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The Soliton Solutions for the Nonlinear Schrödinger Equation with Self-consistent Source

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Abstract. In this paper by using Hirota's method, the one and two soliton solutions of nonlinear Schrödinger equation with self-consistent source are studied. We have shown the evolution of the one and two soliton solutions in detail by using graphics.

Keywords: soliton solution, Schrödinger equation, nonlinear equations, Hirota's method.

1. Introduction

Integrable nonlinear evolution equations have various applications in many fields. It is known that the existence of multi-soliton solutions is an important feature of integrable nonlinear evolution equations, which play a main role in science. They describe nonlinear waves and have important applications in solid state physics, plasma physics and etc.

The nonlinear Schrödinger equation with self-consistent source (nlSES CS) describes the soliton propagation in a medium with both resonant and nonresonant nonlinearities [10]. It is also indicated as the nonlinear interaction of high-frequency electrostatic waves with ion acoustic waves in plasma [1]. Soliton equations with self-consistent source have important physical applications. Therefore, it is always interesting to find its soliton solutions. In 1971, Hirota [3] proposed the Hirota direct method for the Korteweg-de Vries (KdV) equation. Soliton equations with self-consistent source (SESCS) were discussed in [5;7;8;9]. In recent years, extensive research has been conducted on SESCS using the Hirota method [2;14;15;16].

Apart from that, other methods exist to find solutions of SESCS, such as inverse scattering method and the special treatments of the singularity in the evolution of eigenfunctions [6;11;12], the binary Darboux transformations for the KdV hierarchies with self-consistent sources have been proposed [13].

Usually one-soliton and two-soliton solutions are found using the Hirota method and the next step the Wronskian technique is used for N-soliton solutions. Soliton solutions can be represented by Wronski determinant. In this work, we study one-soliton and two-soliton solutions of the nlSESCS through Hirota's method.

We consider the integration of the following system of equations

$$iu_t + 2\left|u\right|^2 u + u_{xx} = 2i\sum_{j=1}^{N}(\varphi_{1j}^2 - \bar{\varphi}_{2j}^2), \tag{1.1}$$

$$\begin{aligned} &\varphi_{1j,x} = -i\xi_j\varphi_{2j} + u\varphi_{1j}, \\ &\varphi_{2j,x} = i\xi_j\varphi_{2j} - \bar{u}\phi_{1j}, \qquad j = 1, 2, ...N, \end{aligned} \tag{1.2}$$

where the bar means complex conjugation and ξ_j, $j = 1, 2, \ldots, N$ are the eigenvalues.

We assume that the solution $u(x,t)$ of the system (1.1)-(1.2) exists possessing the required smoothness and tends to its limits sufficiently rapidly as $|x| \to \infty$, i.e., for all $t \geq 0$ satisfies the condition

$$\int_{-\infty}^{0}(1+|x|)\left|u(x,t)\right|dx + \int_{-\infty}^{\infty}\sum_{k=1}^{2}\left|\frac{\partial^k u(x,t)}{\partial x^k}\right|dx < \infty. \tag{1.3}$$

As shown in [4], under the condition shown below the system of equations (1.2) has a finite number of eigenvalues. In general, these eigenvalues can be multiples. Here, we assume that all the eigenvalues are simple and their numbers are equal to N. We also assume that the eigenfunctions $\Phi_j = (\varphi_{1j}, \varphi_{2j})^T$ corresponding to this eigenvalues satisfy the following normalizing conditions

$$\int_{-\infty}^{\infty}\varphi_{1j}\varphi_{2j}dx = \beta_j^2(t), \ j = 1, 2, ..., N. \tag{1.4}$$

Here $\beta_j(t)$, $j = 1, 2, ..., N$ are given and the continuous functions of t.

2. Bilinear form for the nlSESCS

We will find the solition solution of the nlSESCS by using of Hirota's method. With the help of the dependent variable transformations

$$u = \frac{g}{f}, \ \varphi_{1j} = \frac{p_j}{f}, \ \varphi_{2j} = \frac{h_j}{f}, \ j = 1, 2, \ldots, N \tag{2.1}$$

the system (1.1)-(1.2) can be transformed into the bilinear forms

$$(iD_t + D_x^2)g \cdot f = 2i \sum_{j=1}^{N} (p_j^2 - \bar{h}_j^2), \tag{2.2}$$

$$D_x^2 f \cdot f = 2g \cdot \bar{g}, \tag{2.3}$$

$$\begin{cases} D_x p_j \cdot f = -i\xi_j p_j f + g h_j, \\ D_x h_j \cdot f = i\xi_j h_j f - \bar{g} p_j, \end{cases} \tag{2.4}$$

where $\bar{g}$ and $\bar{h}$ are the complex conjugation of the functions g and h, respectively and Hirota's bilinear operators D_t and D_x are defined by

$$D_x^m D_t^n g(x,t)\cdot f(x,t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n g(x,t) f(x',t')|_{x=x',t=t'}. \tag{2.5}$$

Here, the subscripts of the functions f and g define the order of the partial derivatives with respect to x and t.

Equations (2.2)-(2.4) can be solved by introducing the following power series expansions for f, g, p_j and h_j:

$$f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)} + \, ... \, , \tag{2.6}$$

$$g = \chi g^{(1)} + \chi^3 g^{(2)} + \, ... \, , \tag{2.7}$$

$$p_j = \chi p_j^{(1)} + \chi^3 p_j^{(2)} + \, ... \, , \tag{2.8}$$

$$h_j = h_j^{(1)}, \tag{2.9}$$

where χ is a formal expansion parameter. Substituting Eqs. (2.6)-(2.9) into Eqs.(2.2)-(2.4) and equating coefficients of the same powers of χ to zero can yield the recursion relation for $f^{(k)}$, $g^{(k)}$, $p_j^{(k)}$ and $h_j^{(1)}$, $k = 1, 2, ...$.

3. One-soliton solution

We will give the analytical expression of one-soliton solution (i.e. in the case $N = 1$) of the system (1.1)-(1.2). According to In the known Hirota's method, we consider for the one-soliton solution of nlSESCS in the form below

$$g = \chi g^{(1)}, \; f = 1 + \chi^2 f^{(1)}.$$

Using the definition (2.5) the above (2.3) equation can be expressed in details. Substituting these expressions into (2.3) and equating the coefficients of the same powers of χ, we have

$$f_{xx}^{(1)} = g^{(1)} \bar{g}^{(1)}, \tag{3.1}$$

$$f_{xx}^{(1)} f^{(1)} - \left(f_x^{(1)}\right)^2 = 0. \tag{3.2}$$

If we take

$$g^{(1)} = e^{\eta_1}, \tag{3.3}$$

then

$$f^{(1)} = e^{\eta_1 + \bar{\eta}_1 + a_{11}} \tag{3.4}$$

satisfies equations (3.1) and (3.2). Here, $\eta_1 = k_1 x + \gamma_1(t)$ and $a_{11} = \ln \frac{1}{(k_1+\bar{k}_1)^2}$, where k_1 is constant and $\gamma_1(t)$ is an arbitrary function of t.

The next step is to find functions p_1 and h_1 in case when one-soliton solution are

$$p_1 = \chi p_j^{(1)}, \quad h_1 = h_j^{(1)}.$$

Based on the above, we collect coefficients of the same power in χ according to the expression (2.4) and we get

$$p_{1x}^{(1)} = -i\xi_1 p_1^{(1)} + g^{(1)} h_1^{(1)}, \tag{3.5}$$

$$p_{1x}^{(1)} f^{(1)} - p_1^{(1)} f_x^{(1)} = -i\xi_1 p_1^{(1)} f^{(1)}. \tag{3.6}$$

Using expressions (3.4), (3.3) and by solving (3.5) and (3.6), we have

$$\begin{aligned} p_1^{(1)} &= e^{(k_1+\bar{k}_1-i\xi_1)x+\Omega_1(t)}, \\ h_1^{(1)} &= (k_1+\bar{k}_1)e^{(\bar{k}_1-i\xi_1)x+\Omega_1(t)-\gamma_1(t)}, \end{aligned} \tag{3.7}$$

where Ω_1 is an arbitrary function of t.

Using expressions (3.4), (3.3) and (3.7), we can rewrite the functions f, g, p_1 and h_1 in the following form:

$$\begin{aligned} f &= 1 + e^{\eta_1+\eta_2+a_{11}}, \\ g &= e^{\eta_1}, \\ p_1 &= e^{(k_1+\bar{k}_1-i\xi_1)x+\Omega_1(t)}, \\ h_1 &= (k_1+\bar{k}_1)e^{(\bar{k}_1-i\xi_1)x+\Omega_1(t)-\gamma_1(t)}. \end{aligned} \tag{3.8}$$

Substituting these expressions into (2.2), we can assure that the functions $\Omega_1(t)$, $\gamma_1(t)$ and the constant k_1 satisfy the following conditions

$$\begin{aligned} (\gamma_1(t))_t &= -2e^{2\bar{\Omega}_1(t)-2\bar{\gamma}_1(t)-\gamma_1(t)-a_{11}} + ik_1^2, \\ k_1 &= -2i\bar{\xi}_1. \end{aligned} \tag{3.9}$$

Also, using transformations (2.1) and conditions (1.4), we obtain

$$\Omega_1(t) = \ln \beta_1(t) + \frac{1}{2}(\bar{\gamma}_1(t) + a_{11}) + \gamma_1(t). \tag{3.10}$$

Using expression (3.10) and solving the differential equation (3.9), we get the following:

$$\gamma_1(t) = -4i\bar{\xi}_1^2 t - 2\int_0^t \bar{\beta}_1^2(\tau)d\tau + \gamma_1(0). \tag{3.11}$$

Thus, taking into account (2.1), (3.8), (3.10) and (3.11) we can write the one-soliton solution of nlSESCS in the following form

$$u = \frac{e^{-2i\bar{\xi}_1 x + \gamma_1(t)}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}, \tag{3.12}$$

$$\phi_{11} = \beta_1(t)\frac{e^{(-2i\bar{\xi}_1 + i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}},$$

$$\phi_{21} = \beta_1(t)\frac{(-2i\bar{\xi}_1 + 2i\xi_1)e^{i\xi_1 x + \bar{\gamma}_1(t) + a_{11}}}{1 + e^{(-2i\bar{\xi}_1 + 2i\xi_1)x + \gamma_1(t) + \bar{\gamma}_1(t) + a_{11}}}.$$

The following figure shows one-soliton solution of the nlSESCS.

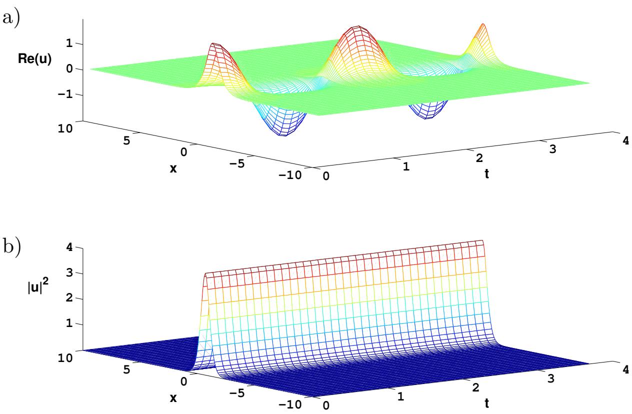

Figure 1. a) real part b) intensity profiles of the one-soliton solution (3.12) for $\xi_1 = i$, $\gamma_1(0) = 0$, $\beta_1(t) = \frac{1}{2}$.

4. Two-soliton solution

In this section, we find two-soliton solution of nlSESCS (i.e. in the case $N = 2$). We take the functions f and g in the following form

$$g = \chi g^{(1)} + \chi^3 g^{(2)}, \; f = 1 + \chi^2 f^{(1)} + \chi^4 f^{(2)}.$$

By applying the same previous procedure, we obtain the set of equations from Eq. (2.3) corresponding to the different power of χ

$$f_{xx}^{(1)} = g^{(1)}\bar{g}^{(1)}, \tag{4.1}$$

$$f_{xx}^{(2)} + f_{xx}^{(1)} f^{(1)} - (f_x^{(1)})^2 = g^{(1)}\bar{g}^{(2)} + g^{(2)}\bar{g}^{(1)}, \tag{4.2}$$

$$f_{xx}^{(1)} f^{(2)} - 2 f_x^{(1)} f_x^{(2)} + f^{(1)} f_{xx}^{(2)} = g^{(2)}\bar{g}^{(2)}, \tag{4.3}$$

$$f_{xx}^{(2)} f^{(2)} - \left(f_x^{(2)}\right)^2 = 0. \tag{4.4}$$

In order to find two-soliton solution, we utilize the superposition principle. We may use this principle since we are dealing with a bilinear equation and not a nonlinear one. As discussed in the one-soliton solution case, we can solve the equations (4.1)-(4.4) for getting the expression of f and g. In order to construct the two-soliton solution of the system (1.1)-(1.2) we assume g_1 has the form

$$g^{(1)} = e^{\eta_1} + e^{\eta_2}, \tag{4.5}$$

where $\eta_j = k_j x + \gamma_j(t)$, $(j = 1, 2)$. Therefore, the solution of the Eq. (4.1) is following

$$f^{(1)} = e^{\eta_1+\bar{\eta}_1+a_{11}} + e^{\eta_1+\bar{\eta}_2+a_{12}} + e^{\eta_2+\bar{\eta}_1+a_{21}} + e^{\eta_2+\bar{\eta}_2+a_{22}}, \tag{4.6}$$

where

$$a_{mn} = \ln \frac{1}{(k_m + \bar{k}_n)^2}, \qquad m, n = 1, 2.$$

With the help of Eqs. (4.2)-(4.4), we can obtain the functions $f^{(2)}$ and $g^{(2)}$

$$f^{(2)} = e^{\eta_1+\bar{\eta}_1+\eta_2+\bar{\eta}_2+r}, \tag{4.7}$$

$$g^{(2)} = e^{\eta_1+\bar{\eta}_1+\bar{\eta}_2+\delta_1} + e^{\eta_1+\bar{\eta}_2+\eta_2+\delta_2}, \tag{4.8}$$

where the constants r, δ_j, $j = 1, 2$, are given by

$$\delta_j = \ln\left(\frac{B_j}{q_j}\right), \qquad j = 1, 2,$$

$$q_1 = (k_1 + \bar{k}_1)(k_2 + \bar{k}_1), \quad q_2 = (k_1 + \bar{k}_2)(k_2 + \bar{k}_2),$$

$$B_1 = \frac{(k_1 + \bar{k}_1 + k_2 + \bar{k}_2)}{(k_1 + \bar{k}_1)^2 (k_2 + \bar{k}_1)^2}, \quad B_2 = \frac{(k_1 + \bar{k}_1 + k_2 + \bar{k}_2)}{(k_1 - \bar{k}_2)^2 (k_2 + \bar{k}_2)^2},$$

$$r = \frac{(k_2 - k_1)^2 (\bar{k}_2 - \bar{k}_1)^2}{(k_1 + \bar{k}_1)^2 (k_1 + \bar{k}_2)^2 (k_2 + \bar{k}_1)^2}.$$

The next step to find two-soliton solution is to determine the functions p_j and h_j $(j = 1, 2)$. The functions p_j and h_j $(j = 1, 2)$ for two-soliton solution are as follows

$$p_j = \chi p_j^{(1)} + \chi^3 p_j^{(2)}, \;\; h_j = h_j^{(1)}.$$

Based on the above, we collect coefficients of the same power in χ according to the the (2.4), we have

$$p_{j,x}^{(1)} = -i\xi_j p_j^{(1)} + g^{(1)} h_j^{(1)}, \tag{4.9}$$

$$p_{j,x}^{(1)} f^{(1)} - p_j^{(1)} f_x^{(1)} + p_{j,x}^{(2)} = -i\xi_j (p_j^{(1)} f^{(1)} + p_j^{(2)}) + g^{(2)} h_1^{(1)}, \tag{4.10}$$

$$p_{j,x}^{(1)} f^{(2)} - p_j^{(1)} f_x^{(2)} + p_{j,x}^{(2)} f^{(1)} - p_j^{(2)} f_x^{(1)} = -i\xi_j (p_j^{(1)} f^{(2)} + p_j^{(2)} f^{(1)}), \tag{4.11}$$

$$p_{j,x}^{(2)} f^{(2)} - p_j^{(2)} f_x^{(2)} = -i\xi_j p_j^{(2)} f^{(2)}. \tag{4.12}$$

We solve differential equations (4.9)-(4.12) by using (4.5)-(4.8) and we get the following expressions

$$\begin{aligned} p_j^{(1)} =& e^{(k_1+\bar{k}_1-i\xi_j)x-\gamma_2(t)-\bar{\gamma}_2(t)+\gamma_1(t)+\bar{\gamma}_1(t)+\Omega_j(t)-r+a_{11}} + \\ & e^{(k_1+\bar{k}_2-i\xi_j)x-\gamma_1(t)-\bar{\gamma}_1(t)+\gamma_2(t)+\bar{\gamma}_2(t)+\Omega_j(t)-r+a_{12}} + \\ & e^{(k_2+\bar{k}_1-i\xi_j)x-\gamma_1(t)-\bar{\gamma}_2(t)+\bar{\gamma}_1(t)+\gamma_2(t)+\Omega_j(t)-r+a_{21}} + \\ & e^{(k_2+\bar{k}_2-i\xi_j)x-\gamma_1(t)-\bar{\gamma}_1(t)+\gamma_2(t)+\bar{\gamma}_2(t)+\Omega_j(t)-r+a_{22}}, \end{aligned} \tag{4.13}$$

$$p_j^{(2)} = e^{(k_1+\bar{k}_1+k_2+\bar{k}_2-i\xi_j)x+\Omega_j(t)}, \tag{4.14}$$

$$\begin{aligned} h_j^{(1)} =& \frac{e^{(k_1+\bar{k}_1-i\xi_j)x-\gamma_2(t)-\bar{\gamma}_2(t)+\gamma_1(t)+\bar{\gamma}_1(t)+\Omega_j(t)-r}}{(k_1+\bar{k}_1)(e^{\eta_1}+e^{\eta_2})} + \\ & \frac{e^{(k_1+\bar{k}_2-i\xi_j)x-\gamma_2(t)-\bar{\gamma}_2(t)+\gamma_1(t)+\bar{\gamma}_1(t)+\Omega_j(t)-r}}{(k_1+\bar{k}_2)(e^{\eta_1}+e^{\eta_2})} + \\ & \frac{e^{(k_2+\bar{k}_1-i\xi_j)x-\gamma_2(t)-\bar{\gamma}_2(t)+\gamma_1(t)+\bar{\gamma}_1(t)+\Omega_j(t)-r}}{(k_2+\bar{k}_1)(e^{\eta_1}+e^{\eta_2})} + \\ & \frac{e^{(k_2+\bar{k}_2-i\xi_j)x-\gamma_2(t)-\bar{\gamma}_2(t)+\gamma_1(t)+\bar{\gamma}_1(t)+\Omega_j(t)-r}}{(k_2+\bar{k}_2)(e^{\eta_1}+e^{\eta_2})}, \\ & j = 1, 2. \end{aligned} \tag{4.15}$$

The time-dependent evolution of the functions $\gamma_j(t)$, $\Omega_j(t)$ $(j = 1, 2)$ can be found similarly as in one-soliton solutions. By substituting the defined functions

$$f = 1 + f^{(1)} + f^{(2)},\ g = g^{(1)} + g^{(2)},\ p_j = p_j^{(1)} + p_j^{(2)},\ h_j = h_j^{(1)} \tag{4.16}$$

into Eq.(2.2), we get the following

$$\begin{aligned} (\gamma_i(t))_t &= -2e^{2\bar{\Omega}_i(t)-2(\bar{\gamma}_1(t)+\bar{\gamma}_2(t))-\gamma_i(t)-r+\delta_i} + ik_i^2, \\ k_i &= -2i\bar{\xi}_i,\ j = 1, 2. \end{aligned} \tag{4.17}$$

We know that, the condition (1.4) is assumed for the functions φ_{1j} and φ_{2j} $(j=1,2)$, so, the function $\Omega_j(t)$ $(j=1,2)$ is defined as

$$\Omega_j(t) = \ln\beta_j(t) + \frac{1}{2}(\bar{\gamma}_j(t) - r + \delta_j) + \gamma_1(t) + \gamma_2(t),\ j=1,2, \tag{4.18}$$

therefore,

$$\gamma_j(t) = -4i\bar{\xi}_j^2 t - 2\int_0^t \bar{\beta}_j^2(\tau)d\tau + \gamma_j(0),\ j=1,2. \tag{4.19}$$

Thus, taking into account (4.5)-(4.8) and (4.13)-(4.15) we can write the solution in the following form

$$u = \frac{g^{(1)} + g^{(2)}}{1 + f^{(1)} + f^{(2)}}, \tag{4.20}$$

$$\varphi_{1j} = \frac{p^{(1)} + p^{(2)}}{1 + f^{(1)} + f^{(2)}},\quad \varphi_{2j} = \frac{h^{(1)}}{1 + f^{(1)} + f^{(2)}},\qquad j=1,2.$$

These functions are two-soliton solutions of the nlSESCS.

The following figure shows two-soliton solution of the nlSESCS.

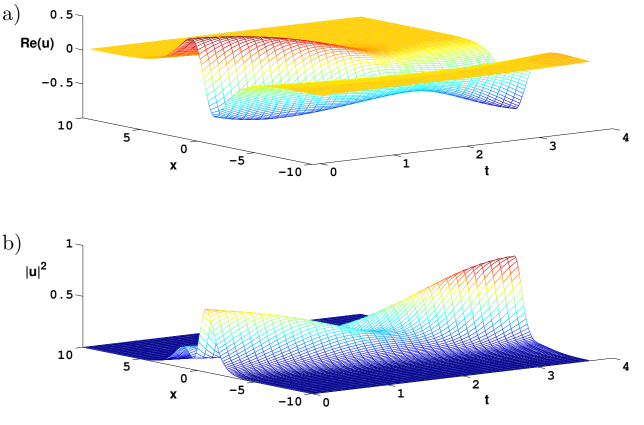

Figure 2. a) real part b) intensity profiles of the two-soliton solution (4.20) for $\xi_1 = \frac{1}{2}i$, $\xi_2 = \frac{1}{2}i + 1$, $\gamma_j(0) = 0$, $\beta_j(t) = \frac{1}{2}$, $(j=1,2)$.

5. Conclusion

In this paper, we have obtained the one-soliton and two-soliton solutions for the nlSESCS, by directly applying Hirota's bilinear method. Besides other soliton solutions can also be got by Hirota's bilinear method.

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Солитонные решения нелинейного уравнения Шредингера с самосогласованным источником

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Аннотация. Нелинейное уравнение Шредингера с самосогласованным источником преобразовано в билинейные формы и найдены односолитонные и двухсолитонные решения прямым билинейным методом Хироты. Подробно обсуждена эволюция солитона с помощью графики.

Ключевые слова: солитонные решения, уравнение Шредингера, нелинейные уравнения, метод Хироты.

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