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Kinds of Pregeometries of Acyclic Theories

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Abstract. The article is devoted to the description of types of pregeometries with an algebraic closure operator for acyclic theories. In these theories we describe conditions of violation of the exchange property for a pregeometry. Taking into account these conditions, we introduce new concepts that do not rely on the exchange property: a-pregeometry, a-modularity, etc. The dependence conditions for an a-modular and a-locally finite a-pregeometry on the number of non-isomorphic trees and special points are established. Sufficient conditions of dependence for a a-local finite a-pregeometry on the vertices of the a-type are established, too.

Keywords: pregeometry, cyclic theory, a-pregeometry, a-modularity, a-locally finite, special vertices, A-special vertices

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Научная статья

Виды предгеометрий ациклических теорий

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Аннотация. Рассмотрены виды предгеометрии с оператором алгебраического замыкания для ациклических теорий. Описаны условия нарушения свойства замены для предгеометрий в этих теориях. Учитывая эти условия, введены новые понятия не опирающиеся на свойство замены: a-предгеометрия, a-модулярность и др. Установлены условия зависимости для a-модулярной и a-локально конечной a-предгеометрии от числа неизоморфных деревьев и специальных точек, а также достаточные условия зависимости для a-локально конечной a-предгеометрии от вершин a-типа.

Ключевые слова: предгеометрия, ациклическая теория, a-предгеометрия, a-модулярность, a-локальная конечность, специальные вершины, A-специальные вершины

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1. Introduction

In the 1970s and 1980s, an active research of the main types of pregeometries and geometries for classes of ω-stable and o-minimal structures has begun. This research includes works by B.I. Zilber [17–20], G. Cherlin, L. Harrington, A. Lachlan [4] and A. Pillay [12]. In the 1970s B.I. Zilber has obtained a series of results and formulated hypotheses on uncountably categorical theories, among which the key one was on the possibility of classifying such theories up to biinterpretibility. In 1986, A. Pillay showed that if an o-minimal theory is modular, then weak elimination of imaginaries follows it [12]. It is known that in the strongly minimal case, again under the modularity, geometric elimination of imaginaries holds [11]. In 1996, E. Hrushovski [7] proposed an original construction of a strongly minimal structure that is not locally modular and for which it is impossible to interpret an infinite group. Today pregeometries and geometries continue to attract the attention of scientists, for example, in the description of the types of geometries of various objects [1–3], in particular, Vamos matroids [10]. Therefore, natural questions arise on the classification of pregeometries and geometries for various significant classes of structures and their theories.

This article contains necessary and sufficient conditions that have been found for the types of pregeometries of acyclic theories with an algebraic closure operator. Definitions of pregeometry and its types have been taken from the paper [11]. It has been also noticed in the article that the exchange property is violated for the pregeometry of acyclic theories. Taking this into account, new concepts that do not rely on this property are introduced: a-pregeometry and a-dimension. Using a-dimension, a new definition of a-modularity was introduced: an analogue of a-modularity for a-pregeometry. For acyclic theories, dependences of the a-modularity and a-local finiteness of the a-pregeometry on the number of non-isomorphic trees and special points were established. Sufficient dependence conditions for the a-local finite a-pregeometries on the vertices of the a-type were also stated.

2. Pregeometry. Types of pregeometry

In this paper we use the definitions from [4–6; 9; 11; 13; 14] and [15].

Definition 1. *A* pregeometry *is a set S together with a certain closure operation* $\mathrm{cl} : P(S) \to P(S)$ *satisfying the following conditions:*

1) for any $X \subseteq S$, $X \subseteq \mathrm{cl}(X)$ is executed;

2) for any $X \subseteq S$, $\mathrm{cl}(\mathrm{cl}(X)) = \mathrm{cl}(X)$ is executed;

3) for any $X \subseteq S$ and for any $a, b \in S$ if $a \in \mathrm{cl}(X \cup \{b\}) - \mathrm{cl}(X)$ then $b \in \mathrm{cl}(X \cup \{a\})$;

4) for any $X \subseteq S$ if $a \in \mathrm{cl}(X)$ then $a \in \mathrm{cl}(Y)$ for some finite $Y \subseteq X$.

Having a pregeometry $\langle S, \mathrm{cl}\rangle$, each subset $X \subseteq S$ has a minimal set $X' \subseteq X$, under inclusion, such that $\mathrm{cl}(X) = \mathrm{cl}(X')$. This minimal set X' is called the basis of the set X. In this case, the cardinality $|X'|$ does not depend on the choice of the basis in the X, and it is called the dimension of the set X in the pregeometry $\langle S, \mathrm{cl}\rangle$, denoted by $\dim(X)$. By the definition we have $\dim(X) = \dim(\mathrm{cl}(X))$, i.e. the dimension is preserved under the transition to the closure of the set X in the pregeometry $\langle S, \mathrm{cl}\rangle$. If $\dim(X) \in \omega$ then set X is called finite-dimensional.

Definition 2. *A set $X \subseteq S$ is called closed if $X = \mathrm{cl}(X)$.*

Definition 3. *A pregeometry $\langle S, \mathrm{cl}\rangle$ is called trivial or degenerate if for every $X \subseteq S$, $\mathrm{cl}(X) = \bigcup\{\mathrm{cl}(\{a\}) \mid a \in X\}$.*

A pregeometry $\langle S, \mathrm{cl}\rangle$ is called modular if for any closed sets $X_0, Y_0 \subseteq S$, X_0 independent from Y_0 with respect to $X_0 \cap Y_0$, i.e. for any finite-dimensional closed sets $X \subseteq X_0$, $Y \subseteq Y_0$ is true

$$\dim(X) + \dim(Y) - \dim(X \cap Y) = \dim(X \cup Y).$$

A pregeometry $\langle S, \mathrm{cl}\rangle$ *is called local modular if, for any* $a \in S$, *the pregeometry* $\langle S, \mathrm{cl}_{\{a\}}\rangle$ *is modular, where* $\mathrm{cl}_{\{a\}}(X) = \mathrm{cl}(X \cup \{a\})$.

A pregeometry $\langle S, \mathrm{cl}\rangle$ *is called projective if it is modular and not-trivial, and is called locally projective if it is locally modular and not-trivial.*

A pregeometry $\langle S, \mathrm{cl}\rangle$ *is called locally finite if for any finite subset* $A \subseteq S$, *the set* $\mathrm{cl}(A)$ *is finite.*

Definition 4. *Let* M *be a model of a theory* T. *Then the algebraic closure operator for the model* M *is an operator* $\mathrm{acl} : P(M) \to P(M)$ *such that for every subset* $X \subseteq S$, $\mathrm{acl}(X) = \{a \in S \mid S \models \exists^{<\omega} x\phi(x, \bar{b}) \wedge \phi(a, \bar{b})$ *for some formula* $\phi(x, \bar{y})$ *and* $\bar{b} \in X\}$.

Throughout we consider $\langle S, \mathrm{acl}\rangle$-type pregeometries only.

3. Kinds of pregeometries of acyclic theories

Definition 5. *An acyclic structure is a graph* $\Gamma = \langle X; R\rangle$, *in which each connected component is a tree. A theory* T *of a graph signature* $\{R^{(2)}\}$ *is called acyclic if* $T = \mathrm{Th}(M)$ *for some acyclic structure* M. *In this case, the structure* M *is called an acyclic model of the theory* T.

Definition 6. In a graph a vertex a is called a *n-vertex* (*∞-vertex*) if it is incident with n edges (infinitely many edges).

Remark 1. For a structure $\langle S, \mathrm{acl}\rangle$ in an acyclic theory T, the exchange property does not always hold, for instance, if a model contains a tree consisting of ∞-vertices only.

Indeed, let the model contain a tree with all infinite degree vertices. Then any three different elements lying on the same path will violate the exchange property.

By virtue of the remark, for a structure $\langle S, \mathrm{acl}\rangle$, the dimension of a trees should be considered as the dimension of a structure and we should talk about the a-modularity of the pregeometry, i.e. about the connection of the dimensions of a trees without relying on the exchange property. In this case, an acyclic structure $\langle S, \mathrm{acl}\rangle$ satisfying the conditions 1), 2), 4) of definitions of a pregeometry will be called an *a-pregeometries.*

Definition 7. *For an acyclic theories* T, *the a-dimension* $dim_a(A)$, *where* $A \subseteq M \models T$, *is considered to be the value* $\mu_A + \sum_{D'} \nu_{A\cap D'}$, *where* μ_A *is the number of a finite trees* $D \subseteq M$ *with condition* $A \cap D \neq \emptyset$, *and* $\nu_{A\cap D'}$ *is the number of vertices of the smallest subtrees of* K *a infinite trees* $D' \subseteq M$ *with condition* $A \cap D' \subseteq K$.

Definition 8. *An a-pregeometry* $\langle S, \mathrm{cl}\rangle$ *is called* a-modular *if for every* acl-*closed sets* $X_0, Y_0 \subseteq S$, X_0 *is independent from* Y_0 *relating to* $X_0 \cap Y_0$, *i.e. for every finite-dimensional* acl-*closed sets* $X \subseteq X_0$, $Y \subseteq Y_0$ *is true:*

1) if there exists an infinite tree D *for which* $X \cap Y \cap D = \emptyset$, $X \cap D \neq \emptyset$, $Y \cap D \neq \emptyset$, *then the equality holds:*

$$\dim_a(X \cap D) + \dim_a(Y \cap D) + \\ +\rho(X \cap D, Y \cap D) = \dim_a((X \cup Y) \cap D), \tag{3.1}$$

where $\rho(X \cap D, Y \cap D)$ *is the number of vertices of the shortest path between the vertices* $x \in X \cap D$ *and* $y \in Y \cap D$ *(without counting the vertices of these sets);*

2) in other cases, for the connectivity components D *is satisfied the following equality:*

$$\dim_a(X \cap D) + \dim_a(Y \cap D) - \dim_a(X \cap Y \cap D) = \dim_a((X \cup Y) \cap D). \tag{3.2}$$

Remark 2. According to the definitions of a-dimension and a-modularity, when summing the relations (3.1) and (3.2) over all connected components of D , some generalized analogue of the modularity formula in pregeometry for a-pregeometry is obtained:

$$\sum_D \dim_a(X \cap D) + \sum_D \dim_a(Y \cap D) - \sum_D \dim_a(X \cap Y \cap D) + \\ + \sum_D \rho(X \cap D, Y \cap D) = \sum_D \dim_a((X \cup Y) \cap D). \tag{3.3}$$

Theorem 1. *Let* T *be an acyclic theory with an infinite saturated model* $M = \langle S, R\rangle$. *Then the a-pregeometry of* $\langle S, \mathrm{acl}\rangle$ *is a-modular.*

Proof. Due to the fact that every non-empty intersection of trees in the model of acyclic theory is again a tree we note that with the exception of the first case in the definition of a-modularity under the conditions of the second point of the theorem, the equality (3.2) holds, defining the modularity.

If there is an infinite tree D: $X \cap Y \cap D = \emptyset$, $X \cap D \neq \emptyset$, $Y \cap D \neq \emptyset$. In this case, $\nu_{X \cap Y} = 0$. $\nu_{(X \cap D) \cup (Y \cap D)}$ is equal to the number of vertices of the smallest subtrees, that containing $X \cap C$, $Y \cap C$ and the number of vertices of the shortest path between the sets $X \cap C$ and $Y \cap D$. Denoted by $\rho(X, Y)$. If the sets X and Y intersect in an infinite tree D, i.e. $X \cap Y \cap D \neq \emptyset$, then $\rho(X, Y) = 0$ and the equality

$$\nu_{X \cap D} + \nu_{Y \cap D} - \nu_{X \cap Y \cap D} = \nu_{(X \cap D) \cup (Y \cap D)}$$

holds.

Thus, the a-pregeometry is a-modular. □

Definition 9. *Let G be a group of substitutions on the universe of the model S, with the condition $\forall g \in G\ \forall a, b \in S\ R(g(a), g(b)) = R(a, b)$. Then a vertex a is called* special*, if its orbit $aG = \{g(a) | \forall g \in G\}$ is finite.*

Definition 10. *Let G_A be a group of substitutions on the universe of the model S, with a fixed set $A \subseteq S$, i.e. $\forall g \in G\ \forall a \in A\ g(a) = a$. And with the condition $\forall g \in G\ \forall a, b \in S\ R(g(a), g(b)) = R(a, b)$. Then a vertex a is called* A-special*, if its orbit $aG = \{g(a) | \forall g \in G\}$ is finite.*

Theorem 2. *Let T be an acyclic theory. Then for the saturated model $M = \langle S, R \rangle$ of the theory T, the following conditions are satisfied:*

1) *if the model does not contain infinite connectivity components, but there are an infinite number of finite connectivity components, then the a-pregeometry $\langle S, \mathrm{acl} \rangle$ is not a-locally finite if only the model contains an infinite number of trees that are isomorphic to a finite number of connectivity components.*

2) *if there is an infinite component in the model, then the a-pregeometry $\langle S, \mathrm{acl} \rangle$ is a-locally finite if only the model does not have an infinite number of special A-special vertices for every finite set $A \subseteq S$.*

Proof. 1) Let the model consist of an infinite number of finite connectivity components.

We note that for any vertex of a finite tree (let's denote it p), you can make a formula for which this vertex will be the solution. Let H be the set of vertices of our tree. Then the formula is compiled according to the following principle:

a) We take all the edges with ends containing the vertices of our tree $a, b \in H$. Denote the set of all such edges E;

b) The formula for the vertex p will be

$$\exists a_1 ... \exists a_n \exists b_1 ... \exists b_m (\bigwedge_{i=1}^{n} R(p, a_i) \bigwedge_{j=1}^{k} e_j) \wedge \forall_{d \in H} d \forall_{q \notin H} q (\neg R(d, q))$$

, where a_i are vertices from H adjacent to p, b_i are vertices from H not adjacent to p and e_j are edges from E not incident to p.

It can be seen that the solution of these formulas (denote it K) will be not only the vertex p, but also all the vertices into which p passes under the action of a group of automorphisms. Moreover, there are no more precise formulas exist. We mean formulas, which solution would be a proper subset of the set K. We will prove the opposite. All formulas are constructed from conjunction, disjunction, negation, quantifiers of existence and universality of the relation R. If we remove the conjunction with relations from the above formulas, then many solutions will expand. If we add a conjunction or disjunction with relations to the formulas, we get a formula for another tree, and we already use these formulas. And in general, any compiled formulas will find the vertices of isomorphic trees. The conjunct of the relations R is a tree. The disjunction of such conjuncts will find the vertices of trees, with each responsible corresponding conjuncture.

The algebraic closure of any nonempty subset $X \subseteq S$ will consist of $acl(\emptyset)$ and the vertices of the connectivity component D, which contain the vertices of the set X, i.e. the vertices of D such that $D \cap X \neq \emptyset$. So whether, the closure of a finite set will be infinite depends only on the dimension $acl(\emptyset)$. Then the algebraic closure of any subset of the universe, in particular the empty set, will be infinite only in the case of presence an infinite number of trees that are isomorphic to a finite number of connectivity components. Hence, only under this condition a-locally finiteness will be violated.

2)Now let's also consider infinite trees. For the vertices of these trees we can obtain formulas similar to the previous case. But removing the conjunct from the formulas $\wedge\forall_{d\in H} d\forall_{q\notin H} q(\neg R(d,q))$. Now the solution of the formula will be not only the vertices of the connected components isomorphic to each other, but also the vertices of the subtrees. Here we note that in an a-pregeometry with such trees the closure of an empty set may not be infinite, and the closure of a finite set of vertices, on the contrary, will violate a-locally finiteness.

By virtue of these formulas, if, as previous, we have an infinite number of finite non-isomorphic connectivity components, then the closure of the empty set is infinite.

All algebraic formulas, that can be composed from the relation R and that do not depend on constants, will describe the vertices of some trees that are isomorphic to each other. Moreover, the solution of the formula (denote it K) will be vertices that pass into each other under the action of a group of automorphisms of the entire model – means, that the solution is the orbit of the element $k \in K$ of the substitution group G with the condition $\forall g \in G\ \forall a, b \in S\ R(g(a), g(b)) = R(a, b)$. If there are a finite number of such vertices, then by definition they are called special. They will always be contained in the closure of an empty set. It turns out that if there is an infinite number of special vertices, then the closure of any finite set will be infinite. Consequently, a-locally finiteness is violated.

Let's analyze one more case. Let us have an infinite connectivity component, in which there are vertices passing into a finite number of vertices under the action of a group of all automorphisms over the universe of this component. If the model has an infinite number of trees, that are isomorphic to this component, then formulas without constants from these components will not have a finite number of solutions. However, if we use the above formulas, which depends from constants (the set of such vertices is denoted by A)from these sets, then the closure of the finite set of vertices

A of these trees will consist of A-special vertices. The closure of such kind set will be infinite, which will violate a-locally finiteness.

Thus, an a-pregeometry $\langle S, \mathrm{acl}\rangle$ is a-locally finite only if the model does not have an infinite number of special and A-special vertices for any finite set $A \subseteq S$. □

Definition 11. *Let's call the* n-neighborhood *of vertex a the set of vertices, connected to it through n edges. This set is denoted by $N_n(a)$.*

Definition 12. *The sequence of integers and the symbol ∞ will be called the* code *of vertex a from vertex b if the i element of the sequence is the degree of i of the vertex on the path from element b to element a.*

Definition 13. *We will say that vertices have the same a-type if each of their neighborhoods has the same number of vertices of the same code.*

Theorem 3. *Let T be an acyclic theory. Then the following conditions are met for any model of $M = \langle S, R\rangle$ theory T:*

1) *if an infinite number of finite sets with vertices of the same a-type can be distinguished in the model, then the a-pregeometry $\langle S, \mathrm{acl}\rangle$ is not a-locally finite.*

2) *if an infinite sequence of neighborhoods can be distinguished in the vertex model, each of which contains a finite set of vertices of the same a-type, then the a-pregeometry $\langle S, \mathrm{acl}\rangle$ is not a-locally finite.*

Proof. The formula for finding the vertices of code $k_0, ..., k_n$ on the n-neighborhood of vertex p has the form:

$$\exists_1^{k_0-1} a_j^{(1)} ... \exists_1^{k_n-1} a_j^{(n)} \exists b^{(1)} ... \exists b^{(n-1)} (\bigwedge_{i=1}^{k_0-1} R(p, a_i^{(1)}) \bigwedge R(p, b^{(1)}) \bigwedge ...$$

$$... \bigwedge_{i=1}^{k_n-1} R(b^{(n-1)}, a_i^{(n-1)}) \bigwedge R(b^{n-1}, x) \bigwedge_{i=1}^{k_n} R(x, a_i^{(n)}))$$

Hence, the formula for the a-type of vertex p is written as conjunctions of formulas for finding the vertices of the corresponding codes with the replacement of x in them by related variables. Also, p is replaced by x.

1. The first condition of the theorem means that an infinite number of formulas will have a finite number of solutions. This means that the closure $acl(\emptyset)$ will be infinite. Hence, the condition a-locally finite is violated.

2. Here, to the already prepared a-type formulas, you need to add more information about the position on the neighborhood through the conjunction

$$\exists p \exists l_1 ... \exists l_{r-1} R(p, l_1) \wedge ... \wedge R(l-1, x)$$

an a-type vertex is located on the r neighborhood of vertex p. The second condition of the theorem says that there are an infinite number of our formulas that have a finite number of solutions. This means that the closure of an empty set or vertex p will be infinite. Hence, the condition is a-locally finite is violated. □

4. Conclusion

For acyclic theories we found conditions for violation of the exchange property and there were introduced new definitions: a-pregeometry and a-modularity. We also established dependences of a-modularity and a-locally finiteness of a-pregeometry on the number of non-isomorphic trees and special points. In addition, sufficient dependence conditions have been established for a-locally finite a-pregeometries on a-type vertices.

References

1. Berenstein A., Vassiliev E. On lovely pairs of geometric structures. *Annals of Pure and Applied Logic*, 2010, vol. 161, no. 7, pp. 866–878.
2. Berenstein A., Vassiliev E. Weakly one-based geometric theories. *J. Symb. Logic*, 2012, vol. 77, no. 2, pp. 392–422.
3. Berenstein A., Vassiliev E. Geometric structures with a dense independent subset. *Selecta Math.*, 2016, vol. 22, no. 1, pp. 191–225.
4. Cherlin G.L., Harrington L., Lachlan A.H. ω-categorical, ω-stable structures. *Annals of Pure and Applied Logic*, 1986, vol. 28, pp. 103–135.
5. Chang C.C., Keisler H.J. *Model theory.* 3rd ed. of XLI 697. Studies in logic and the foundations of mathematics, North-Holland, Amsterdam, New York, 1990, vol. 73, 650 p.
6. Hodges W. *Model theory.* Encyclopedia of Mathematics and its Applications, 1994, vol. 42, Cambridge University Press, 772 p.
7. Hrushovski E. A new strongly minimal set. *Annals of Pure and Applied Logic*, 1993, vol. 62, pp. 147–166.
8. Malyshev S.B. Kinds of pregeometries of cubic theories. *The Bulletin of Irkutsk State University. Series Mathematics*, 2022, vol. 41, pp. 140–149. https://doi.org/10.26516/1997-7670.2022.41.140
9. Markhabatov N.D., Sudoplatov S.V. Topologies, ranks, and closures for families of theories. I. *Algebra and Logic*, 2021, vol. 59, no. 6, pp. 437–455.
10. Mukhopadhyay M.M., Vassiliev E. On the Vamos matroid, homogeneous pregeometries and dense pairs. *Australian Journal of Combinatorics*, 2019, vol. 75, no. 1, pp. 158–170.
11. Pillay A. *Geometric Stability Theory.* Oxford, Clarendon Press, 1996, 361 p.
12. Pillay A. Some remarks on definable equivalence relations in o-minimal structures. *The Journal of Symbolic Logic*, 1986, vol. 51, no. 3, pp. 709–714 .
13. Sudoplatov S.V. *Group polygonometries.* Novosibirsk, Novosibirsk St. Tech. Univ. Publ., 2013, 302 p.
14. Sudoplatov S.V. Models of cubic theories. *Bulletin of the Section of Logic*, 2014, vol. 43, no. 1–2, pp. 19–34.

15. Sudoplatov S.V. Closures and generating sets related to combinations of structures. *The Bulletin of Irkutsk State University. Series Mathematics*, 2016, vol. 16, pp. 131–144.
16. Emelichev V.A., Melnikov O.I., Sarvanov V.I., Tyshkevich R.I. Lectures on graph theory. Moscow, Science Publ., 1990, 384 p. (in Russian))
17. Zilber B.I. The structure of models of ω_1-categorical theories. *Proceedings of International Congress of Mathematicians*, 35968. PWN, Warsaw, 1983.
18. Zilber B.I. Uncountably categorical theories. *Translations of Mathematical Monographs, vol. 117*, AMS, 1993, 122 p.
19. Zilber B.I. Strongly minimal countably categorical theories. *Siberian Mathematical Journal*, 1980, vol. 21, no. 2, pp. 219–230. https://doi.org/10.1007/BF00968268
20. Zilber B.I. Strongly minimal countably categorical theories II. *Siberian Mathematical Journal*, 1984, vol. 25, no. 3, pp. 396–412. https://doi.org/10.1007/BF00968979

Список источников

1. Berenstein A., Vassiliev E. On lovely pairs of geometric structures // Annals of Pure and Applied Logic. 2010. Vol. 161, N 7. P. 866–878.
2. Berenstein A., Vassiliev E. Weakly one-based geometric theories // J. Symb. Logic. 2012. Vol. 77, N 2. P. 392–422.
3. Berenstein A., Vassiliev E. Geometric structures with a dense independent subset // Selecta Math. 2016. Vol. 22, N 1. P. 191–225.
4. Cherlin G.L., Harrington L., Lachlan A.H. ω-categorical, ω-stable structures // Annals of Pure and Applied Logic. 1986. Vol. 28. P. 103–135.
5. Chang C.C., Keisler H.J. Model theory. 3rd ed. of XLI 697. North-Holland, Amsterdam, New York, 1990. 650 p. (Studies in logic and the foundations of mathematics ; vol. 73)
6. Hodges W. Model theory. Cambridge University Press, 1994. 772 p. (Encyclopedia of Mathematics and its Applications ; vol. 42).
7. Hrushovski E. A new strongly minimal set // Annals of Pure and Applied Logic. 1993. Vol. 62. P. 147–166.
8. Малышев С.Б. Виды предгеометрий кубических теорий // Известия Иркутского государственного университета. Серия Математика. 2022. Т. 41. С. 140–149. https://doi.org/10.26516/1997-7670.2022.41.140
9. Markhabatov N. D., Sudoplatov S. V. Topologies, ranks, and closures for families of theories. I // Algebra and Logic. 2021. Vol. 59, N 6. P. 437–455.
10. Mukhopadhyay M. M., Vassiliev E. On the Vamos matroid, homogeneous pregeometries and dense pairs // Australian Journal of Combinatorics. 2019. Vol. 75, N 1. P. 158–170.
11. Pillay A. Geometric Stability Theory. Oxford : Clarendon Press, 1996. 361 p.
12. Pillay A. Some remarks on definable equivalence relations in o-minimal structures // The Journal of Symbolic Logic. 1986. Vol. 51, N 3. P. 709–714 .
13. Судоплатов С. В. Полигонометрии групп. Новосибирск : НГТУ, 2013. 302 с.
14. Sudoplatov S. V. Models of cubic theories // Bulletin of the Section of Logic. 2014. Vol. 43, N 1–2. P. 19–34.
15. Судоплатов С. В. Замыкания и множества образующих, связанные с комбинациями структур // Известия Иркутского государственного университета. Серия Математика. 2016. Т. 16. С. 131–144.
16. Лекции по теории графов / В. А. Емеличев, О. И. Мельников, В. И. Сарванов, Р. И. Тышкевич. М. : Наука, 1990. 384 с.

17. Zilber B.I. The structure of models of ω_1-categorical theories // Proceedings of International Congress of Mathematicians. 35968. PWN, Warsaw, 1983.
18. Zilber B.I. Uncountably categorical theories. AMS, 1993. 122 p. (Translations of Mathematical Monographs ; vol. 117).
19. Zilber B.I. Strongly minimal countably categorical theories // Сибирский математический журнал. 1980. Vol. 21, N 2. P. 98–112.
20. Zilber B.I. Strongly minimal countably categorical theories II // Сибирский математический журнал. 1984. Vol. 25, N 3. P. 71–88.

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