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On the Solvability of a Class of Nonlinear Urysohn Integral Equations on the Positive Half-line*

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Abstract. The paper investigates the Urysohn's nonlinear integral equation on the positive half-line. Some special cases of this equation have specific applications in different areas of modern natural science. In particular, such equations arise in the kinetic theory of gases, in the theory of p-adic open-closed strings, in mathematical theory of the spatio-temporal spread of the epidemic, and in theory of radiative transfer in spectral lines. The existence theorem for nonnegative nontrivial and bounded solutions is proved. Some qualitative properties of the constructed solution are studied. Specific applied examples of the Urysohn's kernel satisfying all the conditions of the approved theorem are provided.

Keywords: Urysohn equation, monotonicity, Caratheodory condition, iterations, bounded solution.

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1. Introduction

We consider the following class of Urysohn's nonlinear integral equations

$$f(x) = \int_0^\infty u(x,t,f(t))dt, \quad x \in \mathbb{R}^+ := [0,+\infty) \tag{1.1}$$

with respect to unknown essentially bounded real function $f(x)$. In equation (1.1) Urysohn's kernel $u(x,t,z)$ is defined on set $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$, accepts real values, satisfies condition of criticality

$$u(x,t,0) \equiv 0, \quad (x,t) \in \mathbb{R}^+ \times \mathbb{R}^+ \tag{1.2}$$

and some other conditions (see formulation of main result).

Note that Urysohn's type nonlinear integral equations have a numerous applications in the most diverse areas of modern natural science. In particular under certain representations of the Urysohn's kernel, the equation (1.1) arises in kinetic theory of gases (in the framework of modified model of nonlinear Boltzmann equation), in mathematical theory of spatial-temporal spread of epidemics, in dynamic theory of p-adic closed-open strings, in radiative transfer theory of spectral lines (see [2–5; 15] and references there in).

In papers [8; 11–13] the equation (1.1) has been investigated in sufficient details in the case of monotone Urysohn's operator, for which as a linear minorant in the sense of M.A. Krasnoselskii is Wiener–Hopf conservative integral operator (or Wiener–Hopf-Hankell substochastic operator). Moreover for some $\eta > 0$ Urysohn's kernel satisfies the following integral inequality

$$\int_0^\infty u(x,t,\eta)dt \le \eta, \quad x \in \mathbb{R}^+. \tag{1.3}$$

In this work under essentially weak constraints on the Urysohn's kernel, the existence of a nonnegative nontrivial solution is proved, as well as some qualitative properties of constructed solution is investigated.

Before proceeding to the formulation of the main result of this work, we introduce some notations.

Let K_0 — is defined on the set $\mathbb{R}^+$, measurable function, satisfying conditions

$$K_0(x) > 0, \quad x \in \mathbb{R}^+, \quad K_0 \in L_1(\mathbb{R}^+) \cap M(\mathbb{R}^+), \tag{1.4}$$

$$\int_0^\infty K_0(x)dx = 1, \tag{1.5}$$

where $M(\mathbb{R}^+)$ — is the space of essentially bounded on $\mathbb{R}^+$ functions with norm

$$\|f\| = \operatorname*{esssup}_{x\in\mathbb{R}^+} |f(x)|. \quad (1.6)$$

Let $\lambda(x)$ — is defined on $\mathbb{R}^+$ continuous function possesses the following properties

$$0 < \varepsilon_0 := \inf_{x\in\mathbb{R}^+} \lambda(x) \le \lambda(x) \le 1, \quad x \in \mathbb{R}^+, \quad \lambda(x) \uparrow \text{ on } \mathbb{R}^+, \quad (1.7)$$

$$\lim_{x\to+\infty} \lambda(x) = 1, \quad (1 - \lambda(x))x \in L_1(\mathbb{R}^+). \quad (1.8)$$

Further, we introduce a function $K_1(x, y)$, that is definite and continuous on $\mathbb{R}^+ \times \mathbb{R}^+$, and

$$0 \le K_1(x, y) \le K_0(y - x), \quad 0 \le x \le y < +\infty, \quad (1.9)$$

$$x \int_x^\infty K_1(x, y)dy \in L_1^0(\mathbb{R}^+), \quad (1.10)$$

where $L_1^0(\mathbb{R}^+)$ — is the space of summable functions on $\mathbb{R}^+$, having zero limit at $+\infty$.

The main result of this work is the following.

Theorem. *Let exist positive numbers ξ and η $(0 < \xi < \eta)$ such that*

a) *the function $u(x, t, z)$ for each fixed $(x, t) \in \mathbb{R}^+ \times \mathbb{R}^+$ monotonically increases by z on interval $[0, \eta]$,*

b) *the function $u(x, t, z)$ satisfies Caratheodory's condition by argument z on set $\mathbb{R}^+ \times \mathbb{R}^+ \times [0, \eta]$, i.e for each fixed $z \in [0, \eta]$ the function $u(x, t, z)$ is measurable by the set of arguments (x, t) on set $\mathbb{R}^+ \times \mathbb{R}^+$, and almost for all $(x, t) \in \mathbb{R}^+ \times \mathbb{R}^+$ is continuous by z on interval $[0, \eta]$,*

c) *for arbitrary measurable function $\varphi(x)$, such that $0 \le \varphi(x) \le \eta$, $x \in \mathbb{R}^+$ the function $\int_0^\infty u(x, t, \varphi(t))dt$ is measurable by x on $\mathbb{R}^+$,*

d) *for kernel $u(x, t, z)$, the minorant in the sense of Krasnoselsky is the function $\lambda(x)(K_0(y - x) - K_1(x, y))z$ at $0 \le x \le y < +\infty$, $z \in [0, \xi]$, i.e*

$$u(x, t, z) \ge \lambda(x)(K_0(y - x) - K_1(x, y))z, \quad 0 \le x \le y < +\infty, \ z \in [0, \xi].$$

Then under conditions (1.2), (1.3), the equation (1.1) possesses nonnegative nontrivial measurable and bounded solution $f(x)$, moreover, $f(x) \le \eta$, $x \in \mathbb{R}^+$.

At the end of the work, specific examples of the Urysohn's kernel are given, which have a direct application in natural science.

2. Proof of the theorem

First of all, along with the equation (1.1), consider the following auxiliary inhomogeneous Volterra integral equation with a variable lower limit

$$F(x) = g(x) + \lambda(x) \int_x^\infty (K_0(y-x) - K_1(x,y))F(y)dy, \quad x \in \mathbb{R}^+ \tag{2.1}$$

and with respect to the unknown measurable function $F(x)$. Here the free term $g(x)$ permits of the form

$$g(x) := 1 - \lambda(x) + \lambda(x) \int_x^\infty K_1(x,y)dy, \quad x \in \mathbb{R}^+. \tag{2.2}$$

Observe that from conditions (1.7)–(1.10) it follows that

$$\int_0^\infty x^j g(x)dx < +\infty, \quad j = 0, 1, \tag{2.3}$$

$$\lim_{x\to+\infty} g(x) = 0. \tag{2.4}$$

Direct checking can be verified that $F_0(x) \equiv 1$ is the solution of the equation (2.1).

Below we show that equation (2.1), in addition to the trivial solution $F_0(x) \equiv 1$, possesses nonnegative summable and bounded solution $F_1(x)$. Furthermore $F_1(x) \leq F_0(x)$ and $\lim\limits_{x\to+\infty} F_1(x) = 0$. To this end, consider the following Volterra conservative nonhomogeneous integral equation with the kernel depending on difference of arguments.

$$\varphi(x) = g(x) + \int_x^\infty K_0(y-x)\varphi(y)dy, \quad x \in \mathbb{R}^+ \tag{2.5}$$

with respect to the sought function $\varphi(x)$. Due to the properties (2.3), (2.4), (1.4) and (1.5) according to the results of the work [1] (see theorem 3.1 and lemma 3.6, pp. 191–193), the equation (2.5) has nonnegative summerable on $\mathbb{R}^+$ solution $\varphi(x)$. Taking into account (1.4), (1.7), (2.2) and (2.4) from (2.5) we get

$$g(x) \leq \varphi(x) \leq g(x) + (\sup_{x\in\mathbb{R}^+} K_0(x)) \int_x^\infty \varphi(t)dt. \tag{2.6}$$

Hence, it can be stated that

$$\varphi \in L_1(\mathbb{R}^+) \cap M(\mathbb{R}^+) \text{ and } \lim_{x\to+\infty} \varphi(x) = 0. \tag{2.7}$$

We introduce the following simple iterations for equation (2.7)

$$\begin{gathered} F^{(n+1)}(x) = g(x) + \lambda(x) \int\limits_x^\infty (K_0(t-x) - K_1(x,t))F^{(n)}(t)dt, \\ F^{(0)}(x) \equiv g(x), \quad n = 0,1,2,\ldots, \quad x \in \mathbb{R}^+. \end{gathered} \tag{2.8}$$

Using (1.9), (1.7), (1.4), (1.5) and (2.5), by the method of mathematical induction it is not difficult to establish the validity of the following statements

$$F^{(n)}(x) \uparrow \text{ by } n, \tag{2.9}$$

$$F^{(n)}(x) \le \varphi(x), \quad n = 0,1,2,\ldots, \quad x \in \mathbb{R}^+, \tag{2.10}$$

$$F^{(n)}(x) \le 1, \quad n = 0,1,2,\ldots, \quad x \in \mathbb{R}^+, \tag{2.11}$$

where $\varphi(x)$ — is the solution of integral equation (2.5) with the properties (2.7).

Therefore there exists pointwise limit of sequences

$$\{F^{(n)}(x)\}_{n=0}^\infty : \lim_{n\to\infty} F^{(n)}(x) =: F_1(x),$$

moreover limit function in accordance to B. Levi's theorem (see [6]), satisfies the equation (2.1). In view of (2.7) from (2.9)–(2.11) it follows that

$$g(x) \le F_1(x) \le \varphi(x), \quad x \in \mathbb{R}^+, \tag{2.12}$$

$$F_1 \in L_1(\mathbb{R}^+), \quad F_1(x) \le 1, \quad x \in \mathbb{R}^+, \quad \lim_{x\to+\infty} F_1(x) = 0. \tag{2.13}$$

Thus, due to (2.12) and (2.13), we can state that $F_1(x) \not\equiv 1, \quad x \in \mathbb{R}^+$. It is obvious that $S^*(x) := 1 - F_1(x) \ge 0$, $S^*(x) \not\equiv 0$ will be solution of the linear homogeneous equation

$$S(x) = \lambda(x) \int\limits_x^\infty (K_0(t-x) - K_1(x,t))S(t)dt, \quad x \in \mathbb{R}^+. \tag{2.14}$$

Moreover, due to (2.12)–(2.13), the function $S^*(x)$ possesses the following properties

$$0 \le S^*(x) \le 1, \quad S^*(x) \not\equiv 0, \quad x \in \mathbb{R}^+, \tag{2.15}$$

$$\lim_{x\to+\infty} S^*(x) = 1, \quad 1 - S^* \in L_1(\mathbb{R}^+). \tag{2.16}$$

Now let's go back to the study of the initial nonlinear integral equation (1.1). We introduce the following special iterations for (1.1):

$$f_{n+1}(x)=\int_0^\infty u(x,t,f_n(t))dt, \quad x\in\mathbb{R}^+,$$
$$f_0(x)=\xi S^*(x), \quad n=0,1,2,\ldots. \tag{2.17}$$

By induction we prove that

$$f_n(x)\uparrow \text{ by } n, \tag{2.18}$$

$$f_n(x)\leq\eta, \quad x\in\mathbb{R}^+, \quad n=0,1,2,\ldots. \tag{2.19}$$

Observe that from (2.15) and $\xi<\eta$ condition follows the inequality $f_0(x)\leq\eta$, $x\in\mathbb{R}^+$. Now we show $f_1(x)\geq f_0(x)$. Indeed, due to monotonicity of Urysohn's kernel $u(x,t,z)$ by z on $[0,\eta]$, and taking into consideration (1.7), (2.14), condition $d)$ from (2.17) we gate

$$f_1(x)=\int_0^\infty u(x,t,\xi S^*(t))dt\geq\int_x^\infty u(x,t,\xi S^*(t))dt\geq$$

$$\geq\xi\lambda(x)\int_x^\infty(K_0(t-x)-K_1(x,t))S^*(t)dt=\xi S^*(x)=f_0(x).$$

Assume that $f_n(x)\geq f_{n-1}(x)$ and $f_n(x)\leq\eta$, $\quad x\in\mathbb{R}^+$ for some natural n. Using monotonicity of the function $u(x,t,z)$ by z, and also inequality (1.3), from (2.17) we obtain

$$f_{n+1}(x)\geq\int_0^\infty u(x,t,f_{n-1}(t))dt=f_n(x), \quad x\in\mathbb{R}^+,$$

$$f_{n+1}(x)\leq\int_0^\infty u(x,t,\eta)dt\leq\eta, \quad x\in\mathbb{R}^+.$$

Thus (2.18) and (2.19) are proved. Using $c)$ and the Caratheodory's conditions, taking into account M.A. Krasnoselskii theorem (see [7]), by induction in n it is easy to check that each element of sequence$\{f_n(x)\}_{n=0}^\infty$ represents measurable function on $\mathbb{R}^+$.

Thus, due to (2.18) and (2.19) the sequence of measurable functions $\{f_n(x)\}_{n=0}^\infty$ has a pointwise limit: $\lim\limits_{n\to\infty}f_n(x)=f(x)$, and

$$\xi S^*(x)\leq f(x)\leq\eta, \quad x\in\mathbb{R}^+. \tag{2.20}$$

Finally, based on the B. Levi's limit theorem and Caratheodory condition, we conclude that $f(x)$ satisfies the equation (1.1). Theorem is proved.

Remark 1. We consider the following particular representation of Urysohn's kernel

$$u(x,t,z) = \lambda(x)(K(x-t) - \frac{1}{2}K_1(x,y))G(z), (x,t,z) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}, \quad (2.21)$$

where $K(x)$ is defined on set $\mathbb{R}$, even and bounded function, moreover

$$0 < K \in L_1(\mathbb{R}), \quad \int_{-\infty}^{\infty} K(x)dx = 1, \quad \int_{0}^{\infty} xK(x)dx < +\infty,$$

$$K(x-y) \geq \frac{1}{2}K_1(x,y), \quad (x,y) \in \mathbb{R}^+ \times \mathbb{R}^+,$$

and function G is the monotonic increasing convex up and continuous function on interval $[0,\eta]$, $G(u) \geq u, u \in [0,\eta]$, $G(\eta) = \eta$ and $G(\xi) = 2\xi$, $0 < \xi < \eta$ (see fig. 1).

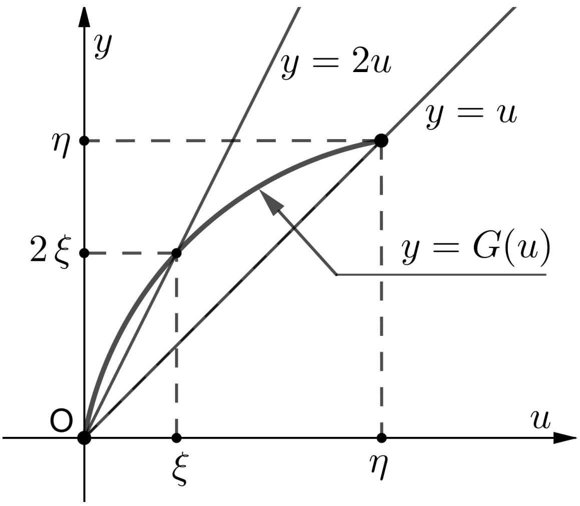

Figure 1.

Note that with such a representation of the Urysohn's kernel, all conditions of the proved theorem are automatically satisfied.

If Urysohn's kernel admits representation (2.21), then from result of the work [14] follows that the solution of equation (1.1) possesses the following additional properties

$$\lim_{x \to +\infty} f(x) = \eta, \quad (2.22)$$

$$\eta - f \in L_1(\mathbb{R}^+). \quad (2.23)$$

Remark 2. It should be also noted that the proved result generalizes and complements the existence theorems for the solution of the Urysohn's equation reported by the works [11]– [13].

3. Application in p-adic string theory

In the dynamic theory of p-adic open-closed strings, the following nonlinear integral equation with Gaussian type kernel depending on sum and difference of arguments are encountered (see [16]):

$$\mathcal{B}^p(x) = \frac{1}{\sqrt{\pi}} \int_0^\infty (e^{-(x-t)^2} - e^{-(x+t)^2}) \mathcal{B}(t) dt, \quad x \in \mathbb{R}^+ \tag{3.1}$$

with respect to sought bounded function $\mathcal{B}(x)$, where $p > 2$ - odd number. In this theory it becomes necessary to construct nonnegative nontrivial and bounded solution for equation (3.1), satisfying the limit relation $\lim\limits_{x \to +\infty} \mathcal{B}(x) = 1$.

By simply substitution $f(x) = \mathcal{B}^p(x)$, the problem of finding such solution is reduced to solve the problem of Hammerstein type integral equation

$$f(x) = \frac{1}{\sqrt{\pi}} \int_0^\infty (e^{-(x-t)^2} - e^{-(x+t)^2}) f^{\frac{1}{p}}(t) dt, \quad x \in \mathbb{R}^+, \tag{3.2}$$

with boundary condition

$$\lim_{x \to +\infty} f(x) = 1. \tag{3.3}$$

By direct verification, one can state that the Urysohn's kernel

$$u(x, t, z) = \frac{1}{\sqrt{\pi}} (e^{-(x-t)^2} - e^{-(x+t)^2}) z^{\frac{1}{p}}$$

satisfies all the conditions of the theorem, which we have proved by choosing as a $\eta = 1$, $\xi = \left(\frac{1}{2}\right)^{\frac{p}{p-1}}$, and

$$K_0(x) = \frac{2}{\sqrt{\pi}} e^{-x^2},\ K_1(x, y) = \frac{2}{\sqrt{\pi}} e^{-(x+y)^2},\ \lambda(x) \equiv 1,\ x \in \mathbb{R}^+,\ y \in \mathbb{R}^+.$$

Based on Remark 1 and on the above proved theorem we can assert that solution of the boundary value problem (3.2), (3.3) possesses property $1 - f \in L_1(\mathbb{R}^+)$.

At the end of the work, we still list some nontrivial examples of the Urysohn's kernels

1) $u(x, t, z) = (K(x - t) - K(x + t)) G(z),\ (x, t, z) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$,
where K and G satisfy conditions of Remark 1,

2) $u(x, t, z) = \lambda(x) K(x - t) G(z),\ (x, t, z) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$,

3) $u(x,t,z) = (1-\lambda(x))K(x-t)G_0(z)+\lambda(x)(K_0(t-x)-K_1(x,t))z\theta(t-x)$, $(x,t,z) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$, where θ — Heaviside function, and $G_0 \in C[0,\eta]$, $G_0 \uparrow$ on $[0,\eta]$, $G_0(0) = 0$, $G_0(\eta) = \eta$,

4) $u(x,t,z) = (1-\lambda(x))K(x-t)G_0(z) + \lambda(x)K_0(t-x)G(z)\theta(t-x)$, $(x,t,z) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$.

4. Conclusion

In this paper, we have obtain sufficient conditions for the existence of a nonnegative and bounded solution for class of Urysohn type nonlinear integral equations on half line. The proved theorem of existence has a constructive character. For one particular representation of Urysohn's kernel the limit of constructed solution at infinite is found, as well as has been shown that difference between limit and solution represents summable function on positive part of numerical axis. The obtained results are applied in dynamic theory of p - adic open-closed strings. The proved results generalize and complement the theorems of existence from works [11] – [13]. Specific and applied examples of the Urysohn's kernel are given. The examples illustrate how all conditions of the proved theorem are fulfilled.

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О разрешимости одного класса нелинейных интегральных уравнений Урысона на положительной полупрямой

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Аннотация. Исследовано нелинейное интегральное уравнение Урысона на положительной полупрямой. Обозначено, что некоторые частные случаи данного уравнения имеют конкретные применения в различных направлениях современного естествознания. В частности, такие уравнения возникают в кинетической теории газов, в теории p-адических открыто-замкнутых струн, в математической теории пространственно-временного распространения эпидемии и в теории переноса излучения в спектральных линиях. Доказана теорема существования неотрицательных нетривиальных и ограниченных решений. Изучены некоторые качественные свойства построенного решения. Приведены конкретные прикладные примеры ядра Урысона, удовлетворяющие всем условиям доказанной теоремы.

Ключевые слова: уравнение Урысона, монотонность, условие Каратеодори, итерации, ограниченное решение.

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