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Displacement Field Construction Based on a Discrete Model in Image Processing Problems

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Abstract. The problem of a displacement field calculation for an image sequence based on a discrete model is being solved. Algorithms for velocity field (displacement field) construction are in demand in various image processing tasks. These methods are used in motion detection, object movement tracking, analysis of complex images, movement correction of medical diagnostic images in nuclear medicine, radiology, etc. An optimization approach to the displacement field construction based on a discrete model is developed in the paper. The approach explores the possibility of taking into account the brightness change along the trajectories of the system. A linear model is considered. Directed optimization methods based on the analytical representation of the functional gradient are constructed to search for unknown parameters. The algorithm for displacement field construction with image partitioning into regions (neighborhoods) is proposed. This algorithm can be used to process a variety of image sequences. The results of the algorithm operation on test radionuclide images are presented.

Keywords: discrete systems, displacement field, functional variation, optimization, image processing

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Научная статья

Построение поля перемещений на основе дискретной модели в задачах обработки изображений

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Аннотация. Решается задача построения поля перемещений для последовательности изображений на основе дискретной модели. Алгоритмы построения поля скоростей (поля перемещений) востребованы в различных задачах обработки изображений. Данные методы применяются для обнаружения перемещений, отслеживания движущихся объектов, анализа сложных изображений с целью коррекции движения на медицинских диагностических изображениях, например радионуклидных и т. д. В работе развивается оптимизационный подход к построению поля перемещений на основе дискретной модели, в которой заложена возможность учитывать изменение яркости вдоль траекторий системы. Рассматривается линейная модель. На основе аналитического представления градиента функционала строятся направленные методы оптимизации для поиска неизвестных параметров. Предлагается алгоритм построения поля перемещений с разбиением изображения на подобласти. Данный алгоритм может применяться для обработки различных последовательностей изображений. Представлены результаты работы алгоритма на тестовых радионуклидных изображениях.

Ключевые слова: дискретные системы, поле перемещений, вариация функционала, оптимизация, обработка изображений

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1. Introduction

The value of application of image processing, pattern recognition and artificial intelligence is constantly growing. The number of areas in which the use of these algorithms becomes necessary is increasing: from the tasks of video surveillance, the construction of inspection complexes, image recognition, to the problems of modern medicine, including a variety of diagnostic images and video support of surgery. The relevance of image processing development dictates the need to create new mathematical methods and approaches based on a variety of subjects, including control theory, linearization, and mathematical modeling. At the same time, new domain problems are constantly emerging in engineering, physics, medicine and other fields, and the requirements to the accuracy of their results are increasing [3; 5–7; 9; 11; 15; 17; 22; 24].

Various authors proposed solutions to the velocity field (displacement field) calculation problem. Some of the first problem definitions relate to the beginning time of the video surveillance systems' use (video sequences analysis). These definitions are known as optical flow determination problems [16]. The key assumption in this case was the brightness constancy along the motion trajectories. Global and local methods, methods of variational calculus, correlation analysis, etc. were considered. [8; 10; 12–14; 16; 20; 23].

An approach that assumes a change in brightness along the motion trajectories was proposed in [8; 21]. In particular, this method was considered for radionuclide images, where the image brightness represents the radiopharmaceutical distribution density. A velocity field calculation method based on a continuous model using an optimization algorithm was proposed in articles [1; 11]. The change of brightness along the movement trajectories was also assumed in these works.

A discrete model was proposed in the article [2] to solve the problem of displacement field calculation for image sequences, and the functional variations being studied are presented in an analytical form. This article expands that approach: the discrete linear model is considered in a general form. A displacement field calculation algorithm for successive images is presented.

2. The discrete model

The following discrete model is considered

$$y\left(k+1\right)=F\left(k,y\left(k\right),u\left(k\right)\right),k=0,...,N-1. \tag{2.1}$$

It is assumed that the system (2.1) describes the movement of points of the object of interest in the sequence of images. Here $y\left(k\right)$ is a m-dimensional state vector that characterizes the change in the image points in the sequence, $u\left(k\right)$ is a r-dimensional vector of parameters. $F\left(k,y\left(k\right),u\left(k\right)\right)$ is a m-dimensional vector function that for every $k\in\{0,1,...,N-1\}$ it is defined and continuous on the set $\Omega_y\times U\left(k\right)$ on its arguments $\left(y\left(k\right),u\left(k\right)\right)$ together with its first and second partial derivatives, Ω_y is the subdomain of R^m, and $U\left(k\right),k=0,1,...,N-1$ are compact subsets of R^r. The Jacobian $J_k=J\left(k,y\left(k\right),u\left(k\right)\right)=|\frac{\partial F(k,y(k),u(k))}{\partial y(k)}|$ is assumed to be nonzero for all of $k,y\left(k\right),u\left(k\right)$. We will further use the notation $y_k=y\left(k\right)$. The displacement field between two consecutive images will be defined as the set of displacement vectors constructed at the original image points.

The initial condition for system (2.1) has the form

$$y\left(0\right)=y_0,y_0\in M_0.$$

Here M_0 is a compact subset of R^m of nonzero measure. $\rho=\rho\left(k\right)=\rho\left(k,y\left(k\right)\right)$ is the image brightness function at the k-th step, and $\rho_0=\rho_0\left(y_0\right)=\rho\left(0,y_0\right)$. It should be noted that it will be a function of the radiopharmaceutical distribution density in case of radionuclide studies [18]. We will assume that the brightness changes $\rho\left(k,y_k\right)$ along the system (2.1) trajectories according to the transfer equation [18; 21]:

$$\rho\left(k+1,y\left(k+1\right)\right)=J_k^{-1}\rho\left(k,y_k\right),k=0,...,N-1, \tag{2.2}$$

with the initial condition $\rho\left(0,y_0\right)=\rho_0\left(y_0\right)$.

Next, we will consider equations (2.1) and (2.2) together with the corresponding initial conditions. This system of equations defines a model for obtaining an image sequence, taking into account the change in brightness along the system (2.1) trajectories. We have to note that if the Jacobian $J_k=1$, then the brightness along the motion trajectories remains constant, and we will have the case of the so-called optical flow.

The sequence of vectors $\{u\left(0\right),u\left(1\right),...,u\left(N-1\right)\}$ we will further call the control vector and denote as u. It defines the right parts of the system (2.1) and needs to be found. The sequence of vectors $\{y\left(0\right),y\left(1\right),...,y\left(N\right)\}$ is denoted as $y=y(y_0,u)$ and will be called the motion trajectory. The state of the trajectory at the k-th step is denoted as $y\left(k\right)=y\left(k,y_0,u\right)$. The set of trajectories $y\left(y_0,u\right)$, which correspond to a given control vector u and various initial states $y_0\in M_0$, will be called the trajectory ensemble [4].

Let's denote the cross-section of the trajectory beam at the k-th step as $M_{k,u}=\{y\left(k\right):y\left(k\right)=y\left(k,y_0,u\left(k\right)\right),y_0\in M_0\}$. The function $\rho\left(k,y\left(k\right)\right)$ for each $k\in\{0,1,...,N\}$ is defined and continuous on the set $M_{k,u}$ with respect to $y\left(k\right)$ together with its partial derivatives. Let us introduce the quality functional in the following form:

$$I\left(u\right)=\Sigma_{k=1}^N\int_{M_{k,u}}g_k\left(y_k,\rho\left(k,y_k\right)\right)dy_k, \tag{2.3}$$

here $g=g\left(y_k,\rho\left(k,y_k\right)\right)$ are continuously differentiable functions of their arguments, y_k is an integration variable over the set $M_{k,u},k=1,...,N$. For the system (2.1) – (2.2) we need to find such an admissible u that minimizes the quality functional (2.3). We will call the control vector admissible if it satisfies the conditions $u\left(k\right)\in U\left(k\right),k=1,...N-1$. Note that finding the control vector u thereby defines the system (2.1), and the displacement vectors at the image points at each step k can be calculated.

The variation of functional (2.3) can be presented in the following form (based on the methodology presented in [4]):

$$\delta I=\sum_{k=0}^{N-1}\int\limits_{M_{k,u}}\left(J\left(k\right)p^{T}\left(k+1\right)\frac{\partial F\left(k\right)}{\partial u\left(k\right)}+J_{k}\xi^{T}\left(k+1\right)\rho\left(k\right)\frac{\partial J^{-1}\left(k\right)}{\partial u\left(k\right)}+\right.$$
$$\left.+\ q\left(k+1\right)\frac{\partial J\left(k\right)}{\partial u\left(k\right)}\right)dy_{k}\Delta u\left(k\right), \tag{2.4}$$

where $\rho\left(k\right),\xi\left(k\right),q\left(k\right)$ are the following auxiliary functions:

$$\begin{gathered}p^{T}\left(N\right)=\left(\frac{\partial g\left(y_{N},\rho_{N}\right)}{\partial y\left(N\right)}\right),\ \xi^{T}\left(N\right)=\left(\frac{\partial g\left(y_{N},\rho_{N}\right)}{\partial\rho\left(N\right)}\right),\\ q\left(N\right)=g\left(y_{N},\rho_{N}\right),\ p^{T}\left(k\right)=J\left(k\right)p^{T}\left(k+1\right)\frac{\partial F\left(k\right)}{\partial y\left(k\right)}+\\ +J_{k}\xi^{T}\left(k+1\right)\rho\left(k\right)\frac{\partial J^{-1}\left(k\right)}{\partial y\left(k\right)}+\ q\left(k+1\right)\frac{\partial J\left(k\right)}{\partial u\left(k\right)},\\ \xi\left(k\right)=\xi\left(k+1\right)+\frac{\partial g\left(k\right)}{\partial\rho\left(k\right)},\ q\left(k\right)=J\left(k\right)q\left(k+1\right)+g\left(k\right),\\ k=1,...,N-1.\end{gathered} \tag{2.5}$$

Let's introduce a special variation of control vector u [4]:

$$\Delta u\left(k\right)=0,\ k=1,...,N-1,\ k\neq j,\ \Delta u\left(j\right)\neq 0.$$

We will call a variation admissible for an admissible control vector u if there exists $\bar{\varepsilon}>0$, such that when $0\leq\varepsilon\leq\bar{\varepsilon}$

$$u\left(j\right)+\varepsilon\Delta u\left(j\right)\in U\left(j\right).$$

Using a special control vector variation and representation of the variation (2.4) of the functional (2.3), we can formulate the following theorem.

Theorem. *In order for the control vector*

$$u^{0}=\left\{u^{0}\left(0\right),u^{0}\left(1\right),...,u^{0}\left(N-1\right)\right\}$$

to be optimal in terms of functional (2.3), it is necessary that the following inequality holds for all permissible control vector variations u^{0}

$$\int\limits_{M_{k,u^{0}}}\left(J\left(k\right)p^{T}\left(k+1\right)\frac{\partial F\left(k\right)}{\partial u\left(k\right)}+J_{k}\xi^{T}\left(k+1\right)\rho\left(k\right)\frac{\partial J^{-1}\left(k\right)}{\partial u\left(k\right)}+\right.$$
$$\left.+\ q\left(k+1\right)\frac{\partial J\left(k\right)}{\partial u\left(k\right)}\right)dy_{k}\Delta u\left(k\right)\geq 0,\ k=1,...,N-1. \tag{2.6}$$

Here $p(k+1), \xi(k+1), q(k+1)$ *satisfy the relations (2.5) with optimal control vector and the corresponding optimal trajectory* $y\left(y_0, u^0\right)$.

Representation of the functional variation (2.4) and the optimality condition (2.6) allow us to construct various directional optimization methods.

3. The optimization algorithm for displacement field calculation

Let's consider an image sequence, i.e. brightness distribution across frames (the brightness function). As an example, we observe the radiopharmaceutical distribution density on each frame in a dynamic radionuclide study [18]. Let's denote it $\bar{\rho}(k, y(k))$, $k=0, ..., N$, and $\bar{\rho}(0, y(0)) = \rho_0(y_0)$. To solve the problem we need to restore some function $F(k, y(k), u(k))$, in which the parameters vector $u(k)$ is the unknown.

Let's consider the functional (2.3) when

$$g_k(y_k, \rho(k, y_k)) = \gamma(k, y_k)\left(\rho(k, y_k) - \bar{\rho}(k, y_k)\right)^2, k = 1, ..., N, \qquad (3.1)$$

where $\gamma(k, y_k)$ are weight functions that allow the selection of image regions of interest. We find the parameters vector $u(k)$ by minimizing the functional (2.3). That allows us to calculate a displacement field in accordance with the equation (2.1).

Considering one step of the process, i.e. two consecutive images ($N = 1$), the system (2.1) will take the form:

$$y(1) = F(0, y(0), u(0)).$$

We will further use the notation $y(0)=y$, $u(0)=u$, $F(0, y, u) = F(y, u)$. As noted earlier [1;2], the type of function F in the system (2.1) is unknown. In this paper we will consider the function F as a linear vector function

$$F(y, u) = Ay + B, \qquad (3.2)$$

where $y = \begin{pmatrix} y^1 \\ y^2 \end{pmatrix}$ is a 2-dimensional vector, and A is a 2×2 matrix, B is a 2-dimensional vector.

Let's consider the case of optical flow, i.e. $detA = 1$. In this case we have

$A = \begin{pmatrix} u_1 & u_2 \\ u_3 & \frac{1+u_2u_3}{u_1} \end{pmatrix}$, $B = \begin{pmatrix} u_4 \\ u_5 \end{pmatrix}$.

Thus, the goal is to find the parameters vector $u = (u_1, u_2, u_3, u_4, u_5)$.

The functional variation (2.4) in this case is written as follows:

$$\delta I(u) = \int_{M_0} p^T(1)\frac{\partial F(y, u)}{\partial u} dy \Delta u, \qquad (3.3)$$

where $p^T(1) = \frac{\partial g(1)}{\partial y(1)}$, $\frac{\partial F(y,u)}{\partial u} = \begin{pmatrix} y^1 & y^2 & 0 & 1 & 0 \\ \frac{-y^2(1+u_2u_3)}{u_1^2} & \frac{u_3}{u_1}y^2 & y^1 + \frac{u_2}{u_1}y^2 & 0 & 1 \end{pmatrix}$.

We denote $p^T(1) = (p_1, p_2)$.

Taking into account the assumptions made in this section and the expression (3.3) for the functional variation we can formulate the following statement.

State 1. *Considering one step of the process ($N = 1$) in a system (2.1) with the right part in the form of a linear vector function (3.2) the gradient of the target functional (2.3) has the following form in the case of an optical flow:*

$$\frac{\partial I}{\partial u_1} = \int_{M_0} \left(p_1 y^1 - p_2 \frac{1 + u_2u_3}{(u_1)^2} y^2\right) dy, \frac{\partial I}{\partial u_2} = \int_{M_0} \left(p_1 y^2 + p_2 \frac{u_3}{u_1} y^2\right) dy,$$
$$\frac{\partial I}{\partial u_3} = \int_{M_0} p_2 \left(y^1 + \frac{u_2}{u_1} y^2\right) dy, \frac{\partial I}{\partial u_4} = \int_{M_0} p_1 dy, \frac{\partial I}{\partial u_5} = \int_{M_0} p_2 dy. \tag{3.4}$$

The obtained analytical representation of the functional gradient allows us to build methods of directed optimization.

Function $\rho(k, y(k))$ with a fixed k is a function of two spatial variables y^1 and y^2. We will consider it in a bounded rectangular region of the image. We introduce a fixed grid of points with a step increment of Δ_1 (size of the pixel along the y^1 axis), and of Δ_2 (size of the pixel along the y^2 axis). Thus, the brightness values (distribution density) of the each image pixel of are $\rho(n_1\Delta_1, n_2\Delta_2) = \rho\left(y^1, y^2\right)|_{y^1=n_1\Delta_1, y^2=n_2\Delta_2}$, and $n_1 \in N_1 \in Z, n_2 \in N_2 \in Z$.

Note that the considered images can include both moving and stationary objects, objects moving at different speeds and in different directions. Therefore, it is possible to divide images into regions, in each one the displacement field is determined by its linear system. The size and number of regions may vary depending on the image content and the specific task goal.

Let's consider the optimization algorithm that finds the vector of unknown parameters u and calculates the displacement field. We assume that the initial state of the system (2.1)-(2.2) is set, i.e. the set M_0 and the initial brightness distribution ρ_0 are defined, and the brightness distribution $\bar{\rho}(1, y(1))$ is known. The main steps of the algorithm are following:

1) Split an image into regions, $j = 0, ..., M - 1$ is a region number, $M = p \times q$, p is a number of partitions on the y^1 axis, q is a number of partitions on the y^2 axis. Set $j = 0$.

2) Set the initial approximation for the vector of unknown parameters u^0. Set *i=0*.

3) Begin the i-th iteration of the algorithm.

4) Calculate the value of the integral functional $I\left(u^i\right)$ by formulas (2.3), (3.1).

5) Calculate the auxiliary function $p^T(1) = (p_1, p_2)$. To determine it, calculate the partial derivatives values of $\frac{\partial \rho}{\partial y^1}$, $\frac{\partial \rho}{\partial y^2}$ in the nodes of the grid using the Sobel operator.

6) Calculate the gradient of the integral functional by formulas (3.4).

7) Calculate the $u^{i+1} = u^i - \bar{\alpha} \cdot gradI\left(u^i\right)$, $\bar{\alpha}$ — optimization parameter. The parameter $\bar{\alpha}$ is found as a result of minimizing the function $f(\alpha) = I\left(u^i - \alpha * gradI\left(u^i\right)\right)$. Meanwhile, any implementation of one-dimensional minimization can be used.

8) Check the algorithm stop conditions: the required accuracy ($|I\left(u^{i+1}\right) - I\left(u^i\right)| < \varepsilon$) or a set number of iterations ($i = \bar{I}$). When performing this step, write down the resulting u^{i+1} for the j-th section of the image and proceed to step 2 of the algorithm for $j = j + 1$ or to step 10 if $j = M - 1$.

9) If the conditions of step 8 are not met, then i=i+1 and return to step 3.

10) Calculate the displacement vectors $(y(1) - y(0))$. The value of vector u defines the right part of the system (2.1) for each region, and the displacement field can be calculated.

4. Examples of the algorithm application

Let's demonstrate the algorithm on images that are obtained during diagnostic radionuclide studies. The studies use radiopharmaceuticals which allow imaging the state of organs and body systems in norm and pathology. The digital processing and analysis of the obtained images is performed. The algorithm proposed in the paper was implemented in Matlab and tested.

The radionuclide images obtained during dynamic scan are considered [18]. Figure 1 shows the radionuclide image sequence obtained as a result of a hepatobiliary system dynamic study. One of the processing stages is to find regions of interest, e.g. the liver, the gallbladder, etc. A contour is usually detected on one frame and then overlaid on the rest, but since this interest area is mobile, the contour may not capture the region of interest on other frames. Therefore, there is a need to correct the contour position on other frames. The study usually contains about 90 frames,

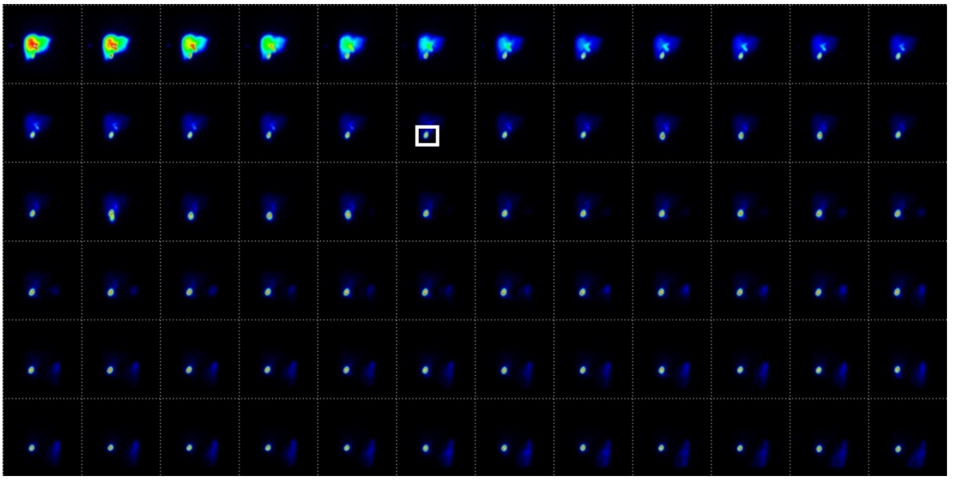

Figure 1. Image sequence of hepatobiliary system radionuclide study

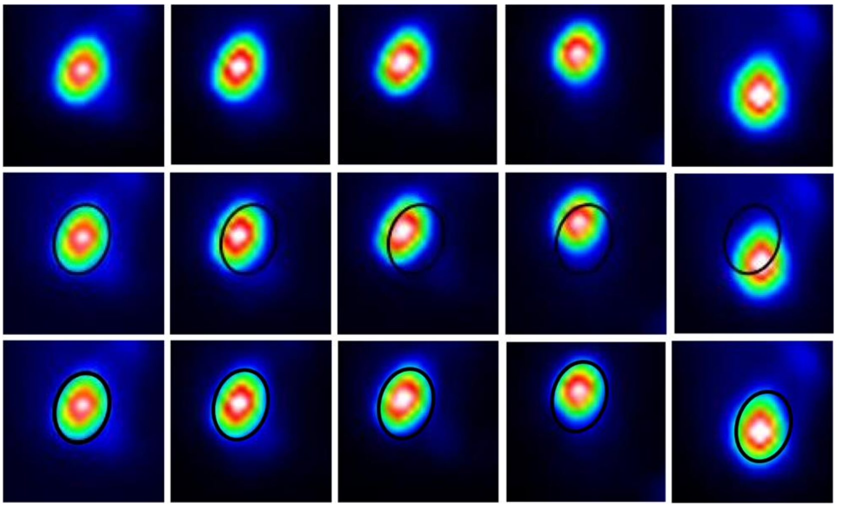

Figure 2. The sequence of source images (top row), a first frame object contour, overlaid to all other frames (middle row), the adjusted position of the contour taking into account the displacement field (bottom row)

so the automation of this correction is highly desirable. The area which contains the region of interest has highlighted by a rectangle on one of the frames.

Figure 2 shows the contour position correction using the proposed algorithm. For each frame, the region of interest indicated above was considered and the contour movement was detected. In the example, the considered images were not divided into local subregions, since it was necessary to determine the resulting displacement of the region of interest.

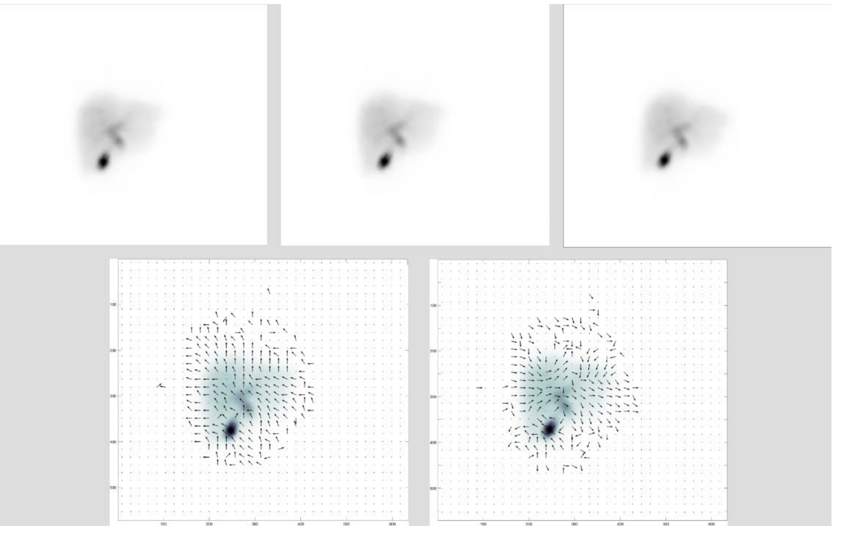

Figure 3. Three consecutive source images (top row), the displacement field between the first and second, and the second and third images (bottom row)

Figure 3 shows three consecutive radionuclide images of the hepatobiliary system. The liver, gallbladder, etc. are visualized. A displacement field is calculated for each pair of consecutive images. In this case, the image was divided into regions, and then the corresponding linear approximation system was determined for each one to construct the displacement field.

5. Conclusion

In the paper, the problem of displacement field calculation in the tasks of image sequences processing was examined. The variational formulation of the problem was considered, an analytical expression of variation was given for the proposed quality functional, and a condition of optimality was formulated. A linear discrete model was used, and the case of optical flow was considered. The article presents a developed and implemented optimization algorithm for displacement field determination with the option of splitting images into regions. The proposed algorithm can be used for image analysis and motion correction in dynamic radionuclide studies. It should be noted that the considered model takes into account changes of brightness along the system trajectories, which expands the scope of this approach. The algorithm was implemented in the Matlab environment. The article provides examples of processing radionuclide images using this method.

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