Серия «Математика»
2014. Т. 8. С. 44—61

Онлайн-доступ к журналу:
http://isu.ru/izvestia

И З В Е С Т И Я
Иркутского государственного университета

УДК 517.977

Optimal control methods for conservation biology: a case of non-harvesting utility [*]

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Abstract. The main purpose of this paper is to retrace the evolution of mathematical models focused on relation and interaction between economic growth, sustainable development and natural environment conservation. The starting point is a simple model of common-property harvesting, where renewable resource grows according to the course of nature. Further, this model is amended with defensive expenditures that favor the species growth. Apart from solely harvesting models, a transition model comprising both harvesting and non-harvesting values of wild biological species is presented. Preponderantly, all these models are designed to seek for long-term optimal and/or sustainable strategies for harvesting, where species preservation guarantees the profit stability for future generations and thus contributes to the economic development.

On the other hand, there is a group of purely non-harvesting models where anthropic activities and economic growth may have positive or negative impact on the natural evolution of wild species. Several scholars have proved that optimal strategies that are relatively good for harvesting purposes are not merely transferrable to the context of conservation of wildlife biological species with no harvesting value. However, existence of long-term conservation policies for all biological species (with or without harvesting value) cannot be guaranteed without having relatively large species populations at initial time. Therefore, all such strategies are incapable to enhance scarce population of endangered species and save them from eventual (local) extinction.

As an alternative, policy makers are compelled to design and implement short-term defensive actions aimed at enhancement of wildlife species populations. The latter is referred to as an emergent area of research in conservation biology.

Keywords: bioeconomic models, endangered species, conservation, non-harvesting utility, optimal control.

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[*] Supported by IRD-PEERS Project “Modeles d’optimisation et de viabilite en ecologie et en economie”, Universite Paris-Est (France) and Universidad del Valle (Colombia).

1. Introduction

Almost all scientists agree that the major factors that contribute to biodiversity loss[1] are: overexploitation, deforestation, invasive species, air and water pollution, soil contamination, and climate change. Despite significant efforts of human society (such as ecosystem stewardship, new common-property ecological policies, legislative initiatives aimed at habitat restoration, increase of protected terrestrial, coastal and maritime areas, etc.), biodiversity continues to decline world-wide (see a thorough analysis in [18] and numerous references therein). Therefore, it is fair to say that global commitments made in 2002, through the Convention on Biological Diversity (CBD), have not been met yet.

A critical review [16] summarizes key features of four basic categories of models that integrate economic theories and strategies aimed at species conservation. The majority of these models are designed in order to help a social planner to define strategies for optimal and/or sustainable *harvesting*, where species preservation guarantees the profit stability for future generations and thus contributes to the economic development. A pioneering work of H. S. Gordon [17] provided the framework for bioeconomical modeling of common-property renewable resources (such as fisheries, for example), and fundamental principles of sustainable bioeconomics were further summarized by C. W. Clark [10] using mathematical modeling.

In recent decades, there has been noted an increasing interest in research on relation and interaction between economic growth, sustainable development and natural environment conservation. Besides direct consumption of renewable resources (i.e. harvesting) there are other factors that may provoke drastic reductions and, possibly, extinction of the species populations. It is worthwhile to note that, according to [15], the *primary cause* of the decay of organic diversity is *not* direct human exploitation or malevolence, but the habitat destruction that inevitably results from expansion of human population and human activities.

Several authors tried to predict the long-run consequences of pollution and natural resource scarcity using macroeconomic growth models. Such models consider man-made capital, natural stock and emissions as necessary factors of production, where pollution directly affects both the growth of renewable resources and social welfare (see, e.g., [11; 23; 24]).

Alternatively, T. M. Swanson [22] had proposed a constructive adjustment to classical harvesting models of Gordon-Schaefer type (thoroughly described in [4; 10] among other similar texts) where harvesting effort is traditionally modeled by a control variable. He had explicitly included another control variable that expresses the allocation of resources required

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[1] For more comprehensive review on imminent threats to biodiversity please refer to [19, pp. 133-135].

for a species' survival. The latter can be treated as an initial attempt to modify the biological dynamics of species growth with defensive actions of policy-makers aimed at species protection.

Swanson's idea was further developed by R. Alexander [2] who also proposed to include in the objective function both consumptive and *non-consumptive*[2] values of the biological species. This approach can be applied to analyze the population dynamics of endangered species that have both consumptive and non-consumptive values in order to design the appropriate conservation policies for their sustainability and bioeconomic viability.

On the other hand, there are many wildlife species with *no harvest value* that are currently in threat by negative side-effects of human activity (urbanization, pollution, habitat loss, etc.). As pointed out by several scholars (see, e.g. [19, p. 133] and [1]), most of the species that are becoming extinct are not “food species” (that is, they are not directly consumed by humans) but their biomass is converted into human food when their habitat is transformed into pasture, cropland, and orchards. A recent study carried out by E. Dumont [14] indicates that we are facing a considerable reduction of the surface area of wild biodiverse land by the year 2050 as a consequence of growing human population on our planet. One may argue that wild species are not absolutely essential for human survival. However, loss of wilderness irreplaceably diminishes an important source of human wellbeing.

To study the evolution of such species, R. Alexander and D. W. Shields [3] had proposed a *non-harvesting* variant of dynamic model for one particular species (New Zealand's yellow-eyed penguin) using as control variable an index of the quantity of land resources which are vital for the species survival. The latter can be viewed as a defensive expenditure of the society aimed at the conservation of the natural habitat of the species. In fact, this non-harvesting model does not explicitly include the negative impact that human activity and aggregated production may have on the natural evolution of species population.

However, other results (such as, [5; 6]) clearly demonstrate that both negative and positive human actions may alter the stability properties of the natural dynamic of biological species. These studies, performed on the basis of dynamic model of two interacting species with linear dynamics, revealed an interesting fact. Namely, if the equilibrium level of the species is high enough, the local stability's properties will be preserved when the natural biological dynamics (without human intervention) is amended with economic and ecological features (that is, negative impact of aggregated production and positive impact of defensive expenditures). Additionally,

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[2] R. Alexander also cites several previous studies that addressed some non-consumptive values (principally, tourism values) of particular biological species; however, none of these studies had explicitly included such non-consumptive values in the mathematical models.

other authors (see [8] and [12]) have come to the same conclusion using as a basis a single-species model with Gompertz-type and logistic population growth, respectively.

All previously mentioned models have been formulated for *infinite time horizon* and are aimed at design of *long-term* policies for species conservation. Their meticulous analysis disclosed that initial abundance of the species population is crucial for existence of sustainable policies for long-term conservation planning. In other words, long-term policies are incapable to enhance initially scarce populations of endangered species.

On the other hand, an emergent study conducted in [13] had revealed the existence of *short-term* decision policies capable to enhance the population of single endangered species within a *finite* period time. However, such policies will require a relatively high level of initial capital to be quickly spent for species conservation. Thus, and aggregated production that generates additional capital surplus may also have a positive effect on species conservation.

2. Harvesting models with consumptive utility

A customary framework of bioeconomic models with harvesting of so-called Schaefer-Gordon type can be found in [4; 10] or similar textbooks. This type of models fits into the following mathematical formulation:

$$\frac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\boldsymbol{x}) - H(\boldsymbol{x}, h), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \tag{2.1}$$

where $\boldsymbol{x}$ can be scalar ($\boldsymbol{x} = x$) or vector $\big(\boldsymbol{x} = (x_1, x_2, \dots, x_n)'\big)$ quantity that stands for the biomass of renewable natural stock consisting of one or more biological species with harvesting value, and $\boldsymbol{x}_0$ denotes an initial stock level (constant). Function $\boldsymbol{F} = (F_1, F_2, \dots, F_n)$ describes the biological recruitment or growth of the species (and possibly an interaction between species in the ecosystem), and $H(\boldsymbol{x}, h)$ is a function of harvest with $h = h(t)$ expressing a stock removal (or harvesting effort) at time t. Variable $h(t)$ is usually exogenous and is chosen by the social planner in order to maximize the present values of its future net revenues (or utility) $U(\boldsymbol{x}, h)$:

$$\max_{h(t)\geq 0} \int_0^\infty e^{-\rho t} U\left(\boldsymbol{x}(t), h(t)\right) dt \qquad \text{s.t. } (2.1) \tag{2.2}$$

where $\rho > 0$ denotes the instantaneous discount rate. The above utility is understood as "consumptive" since it express the net revenue obtained from direct consumption (or harvesting) of the biological resource:

$$U(\boldsymbol{x}, h) = \text{ [total income] } - \text{ [total cost]}.$$

In case of single species, $\boldsymbol{x} = x$, scalar function F is usually supposed to be a hump-shaped function satisfying the following conditions:

$$\begin{array}{rcl} F(x) & > 0 \quad \text{for} & 0 < x < K \\ F(0) & = F(K) & = \quad 0 \\ F''(x) & < 0 \quad \text{for} & 0 < x < K \end{array}$$

where $K > 0$ represents the maximal stock level sustainable by the environment which is usually called “carrying capacity.” This term reflects the fact that without harvesting ($H \equiv 0$) the natural stock x is bounded from above:

$$\lim_{t\to\infty} x(t) = K.$$

A classical example of F is Verhulst’s logistic function:

$$F(x) = rx\left(1 - \frac{x}{K}\right)$$

(as well as its numerous modifications), where $r > 0$ stands for intrinsic growth rate of the stock. Other useful examples are Gompertz growth function

$$F(x) = rx \ln\left(\frac{K}{x}\right)$$

(see, e.g., [20]) and

$$F(x) = \frac{rx(K - x)}{K + (r/a)x}$$

attributed to F. E. Smith [21].

It was fairly stated by C. W. Clark [10] that low growth rate of biological stock relative to discount rate (that is, $\rho > r > 0$) and/or gradual decay of the carrying capacity K will inevitably lead to stock extermination in common-property models with harvesting. In this situation, there are two possible scenarios, namely:

1) *Stock extermination or resource mining.* If a biological resource is unable to generate a competitive return, a social planner may choose so-called “optimal extinction” (see more details in [9]). This option is considered *economically* viable when

   (a) the discount rate sufficiently exceeds the maximum reproductive potential of the population, and

   (b) an immediate profit can be made from harvesting the last remaining stock.

   T. M. Swanson [22] provides a valid example concerned with deforestation of tropical hardwood forests. He points out: “These trees represent substantial amount of standing value, but they have very low growth potential. Thus it is economically rational to “cash in” the hardwoods and invest the return in other, more productive assets”.

2) *Avoiding extermination.* If the natural growth rate of biological stock is decelerated by limited base resources required for species survival (such as land, water, etc.) then social planner should provide them even by incurring to additional costs. This option (also suggested by T. M. Swanson [22] in attempt to avoid species extinction) requires to make some adjustment to the traditional harvesting model (2.1)-(2.2), namely:

$$\max_{h(t),R(t)\geq 0} \int_0^\infty e^{-\rho t}\left[U\left(x(t),h(t)\right)-\rho C_R R(t)\right]dt \tag{2.3}$$

$$\text{s.t.}\quad \frac{dx}{dt}=F(x,R)-H(x,h),\quad x(0)=x_0 \tag{2.4}$$

where C_R is the price of a unit of base resource R. These alteration in the model indicate that, given sufficient resources for the species survival and reproduction, the biological stock will give a competitive return even after discounting the total cost of allocation of additional base resource (expressed by the term $\rho C_R R(t)$ in (2.3)).

It is worthwhile to note that common-property models with harvesting described by (2.1)–(2.2) are derived under implicit assumption that biological resources are naturally “free goods” and do not require (external) investments. However, Swanson’s model (2.3)–(2.4) does require societal investment R in allocation of base resources to the extent that the species is able to generate a competitive return on its own stock value and to provide an additional surplus which compensates the investments in its sustenance.

3. Consumptive and non-consumptive utility

Effectively, Swanson’s idea can be viewed as an initial attempt to modify the biological dynamics of species growth with defensive actions of policymakers aimed at species protection. This idea was further developed by R. Alexander [2] who proposed to include in the objective functional both consumptive and *non-consumptive* values of the biological species. His pioneering term “non-consumptive utility” principally refers to “non-consumptive use values of endangered species” (in the sense of Boyle and Bishop [7]) and can be perceived as species existence values[3].

Traditional harvesting models fail to account for existence values of endangered biological species even though some particular species may have more significant existence values than their consumptive valuation. Using the African elephant (*Loxodonta africanus*) as an example, R. Alexander

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[3] The term “non-consumptive utility” also stands for “non-harvesting utility” for biological species with no harvest value.

had proposed another bioeconomic model that accounts for both consumptive and non-consumptive utility of this particular species and highlights the incentives faced by economic agents who make decisions affecting endangered species:

$$\max_{h(t)\geq 0} \int_0^\infty e^{-\rho t} \left[U\left(x(t), h(t)\right) - C_R R x + U_{nc}(x) \right] dt \tag{3.1}$$

$$\text{s.t.} \quad \frac{dx}{dt} = F(x) - H(x,h), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0. \tag{3.2}$$

In the above model, $U(x,h)$ expresses the utility of harvesting (ivory and non-ivory products, revenues from safari hunting minus the total cost of harvesting), C_R is the unit value of land resources used by elephants, Rx is quantity of land resources used by elephants as a constant proportion of their population, and $U_{nc}(x)$ stands for *non-consumptive* utility. The latter can be displayed as

$$U_{nc}(x) = P_T T(x) + N(x), \tag{3.3}$$

where P_T is the unit price of one tourist day, $T(x)$ is tourist-days as function of population, and $N(x)$ is the non-market existence value of elephants as function of population[4]. The non-consumptive utility (3.3) displays revenues obtained from tourism and contributes to people's awareness that this species exists (for both harvest and tourism purposes).

Remark 1. It is worthwhile to note that in contrast to Swanson's model (2.3)-(2.4), land resources allocated to elephants are not expressed as a control variable in the model (3.1)-(3.2). Instead, R. Alexander suggests that if the correct incentives are put in place in society, further transfers of land resources from alternative uses may naturally arise through the market.

Inclusion of non-consumptive values of a wildlife resource illustrates social values and benefits of maintaining that resource. Such benefits become particularly important for studying endangered species with no consumptive (or harvesting) value.

Many scholars agree that the principal cause of (local) extinction of wildlife species with no harvesting value is merely incidental. In other words, local extinction is frequently provoked by habitat reduction due to urbanization, farming, and other anthropic activities, that are not directly intended to harm the species. The obvious questions that arise are:

— Do such wildlife species have a chance to avoid eventual extinction?

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[4] R. Alexander does not provide explicit forms for $T(x)$ and $N(x)$ and merely states that they are strictly increasing and concave. The latter makes sense since the non-consumptive utility (3.3) augments as x increases while its marginal return decreases due to saturation or overpopulation.

— Can human external intervention contribute to conservation of such species?

In attempt to answer these questions, R. Alexander and D. Shields [3] had proposed a *non-harvesting* bioeconomic model for conservation of the New Zealand yellow-eyed penguin (*Megadyptes antipodes*). This model is significantly different from those revised above, and its objective functional represents the net returns to society derived from tourism, in other word, there is no harvesting value involved. This model is formulated as follows:

$$\max_{L(t)\geq 0} \int_0^\infty e^{-\rho t} U_{tour}\left(x(t), L(t)\right) dt, \tag{3.4}$$

$$\text{s.t.} \quad \frac{dx}{dt} = F(x, L), \quad x(0) = x_0. \tag{3.5}$$

where $U_{tour}(x, L)$ expresses the *non-consumptive* utility obtained from tourism business[5], x is the population of yellow-eyed penguin, and L represents an index of the quality of land resources used by the penguins (control variable). The resource owner may reallocate his land resources $L(t)$ available for penguins, and the latter will be reflected in the population growth, for example,

$$F(x, L) = rx\left(1 - \frac{x}{K \cdot L}\right)$$

assuming logistic growth of the species. Within the mathematical framework of the optimal control theory, R. Alexander and D. Shields [3] derived a *non-harvesting* variant of so-called "golden rule" which is widely used in models with harvesting[6]. Further argument, based on this relationship, results in rather logical conclusion. Namely, if the population is relatively small (less than a half of current carrying capacity) and the intrinsic growth rate of the species is below the discount rate (i.e., $r < \rho$), then it will be optimal to allocate more land resources to the species. On the other hand, if the population is relatively small and the intrinsic growth rate of the species exceeds the discount rate (i.e., $r > \rho$), then it will be optimal to decrease the land allocated to the species up to some "sustainable" level. This implies that the initial population density was too low on the land initially available, and that by increasing that density (that is, by reducing the carrying capacity) one may guarantee an economically viable income from tourism. Impartially speaking, a tourist may choose not to pay for a visit when the probability

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[5] According to [3], $U_{tour}(x, L) = \left[P_T T(x) - C_L - C_O\right] L$ where $P_T T(x)$ represents the income generated by tourist visits, C_L is the cost per unit of land resources used by the penguins, and C_O is the cost of operation of tourism enterprise.

[6] In fishery models of the type (2.1)-(2.2), the "golden rule" is a relationship that specifies an "economically optimal" equilibrium biomass level x^*. Therefore, the control variable of fishing effort, $h(t)$, is chosen in order to move the initial stock x_0 towards x^*. For more detail, the reader may refer to [10].

of sighting is small, but once that probability becomes close to certainty, additional population is unlikely to generate additional visits. Therefore, there must be an economically "sustainable" level of population and an "economically viable" amount of land resources should be allocated to the species.

4. Conservation policies under economic growth

In a broader sense, the control variable L in the model (3.4)-(3.5) can be viewed as a defensive expenditure of the society aimed at the conservation of the natural habitat of the species. On the other hand, there are many wild species with rather low intrinsic growth rate that may require not only the land but other resources (such as additional water, food, special conditions for breeding, etc.) for their survival and sustainable growth. Additionally, the model (3.4)-(3.5) does not explicitly include any impact (positive or negative) that anthropic activity and aggregated production may have on the natural evolution of species population.

A study conducted by A. Antoci et al. [5; 6] claims that defensive actions may deeply alter the natural ecological dynamics and modify its equilibria. In support of this argument, the authors of [5; 6] had proposed another variant of bioeconomic model with two interacting species $\boldsymbol{x} = (x_1, x_2)$ and non-harvesting utility $\hat{U}_{nh}$ which is stated as follows:

$$\max_{\substack{c(t)\,>\,0\\ d(t)\,\geq\,0}} \int\limits_0^\infty e^{-\rho t}\hat{U}_{nh}\left(\boldsymbol{x}(t), c(t)\right) dt, \tag{4.1}$$

$$\text{s.t.} \quad \begin{cases} \dfrac{d\boldsymbol{x}}{dt} = \boldsymbol{F}(\boldsymbol{x}) - k^{\alpha}(t)\boldsymbol{\epsilon} + d^{\mu}(t)\boldsymbol{\sigma}, & \boldsymbol{x}(0) = \boldsymbol{x}_0 > 0 \\[2ex] \dfrac{dk}{dt} = pk^{\alpha}(t) - c(t) - d(t), & k(0) = k_0 > 0 \end{cases} \tag{4.2}$$

where $\boldsymbol{F}(\boldsymbol{x}) = (F_1(x_1, x_2), F_2(x_1, x_2))'$ is a linear affine vector function of $(x_1, x_2)'$ while the positive components of $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2)'$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2)'$ measure, respectively, the negative impact of aggregated production k and the positive impact of generic defensive expenditure d on the target populations x_1 and x_2. The second equation in (4.2) describe the accumulation of capital $k(t)$ which is considered as a sole product of global economy. The capital output can be used for re-investment (expressed by the production function $pk^{\alpha}(t)$, $0 < \alpha < 1$ of Cobb-Douglas type), consumption $c(t)$, and defensive expenditures $d(t)$.

Economic activity modeled by the second equation in (4.2) induces obvious changes in natural biological dynamics of the species given by the first equation in (4.2). Namely, the pollution, habitat reduction and other consequences attributed to aggregated production $k^{\alpha}(t)$, will have *negative* effect for both populations $x_1(t)$ and $x_2(t)$. On the other hand, defensive expenditures d will constitute *positive* effect for $x_1(t)$ and $x_2(t)$. Thus, the natural biological dynamics becomes amended with both negative and positive effects of anthropic activity.

It should be noted that parameter $\mu \in (0,1)$ is introduced in order to emphasize that the positive effect of defensive investment on the specie evolution *is not* directly proportional to population growth; in other words, extra-spending on species conservation (increase in $d(t)$) may decrease the positivity of such impact on the species evolution due to the carrying capacity limitations of the environment.

It is assumed that there is a representative agent in the economy whose welfare $\hat{U}_{nh}\left(\boldsymbol{x}(t), c(t)\right)$ depends, in each instant of time t, on the consumption $c(t)$ and on the present amounts of both species $\boldsymbol{x} = (x_1, x_2)$. It should be emphasized that these species have no harvesting value. In [5], two alternative forms of $\hat{U}_{nh}\left(\boldsymbol{x}(t), c(t)\right)$ had been proposed:

$$U_1(x_1, x_2, c) \;=\; q_1 x_1 + q_2 x_2 + q \ln c, \tag{4.3}$$

$$U_2(x_1, x_2, c) \;=\; q_1 \ln x_1 + q_2 \ln x_2 + q \ln c, \tag{4.4}$$

where q_1, q_2, and q are strictly positive weight parameters. These utility functions U_1 and U_2 clearly reflect the priorities of decision-making. Namely, function U_1 expresses that the *non-harvesting utility* of the species is directly proportional to the species abundance and has constant utility gain, while the utility of consumption (logarithmic term) has decreasing utility gain. Thus, the maximization problem (4.1)-(4.2) with function (4.3) may yield an optimal policy $(c^*(t), d^*(t))$ which is not adverse to species' extinction since $x_1 = 0$ and/or $x_2 = 0$ can be paid off by an aggregated consumption level.

Alternatively, the maximization problem (4.1)-(4.2) with function (4.4) requires that $x_1 > 0, x_2 > 0$ since otherwise the representative agent may suffer an infinite loss of utility when either $x_1 \to 0^+$ or $x_2 \to 0^+$. Therefore, this definition of utility function should yield an optimal policy $(c^*(t), d^*(t))$ that favors the conservation of the species.

The analysis of the model (4.1)-(4.2) (see [5]) suggested that if the agents get constant marginal utility from x_1 and x_2 (i.e. function (4.3) is applied), then the stability features of natural equilibrium (that is, of the ecosystem $d\boldsymbol{x}/dt = \boldsymbol{F}(\boldsymbol{x})$ with no human intervention) will be preserved in the integrated bioeconomic system regardless of both populations' initial or fixed-point levels. In other words, if the fixed point $\bar{\boldsymbol{x}}$ of natural system $(\boldsymbol{F}(\bar{\boldsymbol{x}}) = 0)$ is a repellor (or saddle point, or attractor) then under optimal policy $(c^*(t), d^*(t))$ it will remain being a repellor (or saddle point, or attractor) of the bioeconomic system (4.2). The latter implies that there is no policy capable to convert an "initial" repellor into a "future" attractor when the agents do not care for the species protection.

On the contrary, if the agents suffer an infinite loss of utility from species extinction (i.e. function (4.4) is applied), the defensive expenditures may alter the natural evolutionary dynamics of the interacting species. A study conducted in [6] revealed an existence of optimal policy $(c^*(t), d^*(t))$ under which the stability can be achieved in the bioeconomic system (4.2) even for "naturally unstable" interacting populations.

The results of Antoci *el al.* [5; 6] comply with a study focused on single-species model with Gompertz population growth under human intervention (see [8]). This bioeconomic model fits into the formal framework of (4.1)-(4.2) when instead of two interacting species x_1, x_2 a single one $(0 \le x \le K)$ is considered, and its biological growth is described by so-called Gompertz function $F(x) = rx \ln(K/x)$ where $K > 0$ stands for carrying capacity of the environment. Since $x = K$ is an attractor of natural dynamics with Gompertz population growth, there exists an optimal policy $(c^*(t), d^*(t))$ capable to preserve stability features in the bioeconomic system (4.2) with either $U_1(x, c) = q_1 x + q \ln c$ or $U_2(x, c) = q_1 \ln x + q \ln c$ substituted in (4.1). However, the latter is possible only if the population value in the fixed point $\bar{x}$ of bioeconomic system (4.2) is relatively high, namely, $(K/e) < \bar{x} \le K$, and if the initial population $x(0) = x_0$ is sufficiently close to $\bar{x}$.

Another pertinent contribution to this strand of research is a single-species bioeconomic model with logistic population growth and non-harvesting utility of the form (4.3) which was thoroughly analyzed in [12]. This bioeconomic model also fits into the formal framework of (4.1)-(4.2) when instead of two interacting species x_1, x_2 a single one $(0 \le x \le K)$ is considered, and its biological growth is described by logistic function $F(x) = rx(1 - x/K)$ where $K > 0$ denotes the carrying capacity.

Besides confirming the general trend on preservation of stability features (claimed in preceding works [5; 6; 8]), this study also addressed an important question: *Can the defensive expenditures mitigate the negative impact of aggregated production on the species population and to what extent?*

To answer this question, the underlying features of logistic dynamics (such as its quadratic nature and symmetry of the fixed points with respect to the axis $x = K/2$) were rather beneficial, and helped to disclose and visualize some essential changes in stability properties of the "amended" ecological dynamics, that is, natural population dynamics modified by human intervention. It was established that such changes principally depend on the model's parameters, including two control variables c and d.

The range of damage possibly caused by the aggregated production (term $-\epsilon k^{\alpha}$ in the first equation of (4.2)) to the species population can be estimated by varying $\epsilon > 0$ while holding other parameters of the model constant. This procedure yields two ϵ-depended fixed points $x^*_-(\epsilon)$ and $x^*_+(\epsilon)$

satisfying the following relationships:

$$0 \le x_-^*(\epsilon) \le \frac{K}{2} \le x_+^*(\epsilon) \le K, \quad x_-^*(\epsilon) + x_+^*(\epsilon) = K.$$

Additionally, it resulted possible to define particular values of the parameter $\epsilon > 0$ such that

$$\epsilon_* : \quad x_-^*(\epsilon_*) = 0, \quad x_+^*(\epsilon_*) = K \quad \text{(logistic case)} \tag{4.5}$$

$$\epsilon_0 : \quad x_-^*(\epsilon_0) = x_+^*(\epsilon_0) = \frac{K}{2} \quad \text{(degenerate case)} \tag{4.6}$$

From the ecological point of view, ϵ_* can be referred to as *opportune reference value* since it characterizes an “ideal situation”, that is, both positive and negative effects of aggregated production do not alter the natural equilibrium of the dynamic system. On the other hand, ϵ_0 can be called *critical reference value* since it characterizes an “unhealthy situation” when the equilibrium of the system is unreachable in infinite time t. Both *reference values* ϵ_* and ϵ_0 can be determined outside of the model in accordance with other constant parameters, and then provide the decision-maker with some useful insights regarding to actual ecological situation described by the model (4.1)-(4.2). Naturally, if the value of ϵ given in the first equation of (4.1) is close to ϵ_*, then we say that ecological situation is “good”; otherwise, if ϵ is proximal to ϵ_0, the situation is “bad”.

Moreover, if $\epsilon \neq \epsilon_0$ and if the initial population $x(0) = x_0$ is situated closer to $x_+^*(\epsilon)$ than to $x_-^*(\epsilon)$ (that is, $x_0 > K/2$) there exists an optimal policy $(c^*(t), d^*(t))$ capable to guarantee stability of the bioeconomic system (4.2) even when the ecological situation is regarded as “bad” (that is, if $\epsilon > 0$ given in the model is closer to ϵ_0 than to ϵ_*). Under such policy, the species population trajectory will eventually reach the fixed point of maximal possible abundance of the species ($K/2 < x_+^*(\epsilon) \le K$).

It is worthwhile to note that all results described so far deal with design of *long-term optimal policies* capable to provide stability of bioeconomic systems when $t \to \infty$ and initial populations are proximal enough to their fixed-point values. Such policies are rather useful when the primary goal is *to maintain* the species population(s) on some “desired” level. However, these policies are incapable to enhance (in finite or even infinite time) initially scarce populations and thus save the species from eventual extinction.

5. Conservation policies aimed at enhancement of scarce populations

In the previous section it was shown that initial abundance of the species population is indispensable for existence of long-term conservation policies.

How crucial it is in case of short-term planning? To answer this questions, a finite-time variant of bioeconomic model (4.1)-(4.2) was proposed for single-species population with logistic biological growth [13], namely:

$$\max_{\substack{c(t)\,>\,0 \\ d(t)\,\geq\,0}} \int\limits_0^T U_1\left(x(t), c(t)\right) dt + x(T) + k(T), \tag{5.1}$$

$$\text{s.t.} \quad \begin{cases} \dfrac{dx}{dt} = F(x) - \epsilon k^{\alpha}(t) + \epsilon d^{\mu}(t), & x(0) = x_0 > 0 \\[2ex] \dfrac{dk}{dt} = p k^{\alpha}(t) - c(t) - d(t), & k(0) = k_0 > 0 \end{cases} \tag{5.2}$$

where $F(x)$ has logistic form and $U_1(x,c) = q_1 x + q \ln c$ expresses the preferences of decision-making that are not adverse to local species extinction (rather cruel but more realistic case). Here the objective functional (5.1) is different from the previous one (4.1) and targets not only to maximize the overall utility within the finite period of time $[0,T]$ but also to enhance the terminal values of both state variables — species population $x(T)$ and capital $k(T)$. Additionally, the discount factor $e^{-+\rho t}$ is suppressed in (5.1) in order to emphasize that the planning task is set for a short period of time. In other words, it is supposed that discount rate ρ is close to zero[7].

Among several scenarios meticulously considered in [13], there is one of particular interest and it deserves to be quoted. This scenario deals with design of optimal policies aimed at enhancement of initially scarce populations (i.e., $x(0) = x_0$ is rather low) when the initial level of production $k(0) = k_0$ is reasonably high.

Under this scenario, there are sufficient capital resources k_0 to be spent immediately for defensive expenditures $d(t)$ in order to obtain a significant increase in the species population $x(t)$ by the final time T. Given the scarcity of biological species at initial time, the social planner must implement the defensive policy from very beginning and continue to spend increasingly up to the mid-point $(T/2)$ of the time lapse. At the same time, the optimal consumption $c(t)$ and capital accumulation $k(t)$ should be maintained strictly increasing within $[0,T]$ in order to guarantee maximization of the integral part and second terminal-value term of the objective (5.1)[8]. The optimal decision policy $(c^*(t), d^*(t))$ proposed in [13] complies with maximization criterion (5.1) in the sense that:

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[7] Strictly speaking, the intertemporal discount rate ρ characterizes the patience of decision-makers when it comes to obtaining the maximum overall utility and lower values of ρ describe rather patient social planning ($U_1(x,c)e^{-\rho t} \approx U_1(x,c)$ when $\rho \approx 0$).

[8] The term $k(T)$ is included in the objective in order to ensure sufficient capital resources for next planning period (e.g., $[T, 2T]$) if a desired level of species population is not achieved during $[0,T]$.

- it guarantees a significant increase of final states $x^*(T), k^*(T)$ and thus impede the species extinction;
- it ensures the growth of utility (expressed by the integral part) since $U(x,c)$ is an increasing function of x and c.

Thus, high initial level of production k_0 plays an essential and *notably positive* role for further enhancement of initially scarce populations. However, initial abundance of capital resources is only necessary (but not sufficient) condition for existence of the optimal policy $(c^*(t), d^*(t))$. The length of planning horizon $T > 0$ also play an essential role. Strictly speaking, optimal policy $(c^*(t), d^*(t))$ obtained for $t \in [0,T]$ cannot be simply "truncated" or "extended" to shorter or longer time intervals (such as $[0,T/2]$ or $[0,2T]$, for example).

Numerical experiments held in [13] disclosed that the variation of the length of planning period $[0,T]$ has significant impact on decision policies and the underlying value of the objective (5.1). Namely, shorter periods are more "expensive" since they result in decline of capital accumulation during the whole period while ensuring steady increase in consumption and providing moderate rise of the species population. Alternatively, longer planning periods guarantee strictly increasing capital accumulation together with significant enhancement of the species population and without neglecting the consumption. Thus, there is coherence between the planning horizon T and the preferences of decision-making.

Effectively, in order to achieve faster the highest level of consumption, one may employ a shorter-time decision policy that disregards the capital accumulation while trying to enhance the species population. Conversely, longer-time decision policy will guarantee significant increase in both states (species population x and capital k) on the cost of lesser overall consumption. Therefore, a social planner may implement successively a sequence of decision policies for $[T_0,T_1] \cup [T_1,T_2] \cup \cdots \cup [T_{N-1},T_N]$ where $T_0 = 0$ and $\Big(x^*_{j+1}(t), k^*_{j+1}(t), c^*_{j+1}(t), d^*_{j+1}(t)\Big)$ are defined as solutions of the optimization problem (5.1) subject to (5.2) for $t \in [T_j, T_{j+1}],\ j = 0,\dots,N-1$ with initial conditions given in the form

$$(x_1(T_0), k_1(T_0)) = (x_0, k_0) \ \text{ when } \ t \in [T_0,T_1] \text{ and } j = 0,$$

$$(x_{j+1}(T_j), k_{j+1}(T_j)) = \big(x^*_j(T_j), k^*_j(T_j)\big)$$
$$\text{when } \ t \in [T_j, T_{j+1}] \text{ and } j = 1,\dots,N-1.$$

Such a "partitioned" planning may turn up rather beneficial due to its capability to reflect different priorities of decision-making within each subinterval $[T_j, T_{j+1}]$.

Finally, it will be fair to assert that economic growth may have two-fold effect on the species evolution. On the one hand, the aggregated production may reduce the species population to dangerous levels (leading to local extinction) if the social planner does not spend at all (or spends too little) on defensive measures, preferring to disburse the whole capital surplus (or a great part of it) solely for consumption.

On the other hand, the aggregated production may provide additional surplus which, being destined for defensive expenditures, may significantly enhance the species population. In this case, the social planner should find and implement an optimal decision policy $(c^*(t), d^*(t))$ which utterly depends on three factors:
– the initial species population x_0,
– the initial capital k_0, and
– the length of policy implementation (that is, final time T)
while other parameters of the model (5.1)-(5.2) are kept unchanged.

Remark 2. In mathematical terminology, if either one or both initial values x_0, k_0 are too small for a chosen final time T, the *optimality system*[9] may simply fail to have *feasible* solution in bioeconomic sense, i.e., such that

$$(x^*(t), k^*(t)) \in \mathbb{R}^2_+, \quad \forall t \in [0, T].$$

To remedy the situation, the social planner will be forced to extend the length of policy implementation period; in other words, to increase the value of T.

6. Further perspectives and conclusions

The controversy between the preservation of biological species and the inevitable proliferation of multiple economic activities has been a central topic for many debates in academic circles. However, the main stream of underlying research has been focused at design of long-term conservation policies seeking to maintain so-called “bioeconomic equilibrium” that permits a “sustainable use” of natural resources. Under this posture, the conservation of wildlife species with no harvesting value has been nearly ignored.

There is no secret that rather common wildlife species worldwide are becoming locally extinct in the surroundings of megalopolis areas not because of animal slaughter but because of lack of food, water or land resources necessary for the species survival and reproduction. To counterbalance this tendency, policy makers are compelled to design and implement defensive actions aimed at enhancement of wildlife species populations. These defensive actions should be set for near future and have a *finite* planning horizon in order to impede local extinction of the species.

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[9] That is, the boundary value problem that results from the application of Pontryagin maximum principle to the model (5.1)-(5.2)

Previous studies (see e.g. [5; 6; 8; 12] and references therein) have revealed that initial abundance of the species population x_0 plays central role in the long-term conservation planning (that is, when planning horizon is set as infinite: $T \to \infty$). In other words, long-term policies are incapable to save initially scarce populations from eventual extinction.

In case of finite-horizon planning, the absolute value of x_0 is of lesser concern than initial capital resources k_0. Thus, the forecast for near future can be rather optimistic and reassuring even for scarce species populations x_0 at initial time. Thereby, it is not enough to have time, it is also necessary to have relatively high level of initial capital and to spend it quickly for conservation.

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О. О. Васильева

Методы оптимального управления в моделях природоохранной биологии с неурожайным критерием

Аннотация. Основной целью данной статьи является проследить эволюцию математических моделей, ориентированных на взаимодействие между экономическим ростом, устойчивым развитием и сохранением окружающей среды. В качестве отправной точки была взята простейшая модель сбора урожая, в которой возобновляемые ресурсы растут согласно природным законам. Далее рассматривается другой вариант этой модели, в которой учитываются расходы на рост биологических видов. В основном, модели такого типа предназначены для поиска долгосрочных оптимальных стратегий сбора урожая, где сохранение биологических видов гарантирует устойчивую прибыль и, следовательно, вносит вклад в экономическое развитие.

С другой стороны, существует группа моделей, где факторы, связанные с человеческой деятельностью, а также экономический рост могут оказать положительное или отрицательное влияние на естественную эволюцию диких видов. Некоторые исследователи доказали, что оптимальные стратегии, которые относительно хороши для урожайных целей, в контексте сохранения биологических видов не переносят-

Известия Иркутского государственного университета.
2014. Т. 8. Серия «Математика». С. 44–61

ся на неурожайные критерии. Однако проведение долгосрочной природоохранной политики для всех биологических видов не может быть гарантировано, если в начальный момент времени популяция не имеет достаточно большой численности. Следовательно, такие стратегии неспособны повысить численность вымирающих видов и спасти их от окончательного (локального) исчезновения. В качестве альтернативы, политики вынуждены разрабатывать и реализовывать краткосрочные природоохранные действия, направленных на улучшение популяций диких видов.

Ключевые слова: био-экономические модели, вымирающие виды, охрана природы, неурожайный критерий, оптимальное управление.

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