPRMs (polarized polynomial forms, PPFs), EXOR Sums of Products, ESOPs (polynomial normal forms, PNFs), EXOR Sums of Pseudo-Products, ESPPs (pseudo-polynomial forms, PSPFs), etc. In series of works, minimization algorithms are devised, and bounds are obtained for the length of functions in these classes of EXOR sums. Herewith, there are different aspects in these researches, in particular, to obtain bounds of the length for the most complex functions of n variables for an arbitrary number n and to find the exact length for functions of a small number of variables.

The present work is devoted to finding the exact length for functions of a small number of variables. EXOR Sums of Pseudo-Products, ESPPs (pseudo-polynomial forms, PSPFs) for Boolean functions are considered in it. An EXOR Sum of Pseudo-Products, ESPP (a pseudo-polynomial form, PSPF) is an expression that is an EXOR sum of products of linear functions. The length of an ESPP is the number of its summands; the length of a function in the class of ESPPs is the smallest length among all ESPPs, representing this function. In the work, the complete classification by the length in the class of ESPPs is obtained for functions of four variables. The largest length and the average length in the class of ESPPs are found for functions of five variables.

Keywords: Boolean function, Zhegalkin polynomial, EXOR Sum of Pseudo-Products, ESPP (pseudo-polynomial form, PSPF), length, classification by the length.

1. Introduction

Minimization of Boolean functions in their various representations is required in logic design of digital devices [10]. Polynomial forms (EXOR sums) are considered among other representations [2; 6]. A polynomial form (an EXOR sum) is an expression that is an EXOR sum of products of factors in a certain form. We can accentuate some classes of polynomial forms such that polarized polynomial forms, PPFs (Fixed Polarized Reed-Muller forms, FPRMs), polynomial normal forms, PNFs (EXOR Sums of Products, ESOPs), pseudo-polynomial forms, PSPFs (EXOR Sums of Pseudo-Products, ESPPs), etc.

The number of summands in a polynomial form (in an EXOR sum) is called its length. The length of a function in a particular class of polynomial forms is the smallest length among all polynomial forms from this class such that these forms represent this function. In [1–3; 5–9; 11], length bounds are obtained for the "most complex" Boolean n-variable functions in various classes of polynomial forms.

In logic design of digital devices, the complexity of functions of a small number of variables is important. In [6], a comparison is adduced for the average length of four-variable Boolean functions in the classes of disjunctive normal forms, DNFs (Sums of Products, SOPs) and of ESOPs. This comparison is adduced for functions with the same number of their minterms. The average length of ESOPs appears to be less than the average length of SOPs. In the same paper [6], the average length is compared for symmetric n-variable functions, where $n = 3, \dots, 7$, in the classes of SOPs and of ESOPs. Again the average length of ESOPs always appears to be less than one of SOPs. Notice that it is shown in [3] that the maximum length of n-variable functions is less in order for the class of ESOPs than for the class of SOPs. In [11], the maximum length of seven-variable functions is evaluated for the class of ESOPs.

Notice also that complexity bounds of functions of a small number of variables are examined for other representations [4].

In [2], polynomial forms and their application capability in design of digital devices are considered. A pseudo-polynomial form, PSPF (an EXOR Sum of Pseudo-Products, ESPP), or a pseudo-polynomial, is an expression that is a sum modulo two (an EXOR sum) of products of linear Boolean functions. In [2], a comparison of PSPFs and pseudo-disjunctive forms, PSDFs (Sums of Pseudoproducts, SPPs) is adduced (a PSDF (a SPP) is an expression that is a disjunction of products of linear functions). It is shown that the average length of symmetric n-variable functions, where $n = 3, \dots, 6$, is always less for the class of ESPPs than for the class of

* The work is supported by the MCFAM-MSU project "Bounds of complexity characteristics for Boolean functions and graphs" (2020 y.)

SPPs. Notice that the order is obtained in [7; 8] for the maximum length of n-variable functions in the class of ESPPs.

2. Basic definitions

Assume $E_2 = \{0, 1\}$, and E_2^n denotes the set of all tuples on zeros and ones of length n, $n \geqslant 1$. If $\alpha \in E_2^n$ then we consider that α_i denotes the ith component of the tuple α. In other words, $\alpha = (\alpha_1, \ldots, \alpha_n)$. A mapping $f : E_2^n \to E_2$ is called a Boolean n-variable function, $n = 0, 1, 2, \ldots$. We denote the set of all Boolean functions by P_2, and the set of all Boolean n-variable functions by $P_2^{(n)}$. Everywhere further in the paper, "a function" means "a Boolean function".

A *monomial* is an expression that is either a product (modulo two) of distinct variables or the constant 1. A *Zhegalkin polynomial* is an expression that is either a sum modulo two of distinct monomials or the constant 0. Any Boolean function can be represented by a Zhegalkin polynomial, and, for any function, there exists a unique Zhegalkin polynomial, representing this function [12]. We will denote the Zhegalkin polynomial of a function f by $P(f)$.

A function $g(x_1, \ldots, x_n)$ is called *linear*, if its Zhegalkin polynomial $P(g)$ has no products of variables, i. e. if

$$P(g) = c_0 \oplus c_1 x_1 \oplus \ldots \oplus c_n x_n$$

for some coefficients $c_0, c_1, \ldots, c_n \in E_2$. A *pseudo-monomial* is an expression that is a product of linear functions. A *pseudo-polynomial form* (a PSPF), or a *pseudo-polynomial*, is an expression that is a sum modulo two of pseudo-monomials. In other words, a PSPF is an expression in the form

$$\bigoplus_{j=1}^{l} \prod_{i=1}^{r_j} g_{j_i},$$

where $g_{j_1}, \ldots, g_{j_{r_j}}$ are linear functions, $r_j \geqslant 1$, $j = 1, \ldots, l$. Notice that any Boolean function is represented by a PSPF, for example, by its Zhegalkin polynomial. The length $l(P)$ of a PSPF P is the number of its summands. The length $l(f)$ of a Boolean function f in the class of PSPFs is the smallest length among all PSPFs, representing this function f. It is true that $l(f) \leqslant 2^n$ for an arbitrary Boolean n-variable function. Define the *length* $l(n)$ and the *average length* $l^*(n)$ of Boolean functions in the class of PSPFs as the largest length in the class of PSPFs among all n-variable functions and as the ratio of the sum of the lengths of all n-variable functions in the class of PSPFs to the number of n-variable functions. In [7; 8], it is proved that $l(n) = \Theta\left(\frac{2^n}{n^2}\right)$.

3. Function classification by the length in the class of PSPFs

In order to get a classification of functions by the length in the class of PSPFs, we wrote a program in C++14 language. The program performs a sorting through all n-variable functions, some restriction of the sorting are realized in the algorithm. The program computes not only the length $l(f)$ in the class of PSPFs for each function f but also a minimum length PSPF of this function f.

Below we describe some results, obtaining by this program.

3.1. Four-variable and five-variable functions

In the following table, we provide the average length in the class of the PSPFs for all 4-variable functions and for all 5-variable ones. In this table, the columns indicate a number n of function variables, the maximum length $l(n)$ of n-variable functions in the class of PSPFs, and the average length $l^*(n)$ of n-variable functions in the class of PSPFs, respectively

n	$l(n)$	$l^*(n)$
4	4	$2,8515015$
5	5	4.115962973795831

In the following table, we show the distribution for 4-variable functions by their length in the class of PSPFs. In this table, the columns indicate a length l, the number $m_4(l)$ of 4-variable functions of the length l in the class of PSPFs, and the proportion $d_4(l)$ of the number of 4-variable functions of the length l in the class of PSPFs to the number of all 4-variable functions, respectively. .

l	$m_4(l)$	$d_4(l)$
1	308	$0,005$
2	16284	$0,248$
3	41776	$0,637$
4	7168	$0,101$

In the following table, we show the distribution for 5-variable functions by their length in the class of PSPFs. In this table, the columns indicate a length l, the number $m_5(l)$ of 5-variable functions of the length l in the class of PSPFs, and the proportion $d_5(l)$ of the number of 5-variable functions of the length l in the class of PSPFs to the number of all 5-variable functions, respectively.

l	$m_5(l)$	$d_5(l)$
1	2452	0
2	1836348	0
3	279379440	0,065
4	3232632384	0,753
5	781116672	0,182

3.2. Classification of four-variable functions

Assume $f(x_1, \ldots, x_n) \in P_2$, where $n \geqslant 1$. If A is a non-degenerate matrix of the size $n \times n$ over the residue field modulo two $\mathbb{Z}_2 = (E_2; \oplus, \cdot)$, i. e. A is a non-degenerate matrix from $(\mathbb{Z}_2)_{n \times n}$, then we say that the function

$$h(x_1, \ldots, x_n) = f((x_1, \ldots, x_n) \cdot A) \in P_2$$

is derived by a *non-degenerate linear transformation of variables* from the function $f(x_1, \ldots, x_n)$. In other words, $h(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$, where

$$y_j = \bigoplus_{i=1}^{n} x_i \cdot a_{i,j}$$

for all $j = 1, \ldots, n$. In this case, we denote the function h by Af. If $b \in E_2^n$, then we say that the function

$$g(x_1, \ldots, x_n) = f(x \oplus b) \in P_2$$

is derived by *negating some variables* from the function $f(x_1, \ldots, x_n)$. In other words, $g(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$, where $y_i = x_i \oplus b_i$ for all $i = 1, \ldots, n$. In this case, we denote the function g by bf.

We say that two functions $f_1, f_2 \in P_2^{(n)}$ are in the relation ρ, if one of them can be derived from other by a finite number of operations, each of which is either a non-degenerate linear transformation of variables or negating some variables. In other words,

$$f_2 = p_s(p_{s-1}(\ldots(p_1 f_1)\ldots)),$$

where p_j is either a non-degenerate matrix from $(\mathbb{Z}_2)_{n \times n}$ or a tuple from E_2^n, $j = 1, \ldots, s$, $s \geqslant 1$. It is easy to see that ρ is an equivalence relation on the set $P_2^{(n)}$. We call any equivalence class of the relation ρ by a *class* of the set $P_2^{(n)}$. It is clear that if Q is a class of the set $P_2^{(n)}$ then $l(f_1) = l(f_2)$ for any functions $f_1, f_2 \in Q$. If Q is a class of $P_2^{(n)}$ then we denote its cardinality $|Q|$ by $m(Q)$, and we denote the length of any function from Q in the class of PSPFs by $l(Q)$.

In the following table, we provide all 32 classes of the set $P_2^{(4)}$. In this table, the columns indicate the number k of a class Q_k, the length $l(Q_k)$, the cardinality $m(Q_k)$, and a function $f \in Q_k$, respectively.

Classification of 4-variable functions

k	$l(Q_k)$	$m(Q_k)$	$f \in Q_k$
1	1	1	0
2	1	16	$x_3x_2x_1x_4$
3	1	120	$x_2x_1x_3$
4	1	140	x_1x_2
5	1	30	x_1
6	1	1	1
7	2	560	$x_1x_2 + (x_1 + x_3)(x_2(x_1(x_3 + x_2 + x_4)))$
8	2	1680	$x_1(x_3 + x_2 + x_1 + x_4) + (x_2 + x_1 + x_3)(x_1(x_3 + x_1 + x_4))$
9	2	1680	$x_3((x_1 + x_2)x_1x_4) + x_1x_2$
10	2	840	$x_1 + (x_1 + x_2)(x_1(x_2 + x_1 + x_3))$
11	2	240	$x_1 + (x_1 + x_3)((x_1 + x_2)(x_1(x_3 + x_2 + x_1 + x_4)))$
12	2	6720	$x_3((x_1 + x_2)x_4) + x_1x_2$
13	2	1920	$(x_1 + x_3)((x_1 + x_2)(x_1 + x_4)) + x_1$
14	2	240	$x_3((x_1 + x_2)x_2x_4) + x_1$
15	2	840	$x_3 + x_2 + x_1 + 1 + (x_3 + x_2 + 1)(x_3 + x_1 + 1)$
16	2	448	$(x_4 + x_3 + x_2 + 1)(x_4 + x_1 + 1) + (x_4 + x_3 + x_1 + 1)(x_4 + x_2 + x_1 + 1)$
17	2	840	$(x_1 + x_2)x_2x_3 + x_1$
18	2	140	$1 + (x_2 + x_1 + 1)(x_2 + 1)$
19	2	120	$1 + (x_3 + x_2 + 1)((x_3 + x_2 + x_1 + 1)(x_3 + 1))$
20	2	16	$1 + (x_4 + x_3 + x_1 + 1)((x_4 + x_3 + x_2 + 1)((x_4 + x_3 + x_2 + x_1 + 1)(x_4 + 1)))$
21	3	2688	$(x_3+x_2+1)(x_3+x_1+1)+(x_4+x_3+x_1+1)((x_4+x_3 + x_2 + 1)((x_4 + x_3 + x_2 + x_1 + 1)(x_4 + 1))) + x_4(x_3 + x_2 + x_1 + x_4)$
22	3	6720	$x_3 + x_2 + x_1 + 1 + (x_3 + x_2 + 1)(x_3 + x_1 + 1) + x_3((x_1 + x_2)(x_2(x_3 + x_4)))$
23	3	4480	$(x_4 + x_3 + x_2 + 1)(x_4 + x_1 + 1) + (x_4 + x_3 + x_1 + 1)(x_4 + x_2 + x_1 + 1) + x_3x_2x_1x_4$
24	3	10080	$x_3 + x_2 + 1 + (x_3 + x_2 + x_1 + 1)(x_3 + 1) + (x_1 + x_2)(x_1(x_3 + x_2 + x_1 + x_4))$
25	3	6720	$x_3 + x_2 + x_1 + 1 + (x_3 + x_2 + 1)(x_3 + x_1 + 1) + (x_1 + x_3)((x_1 + x_2)x_1x_4)$
26	3	6720	$1 + (x_2 + x_1 + 1)(x_2 + 1) + (x_2 + x_1 + x_3)((x_1 + x_2)(x_3 + x_2 + x_1 + x_4))$

continued on the next page

k	$l(Q_k)$	$m(Q_k)$	$f \in Q_k$
27	3	1680	$1+(x_2+x_1+1)(x_2+1)+(x_2+x_3)((x_1+x_2)(x_2(x_3+x_2+x_1+x_4)))$
28	3	448	$1+(x_2+x_1+1)(x_2+1)+(x_2+x_1+x_3)(x_2+x_1+x_4)$
29	3	1680	$1+(x_2+x_1+1)(x_2+1)+(x_2+x_1+x_3)((x_2+x_3)(x_2+x_4))$
30	3	560	$1+(x_2+x_1+1)(x_2+1)+(x_2+x_3)((x_1+x_3)x_3x_4)$
31	4	4480	$1+(x_2+x_1+1)(x_2+1)+(x_4+x_3+1)(x_4+1)+(x_4+x_3+x_1+1)((x_4+x_3+x_2+1)((x_4+x_3+x_2+x_1+1)(x_4+1)))$
32	4	2688	$1+(x_2+x_1+1)(x_2+1)+(x_2+x_1+x_3)(x_2+x_1+x_4)+x_3x_2x_1x_4$

4. Conclusion

In the paper, the complete classification by the length in the class of pseudo-polynomials is obtained for Boolean functions of four variables. The maximum length and the average length in the class of pseudo-polynomials are found for Boolean functions of five variables.

References

1. Baliuk A. S. Complexity lower bound for Boolean functions in the class of extended operator forms. *The Bulletin of Irkutsk State University. Series Mathematics*, 2019, vol. 30, pp. 125–140. https://doi.org/10.26516/1997-7670.2019.30.125
2. Ishikawa R., Hirayama T., Koda G., Shimizu K. New three-level Boolean expression based on EXOR gates. *IEICE Trans. Inf. & Syst.*, 2004, vol. E87-D, no 5, pp.214–1222.
3. Kirichenko K. D. An upper bound for complexity of polynomial normal forms of Boolean functions. *Discrete Math. Appl.*, 2005, vol. 15, no. 4, pp. 351–360. https://doi.org/10.1515/156939205774464891
4. Lozhkin S. A., Shupletsov M. S., Konovodov V. A., Danilov B. R., Zhukov V. V., Bagrov N. Yu. Raspredelennaya sistema i algoritmy poiska minimal'nyh i blizkih k nim kontaktnyh shem dlya bolevyh funkthiy malogo chisla peremennyh. *Sbornik trudov vserossiyskih nauchno-tehnicheskih konferentsiy "Problemy razrabotki perspektivnyh mikro- i nanoelektronnyh sistem (MES)"*, 2016, vol. 1, pp. 40–47. (in Russian)
5. Peryazev N. A. Complexity of Boolean functions in the class of polarized polynomial forms. *Algebra and Logic*, 1995, vol. 34, pp. 177–179. https://doi.org/10.1007/BF02341875
6. Sasao T., Besslich P. On the complexity of mod-2 sum PLA's. *IEEE Trans. on Comput.*, 1990, vol. 39, no. 2, pp. 262–266.

7. Selezneva S. N. On the length of Boolean functions in the class of exclusive-OR sums of pseudoproducts. *Moscow University Computational Mathematics and Cybernetics*, 2014, vol. 38, no. 2, pp. 64–68. https://doi.org/10.3103/S0278641914020083
8. Selezneva S. N. Order on the length of Boolean functions in the class of exclusive-OR sums of pseudoproducts. *Moscow University Computational Mathematics and Cybernetics*, 2016, vol. 40, no. 3, pp. 123–127. https://doi.org/10.3103/S0278641916030043
9. Suprun V. P. Complexity of Boolean functions in a class of canonical polarized polynomials. *Discrete Math. Appl.*, 1994, vol. 4, no. 3, pp. 273–277.
10. Ugryumov E. P. *Tsifrovaya shemotehnika* [Digital Circuitry]. Saint-Petersburg, BHV-Petersburg Publ., 2004. 528 p.
11. Vinokurov S. F., Kazimirov A. S. On the complexity of one class of Boolean functions. *The Bulletin of Irkutsk State University. Series Mathematics*, 2010, vol. 4, pp. 2–6. (in Russian)
12. Yablonski S. V. *Vvedeniye v diskretnuyu matematiku* [Introduction in Discrete Mathematics]. Moscow, Higher School Publ., 2001. 384 p.

Svetlana Selezneva, Doctor of Sciences (Physics and Mathematics), professor, Lomonosov Moscow State University, build. 52, 1, Leninskiye Gory, 119991, Moscow, Russian Federation, tel.: (495)9395392, email: selezn@cs.msu.ru, ORCID iD https://orcid.org/0000-0002-9918-5337.

Aleksey Lobanov, student, Lomonosov Moscow State University, build. 52, 1, Leninskiye Gory, 119991, Moscow, Russian Federation, tel.: (495)9395392, email: alex_cmc@likemath.ru,
ORCID iD https://orcid.org/0000-0002-4983-8985.

Received 27.07.20

О длине функций алгебры логики малого числа переменных в классе псевдополиномов

С. Н. Селезнева[1], А. А. Лобанов[1]

[1] *Московский государственный университет им. М. В. Ломоносова, Москва, Российская Федерация*

Аннотация. Минимизация функций алгебры логики при различных их представлениях востребована при логическом проектировании цифровых устройств. Среди других представлений рассматриваются полиномиальные формы. Полиномиальной формой называется выражение, являющееся суммой по модулю два произведений сомножителей определенного вида. Можно выделить такие классы полиномиальных форм, как поляризованные полиномиальные, полиномиальные нормальные, псевдополиномиальные и др. В ряде работ разработаны алгоритмы минимизации и получены оценки длины функций в этих классах полиномиальных форм. При этом исследования ведутся в нескольких направлениях, в частности, получение оценок длины самых сложных функций n переменных для произвольных n и нахождение точной длины функций малого числа переменных.

Настоящая работа посвящена нахождению точной длины функций малого числа переменных. В ней рассматриваются псевдополиномиальные формы для функций алгебры логики. Под псевдополиномиальной формой (ПСПФ), или псевдополиномом, понимается выражение, являющееся суммой по модулю двух произведений линейных функций. Длиной ПСПФ называется число ее слагаемых; длиной функции алгебры логики в классе ПСПФ — наименьшая длина среди всех ПСПФ, представляющих эту функцию. В работе получена полная классификация по длине в классе ПСПФ функций, зависящих от четырех переменных. Для функций, зависящих от пяти переменных, найдена наибольшая и средняя длина в классе ПСПФ.

Ключевые слова: функция алгебры логики (булева функция), полином Жегалкина, псевдополиномиальная форма (ПСПФ), длина, классификация по длине.

Список литературы

1. Baliuk A. S. Complexity lower bound for Boolean functions in the class of extended operator forms // Известия Иркутского государственного университета. Серия Математика. 2019. Т. 30. С. 125–140. https://doi.org/10.26516/1997-7670.2019.30.125
2. New three-level Boolean expression based on EXOR gates / R. Ishikawa, T. Hirayama, G. Koda, K. Shimizu // IEICE Trans. Inf. & Syst. 2004. Vol. E87-D, N 5. P. 1214–1222.
3. Кириченко К. Д. Верхняя оценка сложности полиномиальных нормальных форм булевых функций // Дискретная математика. 2005. Т. 17, вып. 3. С. 80–88. https://doi.org/10.1515/156939205774464891
4. Распределенная система и алгоритмы поиска минимальных и близких к ним контактных схем для булевых функций от малого числа переменных / С. А. Ложкин, М. С. Шуплецов, В. А. Коноводов, Б. Р. Данилов, В. В. Жуков, Н. Ю. Багров // Сборник трудов Всероссийских научно-технических конференций «Проблемы разработки перспективных микро- и наноэлектронных систем (МЭС)». 2016. Т. 1. С. 40–47.
5. Перязев Н. А. Сложность булевых функций в классе полиномиальных поляризованных форм // Алгебра и логика. 1995. Т. 34, вып. 3. С. 323–326. https://doi.org/10.1007/BF02341875
6. Sasao T., Besslich P. On the complexity of mod-2 sum PLA's // IEEE Trans. on Comput. 1990. Vol. 39, N 2. P. 262–266.
7. Селезнева С. Н. О длине булевых функций в классе полиномиальных форм с аффинными множителями в слагаемых // Вестник Московского университета. Серия 15, Вычислительная математика и кибернетика. 2014. Вып. 2. С. 34–38. https://doi.org/10.3103/S0278641914020083
8. Селезнева С. Н. Порядок длины функций алгебры логики в классе псевдополиномиальных форм // Вестник Московского университета. Серия 15, Вычислительная математика и кибернетика. 2016. Вып. 3. С. 27–31. https://doi.org/10.3103/S0278641916030043
9. Супрун В. П. Сложность булевых функций в классе канонических поляризованных полиномов // Дискретная математика. 1993. Т. 5, вып. 2. С. 111–115.
10. Угрюмов Е. П. Цифровая схемотехника. СПб. : БХВ-Петербург, 2004. 528 с.
11. Винокуров С. Ф., Казимиров А. С. О сложности одного класса булевых функций // Известия Иркутского государственного университета. Серия Математика. 2010. Т. 3, вып. 4. С. 2–6.
12. Яблонский С. В. Введение в дискретную математику. М. : Высшая школа, 2001. 384 с.

Известия Иркутского государственного университета.
2020. Т. 33. Серия «Математика». С. 96–105

Светлана Николаевна Селезнева, доктор физико-математических наук, профессор, факультет вычислительной математики и кибернетики, Московский государственный университет им. М. В. Ломоносова, Российская Федерация, 119991, г. Москва, Ленинские горы, 1, стр. 52, тел.: (495)9395392, email: selezn@cs.msu.ru,
ORCID iD https://orcid.org/0000-0002-9918-5337.

Алексей Андреевич Лобанов, студент, факультет вычислительной математики и кибернетики, Московский государственный университет им. М. В. Ломоносова, Российская Федерация, 119991, г. Москва, Ленинские горы, 1, стр. 52, тел.: (495)9395392,
email: alex_cmc@likemath.ru,
ORCID iD https://orcid.org/0000-0002-4983-8985.

Поступила в редакцию 27.07.2020