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A Method for Semidefinite Quasiconvex Maximization Problem

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Abstract. We introduce so-called semidefinite quasiconvex maximization problem. We derive new global optimality conditions by generalizing [9]. Using these conditions, we construct an algorithm which generates a sequence of local maximizers that converges to a global solution. Also, new applications of semidefinite quasiconvex maximization are given. Subproblems of the proposed algorithm are semidefinite linear programming.

Keywords: Semidefinite linear programming, global optimality conditions, semidefinite quasiconvex maximization, algorithm, approximation set.

1. Introduction

Semidefinite linear programming can be regarded as an extension of linear programming and solves the following problem

$$\begin{cases} \min\langle C, X\rangle_F, \\ \langle A_j, X\rangle_F \leqslant b_j, j = 1, 2, .., s, \\ X \succcurlyeq 0, \end{cases} \tag{1.1}$$

Here $X \in \mathbb{R}^{n\times n}$ is a matrix of variables and $A_j \in \mathbb{R}^{n\times n}, j = 1, 2, ..., s$. $X \succcurlyeq 0$ means that X is a positive semidefinite matrix. We denote by $\langle\cdot,\cdot\rangle_F$ the Frobenius scalar product of two matrices X and Y defined by:

$\langle X, Y\rangle_F = trace(X^T Y)$ where $trace(Z)$ denotes the trace of the square matrix Z. The corresponding norm is the well known Frobenuis norm defined by $\|X\|_F = \sqrt{\langle X, X\rangle_F}$.
Semidefinite programming finds many applications in engineering and optimization [7]. Most interior-point methods for linear programming have been generalized to semidefinite convex programming [13; 7]. There are many works devoted to the semidefinite convex programming problem but less attention so far has been paid to the semidefinite quasiconvex maximization problem.

2. Quasiconvex function and its properties

Let $X = [x_{ij}]$ be a matrix in $\mathbb{R}^{n\times n}$, and define a scalar matrix functions f as follows

$$f : \mathbb{R}^{n\times n} \to \mathbb{R}.$$

Definition 2.1. *Let $f(X)$ be a differentiable function of the matrix X. Then*

$$f'(X) = \left(\frac{\partial f(X)}{\partial x_{ij}}\right)_{n\times n}.$$

If $f(\cdot)$ is differentiable, then it can be checked that

$$f(X + H) - f(X) = \langle f'(X), H\rangle_F + o(\|H\|_F).$$

Definition 2.2. *A set $\mathbb{D} \subset \mathbb{R}^{n\times n}$ is convex if $\alpha X + (1-\alpha)Y \in \mathbb{D}$ for all $X, Y \in \mathbb{D}$ and $\alpha \in [0,1]$.*

Definition 2.3. *The function $f : \mathbb{D} \to \mathbb{R}$ is said to be quasiconvex on $\mathbb{D}$ if $f(\alpha X + (1-\alpha)Y) \leqslant \max\{f(X), f(Y)\}$ for all $X, Y \in \mathbb{D}$ and $\alpha \in [0,1]$.*

The well known property of a convex function [8] can be easily generalized as follows:

Lemma 2.1. *A function $f : \mathbb{R}^{n\times n} \to \mathbb{R}$ is quasiconvex if and only if the set*

$$L_c(f) = \{X \in \mathbb{R}^{n\times n} \mid f(X) \leqslant c\}$$

is convex for all $c \in \mathbb{R}$.

Proof. Necessity. Suppose that $c \in \mathbb{R}$ is an arbitrary number and $X, Y \in L_c(f)$. By the definition of quasiconvexity, we have

$$f(\alpha X + (1-\alpha)Y) \leqslant \max\{f(X), f(Y)\} \leqslant c \quad \text{for all } \alpha \in [0,1],$$

which means that the set $L_c(f)$ is convex.
Sufficiency. Let $L_c(f)$ be a convex set for all $c \in R$. For arbitrary $X, Y \in \mathbb{R}^n$, define
$c^o = \max\{f(X), f(Y)\}$. Then $X \in L_{c^o}(f)$ and $Y \in L_{c^o}(f)$. Consequently, $\alpha X + (1 - \alpha Y) \in L_{c^o}(f)$, for any $\alpha \in [0,1]$. This completes the proof.

Lemma 2.2. *Let $f : \mathbb{R}^{n\times n} \to \mathbb{R}$ be a quasiconvex and differentiable function. Then the inequality $f(X) \leqslant f(Y)$ for $X, Y \in \mathbb{R}^{n\times n}$ implies that $\langle f'(Y), X-Y\rangle_F \leqslant 0$*

Proof. Since f is quasiconvex,

$$f(\alpha X + (1-\alpha)Y) \leqslant \max\{f(X), f(Y)\} = f(Y)$$

for all $\alpha \in [0,1]$ and $X, Y \in \mathbb{R}^{n\times n}$ such that $f(X) \leqslant f(Y)$. By Taylor's formula, there is a neighborhood of the point Y on which:

$$f(Y + \alpha(X-Y)) - f(Y) =$$

$$\alpha\left(\langle f'(Y), X-Y\rangle_F + \frac{o(\alpha\|X-Y\|_F)}{\alpha}\right) \leqslant 0,\ \alpha > 0.$$

From the fact that $\frac{o(\alpha\|x-y\|_F)}{\alpha} \overset{\alpha\to o}{\longrightarrow} 0$, we obtain $\langle f'(Y), X-Y\rangle_F \leqslant 0$ which completes the proof.

3. Semidefinite quasiconvex maximization problem

3.1. Global optimality conditions

Consider the problem of maximizing a differentiable quasiconvex matrix function subject to constraints.

$$\begin{cases} \max f(X) \\ \text{subject to :} \\ \langle A_j, X\rangle_F \leqslant b_j, j = 1, 2, .., s, \\ X \succcurlyeq 0, \end{cases} \tag{3.1}$$

where $A_j \in \mathbb{R}^{n\times n}, j = 1, 2, ..., s$ and $b_j \in \mathbb{R}$. We call the problem (3.1) as the semidefinite quasiconvex maximization problem or equivalently, semidefinite quasiconcave programming.
Denote by $\mathbb{D}$ the set corresponding to the constraints of the problem:

$$\mathbb{D} = \{X \in \mathbb{R}^{n\times n} | \langle A_j, X\rangle_F \leqslant b_j, j = 1, 2, .., s; X \succcurlyeq 0\}.$$

Then the problem (3.1) reduces to

$$\max_{X\in\mathbb{D}} f(X) \tag{3.2}$$

It can be checked that the set $\mathbb{D}$ is convex. Problem (3.2) is nonconvex and belongs to a class of global optimization problems.

Introduce the level set $\mathrm{E}_{f(Z)}(f)$ of the function $f : \mathbb{R}^{n\times n} \to \mathbb{R}$ at a point $Z \in \mathbb{R}^{n\times n}$:

$$\mathrm{E}_{f(Z)}(f) = \{Y \in \mathbb{R}^{n\times n} | f(Y) = f(Z)\}.$$

It can be checked that a space $\mathbb{R}^{n\times n}$ is a Hilbert space equipped with a norm $\|\cdot\|_F$. We now compute a gradient of the function $g(X)$ defined as:

$$g(X) = \frac{1}{2}\|X - U\|_F^2, X \in \mathbb{R}^{n\times n}.$$

Indeed,

$$\triangle g(X) = g(X + \triangle X) - g(X) = \frac{1}{2}\|X + \triangle X - U\|_F^2 - \frac{1}{2}\|X - U\|_F^2 =$$

$$\frac{1}{2}\langle X + \triangle X - U, X + \triangle X - U\rangle_F - \frac{1}{2}\langle X - U, X - U\rangle_F =$$

$$\frac{1}{2}\langle X - U, X - U\rangle_F + \langle X - U, \triangle X\rangle_F + \langle \triangle X, \triangle X\rangle_F - \frac{1}{2}\langle X - U, X - U\rangle_F$$

$$\triangle g(X) = \langle X - U, \triangle X\rangle_F + \frac{1}{2}\| \triangle X\|_F^2$$

Hence, we get

$$g'(X) = (X - U) \tag{3.3}$$

The global optimality condition for the problem (3.2) can be formulated in the following theorem.

Theorem 3.1. *[5] If $Z \in \mathbb{D}$ is a global solution to the problem*(3.2) *then*

$$\langle f'(Y), X - Y\rangle_F \leqslant 0 \tag{3.4}$$

holds for all $Y \in \mathrm{E}_{f(Z)}(f)$ and $X \in \mathbb{D}$. If in addition, $f'(Y) \neq 0$ holds for all $Y \in E_{f(Z)}(f)$, then condition (3.4) *is sufficient for $Z \in D$ being a solution to the problem* (3.2)*.*

Proof. Necessity. Assume that Z is a solution of problem (3.2) and let $Y \in E_{f(Z)}(f)$ and $X \in D$. Then we have $f(X) \leqslant f(Y)$. Applying Lemma 2.2, we obtain $\langle f'(Y), X - Y\rangle_F \leqslant 0$.

Sufficiency. Suppose, on the contrary, that Z is not a solution to the problem (3.2), i.e, there exists an $U \in \mathbb{D}$ such that $f(U) > f(Z)$. The closed set $\mathrm{L}_{f(Z)}(f) = \{X \in \mathbb{R}^{n\times n} | f(X) \leqslant f(Z)\}$ is convex by Lemma 2.1. Let Y be the projection of U onto $\mathrm{L}_{f(Z)}(f)$ such that

$$\|Y - U\|_F = \min_{X\in \mathrm{L}_{f(Z)}(f)} \|X - U\|_F.$$

Obviously,

$$\|Y - U\|_F > 0, \tag{3.5}$$

holds since $U \notin \mathrm{L}_{f(Z)}(f)$. The point Y can be considered as a solution of the convex minimization problem:

$$\min_{X \in \mathrm{L}_{f(Z)}(f)} \{g(X) = \frac{1}{2}\|X - U\|_F^2\} \tag{3.6}$$

Taking into account (3.3) applying the lagrange method [3] to problem (3.6) defined on a Hilbert space, we obtain the following optimality conditions at the point Y:

$$\begin{cases} \lambda_0 \geqslant 0, \quad \lambda \geqslant 0, \quad \lambda_0 + \lambda > 0, \\ \lambda_0 g'(Y) + \lambda f'(Y) = 0, \\ \lambda(f(Y) - f(Z)) = 0 \end{cases} \tag{3.7}$$

or equivalently,

$$\begin{cases} \lambda_0 \geqslant 0, \quad \lambda \geqslant 0, \quad \lambda_0 + \lambda > 0, \\ \lambda_0 (Y - U) + \lambda f'(Y) = 0, \\ \lambda(f(Y) - f(Z)) = 0. \end{cases} \tag{3.8}$$

If $\lambda = 0$, then (3.8) implies that $\lambda > 0, f(Y) = f(Z)$, and $f'(Y) = 0$ which contradicts the assumption in the theorem. If $\lambda = 0$, then we have $\lambda_0 > 0$, and $g'(Y) = Y - U = 0$ which also contradicts (3.5). So, without loss of generality, we can set $\lambda_0 = 1$ and $\lambda > 0$ in (3.8). Hence, we have

$$Y - U + \lambda f'(Y) = 0, \quad \lambda > 0.$$

From this, we can conclude that

$$\lambda f'(Y) = U - Y$$

and

$$\lambda \langle f'(Y), U - Y \rangle_F = \|U - Y\|_F^2 > 0$$

which contradicts (3.4). Last contradiction implies that the assumption that Z is not a global solution to problem (3.2) must be false which completes the proof.

Remark 3.1. *For a fixed* $Y \in \mathrm{E}_{f(Z)}(f)$ *checking condition* (3.4) *reduces to*

$$\max_{x \in \mathbb{D}} \langle f'(Y), X \rangle_F \leqslant \langle f'(Y), Y \rangle_F$$

or equivalently to semidefinite linear programming:

$$\begin{cases} \max \langle f'(Y), X \rangle_F, \\ \text{subject to :} \\ \langle A_j, X \rangle \leqslant b_j, j = 1, 2, ..., s, \\ X \succcurlyeq 0. \end{cases} \tag{3.9}$$

Remark 3.2. *In order to conclude that a point $\widetilde{Z} \in \mathbb{D}$ is not a global solution to problem* (3.2)*, we need to find a pair* $(U, \widetilde{Y})$ *such that*

$$\langle f'(\widetilde{Y}), U - \widetilde{Y}\rangle_F \geq 0, \widetilde{Y} \in \mathrm{E}_{f(Z)}(f), U \in \mathbb{D}.$$

The following example illustrates the use of this property.

Example 3.1. *Consider the problem*

$$\max_{x \in \mathbb{D}} \|CX\|_F^2,$$

$$\mathbb{D} = \{X \in \mathbb{R}^{2\times 2} | \underline{X} \leqslant X \leqslant \overline{X}, X \succcurlyeq 0\},$$

where

$$C = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}, \qquad \underline{X} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \qquad \overline{X} = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}.$$

We can evaluate the gradient of f as:

$$f'(X) = 2C^T CX.$$

We check whether a point X^0

$$X^0 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

is a global solution or not. We have

$$f(X^0) = 254.$$

Consider the matrix $U \succcurlyeq 0$ in $\mathbb{D}$ defined by

$$U = \begin{pmatrix} 2.7 & 1.2 \\ 2.9 & 5.7 \end{pmatrix}.$$

Let $\widetilde{Y}$ so that $\widetilde{Y} \in \mathrm{E}_{f(X^0)}(f)$ and defined by

$$\widetilde{Y} = \begin{pmatrix} 0.0027 & 0.01264 \\ 0.0090 & 0.03723 \end{pmatrix}$$

If we evaluate $\langle f'(\widetilde{Y}), U - \widetilde{Y}\rangle_F$, then $\langle f'(\widetilde{Y}), U - \widetilde{Y}\rangle_F = 6.7218 > 0$ which means that X^0 is not a global solution. Therefore, we obtain the point U such that $f(U) > f(X^0)$. Similarly, continuing this process we can get the global solution X^* which is :

$$X^* = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$$

3.2. The SDMAX Algorithm

As we have seen in Subsection 3.1 that in order to check condition (3.4), we need to solve the following semidefinite linear programming for each given $Y \in \mathrm{E}_{f(Z)}(f)$.

$$\max_{x\in\mathbb{D}} \langle f'(Y), X\rangle_F. \tag{3.10}$$

For this purpose, we need to approximate the level set of the function f with a finite number of points so that one could solve a finite number of problems (3.10).

Definition 3.1. *The set A_Z^m defined for a given $m \in \mathbb{N}$ and $Z \in \mathbb{R}^{n\times n}$ by*

$$A_Z^m = \{Y^1, Y^2, ..., Y^m | Y^i \in \mathrm{E}_{f(Z)}(f), i = 1, 2, ..., m\} \qquad (3.12).$$

is called an approximation set to the level set $\mathrm{E}_{f(Z)}(f)$ at the point Z.

Assume that A_Z^m is given and $\mathbb{D}$ is compact in $\mathbb{R}^{n\times n}$. Let $U^i, i = 1, 2, ..., m$ be the solutions to the following problems:

$$\langle f'(Y^i), U^i\rangle_F = \max_{X\in\mathbb{D}}\langle f'(Y^i), X\rangle_F. \tag{3.13}$$

Define θ_m as follows:

$$\theta_m = \max_{i=1,2..,m} \langle f'(Y^i), X\rangle_F.$$

Lemma 3.1. *If there is a point $Y^i \in A_Z^m$ for $Z \in \mathbb{D}$ such that $\langle f'(Y^i), U^i - Y^i\rangle_F > 0$, where U^i satisfies (3.13),then*

$$f(U^i) > f(Z).$$

Proof. By the definition of U^i, we have

$$\langle f'(Y^i), U^i - Y^i\rangle = \max_{X\in\mathbb{D}}\langle f'(Y^i), X - Y^i\rangle$$

Since f is quasiconvex, then by Lemma 2 we have $f(U^i) \leqslant f(Y^i)$ which implies that $\langle f'(Y^i), U - Y^i\rangle_F \leqslant 0$ for all $U, Y \in \mathbb{R}^{n\times n}$. The proof is complete.

Now we can formulate an algorithm for finding an approximate solutions for problem (3.2)

Algorithm Semidefinite quasiconvex maximization SDMAX

Input: A quasiconvex differentiable function f and a compact set $\mathbb{D}$ in $\mathbb{R}^{n\times n}$

Output: An approximate solution X to (3.2).

Step 1. Choose a point $X^0 \in \mathbb{D}$. Set $k := 0$.

Step 2. Find a local maximizer $Z^k \in \mathbb{D}$ of problem (3.2) for example by a gradient method of semidefinite nonconvex programming proposed in [16]
Step 3. Construct an approximation set $A^m_{Z^k}$ at the point Z^k.
Step 4. For each $Y^i \in A^m_{Z^k}$ solve semidefinite linear programming

$$\max_{X \in \mathbb{D}} \langle f'(Y^i), X \rangle_F.$$

Let $U^i, i = 1, 2, ..., m$ be solutions, i.e.,

$$\langle f'(Y^i), U^i \rangle_F = \max_{X \in \mathbb{D}} \langle f'(Y^i), X \rangle_F.$$

Step 5. Find a number $j \in 1, 2, ..., m$ such that

$$\theta_m = \langle f'(Y^j), U^j - Y^j \rangle_F = \max_{j=1,2..,m} \langle f'(Y^i), U^i - Y^i \rangle_F$$

Step 6. If $\theta^k_m \leqslant 0$ then terminate and Z^k is an approximate solution.
Step 7. Set $X^{k+1} := U^i, k := k + 1$ and go to step 2.
We notice that Algorithm SDMAX generates a sequence of local maximizers $\{Z^k\}$ of the problem (3.2) such that

$$f(Z^{k+1}) \geqslant f(Z^k), k = 0, 1...$$

Also, local maximizers can be found by semidefinite linear programming relaxations similar to [10]. This gives us an opportunity to approach the global solution in (3.2) using standard approach of semidefinite programming.
As we can see that in Algorithm SDMAX, in order to run the algorithm we need to specify how to to construct an approximation set A^m_Z. In general, construction of such approximation sets depends on the objective function f and structure of a feasible set $\mathbb{D}$ in $\mathbb{R}^{m \times n}$. Let us show this on the following example.
Consider the quadratic function f:

$$f(X) = \|CX - XB - E\|^2_F, C, B, E \in \mathbb{R}^{m \times n}.$$

It can be checked that the gradient of f is evaluated as follows

$$f'(X) = 2C^T[(CX - XB - E)] - 2(CX - XB - E)B^T$$

Lemma 3.2. *Let a point* $\mathbb{Z} \in \mathbb{D}$ *and a vector* $H \in \mathbb{R}^{m \times n}$ *satisfy*

$$\langle f'(Z), H \rangle_F < 0.$$

Then there exists a positive number α *such that* $Z + \alpha H \in \mathrm{E}_{f(Z)}(f)$.

Proof. With $Y_\alpha = Z + \alpha H$, solve the equation $f(Y_\alpha) = f(Z)$
In fact, we have
$f(Z + \alpha H) = \|C(Z + \alpha H) - (Z + \alpha H)B - E\|_F^2 = \|(CZ - ZB - E) + \alpha(CH - HB)\|_F^2 = \|(CZ - ZB - E\|_F^2 + 2\alpha\langle CZ - ZB - E, CH - HB\rangle_F + \alpha^2\|CH-HB\|_F^2 = f(Z) + 2\alpha\langle C^T(CZ-ZB-E)-(CZ-ZB-E)B^T, H\rangle_F + \alpha^2\|CH - HB\|_F^2$.
Now the equation $f(Y_\alpha) = f(Z)$ gives us

$$\alpha = -\frac{2\langle C^T(CZ - ZB - E) - (CZ - ZB - E)B^T, H\rangle_F}{\|CH - HB\|_F^2} > 0, \qquad (3.14)$$

which completes the proof.
Remark 1. If Z is a local maximizer of problem (3.2), then by [8] we have

$$\langle f'(Z), X - Z\rangle_F \leqslant 0, \forall X \in \mathbb{D}.$$

If we take $H = U - Z, \quad U \in \mathbb{D}$, then $\langle f'(Z), H\rangle \leqslant 0$ which satisfies condition of the lemma.

For this reason, in a computational experiment points $Y^i \in A^m_{Z^k}$ should be constructed as

$$Y^i = Z^k + \alpha_i H^i, \quad i = 1, 2, ..., m,$$

where Z^k is a current local maximizer to problem at k-th iteration (2,4), and H^i is random matrix in $\mathbb{R}^{n\times n}$, α_i is computed by formula (3.14).

4. Application of semidefinite quasiconvex maximization

4.1. Maximum sum of mutual information in MIMO interference networks

In communication theory, multiple-input multiple-output (MIMO) refers to radio links with multiple antennas at the transmitter and the receiver side. The system to model consists to k user-MIMO where the transmitter has M antennas and each receiver has N antennas. A wide range of studies in this area end up to solve difficult optimization problems [15; 12]. Such problems do not admit a closed from solution and in general it is very difficult to solve them numerically. For instance, in[1], multiuser MIMO system with in general it is neither concave nor convex. Without going into details, mathematically the problem in question can be formulated as follows:

$$\begin{cases} \max F(Q_1, Q_2, ..., Q_k), \\ \text{subject to} \\ \sum_{i=1}^{k} trace(Q_i) \leq p_T, \\ Q_i \succeq 0, \end{cases}$$

where p_T is the total power constraint and the objective function F is a nonlinear function defined by:

$$F(Q_1, Q_2, ..., Q_k) = \sum_{l=1}^{k} \log_2 \|I + p_l H_{l,l} Q_l H_{l,l}^* R_l^{-1}\|_F,$$

whith $R_l = I + \sum_{j=1,j\neq l}^{k} \eta_{l,j} H_{l,j} Q_j H_{l,j}^*$, $H_{l,j} \in \mathbb{R}^{N\times M}$ denotes the channel matrix between the receive antennas of user l and the transmit antennas of user j. The parameters p_l and $\eta_{l,j}$ are , respectively, the signal-to-noise ratio (SNR) of user l and the interference- to noise ratio (INR) of the interference which is generated by user j and received by user l's receiver.
The maximization is performed over covariance matrices of all transmitter $Q_1, Q_2, ..., Q_k$ each of which is an $M \times M$ positive semi-definite matrix. The goal covariance matrices that achieve this maximum. In general, this problem seems not to be a standard semidefinite programming. It has been shown in [1] that when the INR is sufficiently large(large interference) for any i, $F(Q_1, Q_2, ..., Q_k)$ is convex function with respect to one variable Q_i, $i = 1, \ldots, k$.

5. Conclusion

We consider the semidefinite convex maximization problem. Unlike semidefinite convex programming, the problem is nonconvex and NP hard. We derived global optimality conditions by extending a result of Strekalovsky [9] for semidefinite quasiconvex maximization problem. Based on the global optimality conditions, we propose an algorithm for solving the problem.

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Полуопределенное квазивыпуклое программирование

Аннотация. Рассматривается задача полуопределенного квазивыпуклого программирования (задача максимизации или минимизации квазивыпуклой функции на выпуклом множестве). Обобщая теорему А. С. Стрекаловского, мы получаем новое условие глобальной оптимальности для рассматриваемого класса задач. Основываясь на условиях глобальной оптимальности, мы строим алгоритм, который генерирует последовательность точек локальных максимумов, сходящуюся к глобальному решению. Вспомогательными задачами предложенного алгоритма являются задачи полуопределенного линейного программирования. Приводятся новые приложения задач полуопределенного квазивыпуклого программирования.

Ключевые слова: полуопределенное линейное программирование, условия глобальной оптимальности, полуопределенная квазивыпуклая максимизация и минимизация, алгоритм.

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