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An Initial Problem for a Class of Weakly Degenerate Semilinear Equations with Lower Order Fractional Derivatives *

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Abstract. An initial value problem is studied for a class of evolutionary equations with a weak degeneration, which are nonlinear with respect to lower order fractional Gerasimov – Caputo derivatives. The linear part of the equations contains a respectively bounded pair of operators. Unique local solvability is proved in the case of a nonlinear operator depending on elements of the degeneration space only. Examples of an equation and a system of partial differential equations are given, the initial-boundary value problems for which are reduced to the initial problem for an equation in a Banach space of the studied class.

Keywords: fractional Gerasimov – Caputo derivative, fractional order differential equation, degenerate evolution equation, semilinear equation.

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1. Introduction

Among the equations of mathematical physics, a special place is occupied by equations and systems of equations that are not solvable with respect to the highest time derivative, called degenerate evolutionary equations. In this paper, we investigate the solvability of a class of degenerate evolution equations with fractional derivatives.

In Banach spaces $\mathcal{X}$, $\mathcal{Y}$, a continuous linear operator L is given (briefly, $L \in \mathcal{L}(\mathcal{X},\mathcal{Y})$), M is a linear closed operator with domain D_M dense in $\mathcal{X}$, ($M \in \mathcal{C}l(\mathcal{X},\mathcal{Y})$) and $N : X \to \mathcal{Y}$ is a nonlinear operator, where X is an open set in $\mathbb{R} \times \mathcal{X}^n$. Consider the equation

$$D_t^{\alpha} Lx(t) = Mx(t) + N(t, D_t^{\alpha_1}x(t), D_t^{\alpha_2}x(t), \dots, D_t^{\alpha_n}x(t)), \tag{1.1}$$

where $0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_n < \alpha$, $m-1 < \alpha \leq m \in \mathbb{N}$, D_t^{β} is the Gerasimov — Caputo derivative.

Note the studies of the solvability of degenerate evolution equations of form (1.1) of integer [3; 4; 20–22] and fractional [2; 5; 6; 8; 9; 15; 16] orders. For various equations resolved with respect to the fractional derivative, as well as for the corresponding integral equations, results on the existence of a unique solution were obtained by such authors as J. Prüss [19], E.G. Bajlekova [1], A.V. Glushak [10], M. Kostic [12], V.E. Fedorov [7].

Solvability conditions for the Cauchy problem to an equation of form (1.1) with $\mathcal{X} = \mathcal{Y}$, $L = I$, $M \in \mathcal{L}(\mathcal{X},\mathcal{Y})$ are examined in [18]. Solvability of initial problems for a degenerate ($\ker L \neq \{0\}$) equation with a relatively bounded pair of operators L and M the authors of the article investigated for two types of constraints on the nonlinear operator [17; 18]: if the image of the nonlinear operator belongs to a subspace without degeneration, or if the nonlinear operator depends on the elements of such subspace only. In this paper, on the contrary, we use the condition that the nonlinear operator depends on the elements of the degeneration subspace only. Using the results of the theory of degenerate evolution equations (see [22]), we investigate the equation (1.1) equipped with the initial conditions

$$x^{(k)}(t_0) = x_k,\ k = 0, 1, \dots, r-1,\ (Px)^{(l)}(t_0) = x_l,\ l = r, \dots, m-1, \tag{1.2}$$

where P is the projector onto the space without degeneration, the number r is determined by the value of α_n (see further). Such a problem is reduced to the Cauchy problem for a system consisting of a linear equation solved with respect to the highest fractional derivative on a subspace without degeneration and a semilinear equation of a lower fractional order on the subspace of degeneration obtained using the implicit function theorem. To illustrate the abstract results obtained, examples of initial-boundary value problems for an equation and a system of partial differential equations that are nonlinear with respect to the lowest fractional time derivatives are given.

2. Equations solvable with respect to the highest fractional derivative

Introduce the notations $g_\delta(t) := \Gamma(\delta)^{-1}t^{\delta-1}$, $\tilde{g}_\delta(t) := \Gamma(\delta)^{-1}(t-t_0)^{\delta-1}$, $J_t^\delta h(t) := \int\limits_{t_0}^{t} g_\delta(t-s)h(s)ds$ for $\delta > 0$, $t > 0$. Let D_t^m be the usual derivative of the order $m \in \mathbb{N}$, J_t^0 be the identity operator. The fractional Gerasimov — Caputo [1, p. 11] derivative of the function h is defined as

$$D_t^\alpha h(t) := D_t^m J_t^{m-\alpha}\left(h(t) - \sum_{k=0}^{m-1} h^{(k)}(t_0)\tilde{g}_{k+1}(t)\right), \quad t > t_0.$$

Let $\mathcal{Z}$ be a Banach space, $A \in \mathcal{L}(\mathcal{Z})$. Consider the Cauchy problem

$$z^{(k)}(t_0) = z_k, \quad k = 0, 1, \ldots, m-1, \tag{2.1}$$

for the inhomogeneous linear equation

$$D_t^\alpha z(t) = Az(t) + f(t), \quad t \in [t_0, T), \tag{2.2}$$

where $T \in (t_0, +\infty]$. A function $z \in C^{m-1}([t_0, T); \mathcal{Z})$ is called a solution of problem (2.1), (2.2), if $J_t^{m-\alpha}\left(z - \sum\limits_{k=0}^{m-1} z^{(k)}(t_0)\tilde{g}_{k+1}\right) \in C^m([t_0, T); \mathcal{Z})$ and equalities (2.1), (2.2) are valid.

Theorem 1. [15]. *Let $A \in \mathcal{L}(\mathcal{Z})$, $f \in C([t_0, T); \mathcal{Z})$. Then for all $z_0, z_1, \ldots, z_{m-1} \in \mathcal{Z}$ there exists a unique solution of problem* (2.1), (2.2).

Let $n \in \mathbb{N}$, Z be an open set in $\mathbb{R} \times \mathcal{Z}^n$, $B : Z \to \mathcal{Z}$ be a nonlinear operator, $z_k \in \mathcal{Z}$, $k = 0, 1, \ldots, m-1$, $0 \le \alpha_1 < \alpha_2 < \cdots < \alpha_n < \alpha$, $m - 1 < \alpha \le m \in \mathbb{N}$/ Consider the semilinear equation

$$D_t^\alpha z(t) = Az(t) + B(t, D_t^{\alpha_1} z(t), D_t^{\alpha_2} z(t), \ldots, D_t^{\alpha_n} z(t)). \tag{2.3}$$

By a solution to problem (2.1), (2.3) on an interval $[t_0, t_1]$ we mean a function $z \in C^{m-1}([t_0, t_1]; \mathcal{Z})$, for which the condition

$$J_t^{m-\alpha}\left(z - \sum_{k=1}^{m-1} z^{(k)}(t_0)\tilde{g}_{k+1}\right) \in C^m([t_0, t_1]; \mathcal{Z})$$

is satisfied, and for any $t \in [t_0, t_1]$ $(t, D_t^{\alpha_1} z(t), D_t^{\alpha_2} z(t), \ldots, D_t^{\alpha_n} z(t)) \in Z$, equalities (2.1) and (2.3) are true for all $t \in [t_0, t_1]$.

Further, the line above the symbol will denote a set of n elements with indices from 1 to n, for example, $\bar{x} = (x_1, x_2, \ldots, x_n)$. Let $S_\delta(\bar{x}) = \{\bar{y} \in \mathcal{Z}^n : \|y_k - x_k\|_\mathcal{Z} \le \delta, k = 1, 2, \ldots, n\}$. A mapping$B : Z \to \mathcal{Z}$ will be called locally Lipschitzian in z if for each $(t,\bar{x}) \in Z$ there are $\delta > 0$ and $l > 0$ for which $[t_0-\delta, t_0+\delta] \times S_\delta(\bar{x}) \subset Z$ and for any $(s,\bar{y}),(s,\bar{v}) \in [t_0-\delta, t_0+\delta] \times S_\delta(\bar{x})$

$$\|B(s,\bar{y}) - B(s,\bar{v})\|_{\mathcal{Z}} \leq l \sum_{k=1}^{n} \|y_k - v_k\|_{\mathcal{Z}}.$$

Using the initial data z_0, $z_1, \ldots, z_{m-1}$ we define

$$\tilde{z} = z_0 + \frac{z_1}{1!}(t-t_0) + \frac{z_2}{2!}(t-t_0)^2 + \cdots + \frac{z_{m-1}}{(m-1)!}(t-t_0)^{m-1},$$

$$\tilde{z}_1 = D_t^{\alpha_1}|_{t=t_0}\tilde{z}(t), \quad \tilde{z}_2 = D_t^{\alpha_2}|_{t=t_0}\tilde{z}(t), \quad \ldots, \quad \tilde{z}_n = D_t^{\alpha_n}|_{t=t_0}\tilde{z}(t).$$

Theorem 2. [18]. *Let $A \in \mathcal{L}(\mathcal{Z})$, a set Z be open in $\mathbb{R} \times \mathcal{Z}^n$, a mapping $B \in C(Z;\mathcal{Z})$ be locally Lipschitzian in z, $z_k \in \mathcal{Z}$, $k = 1,2,\ldots,m-1$ be such that $(t_0, \tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_n) \in Z$. Then problem* (2.1), (2.3) *has a unique solution on a segment $[t_0,t_1]$ for some $t_1 > t_0$.*

3. Semilinear equation with a weak degeneration

Let $\mathcal{X}, \mathcal{Y}$ are Banach spaces, $L \in \mathcal{L}(\mathcal{X};\mathcal{Y})$, $\ker L \neq \{0\}$, $M \in \mathcal{C}l(\mathcal{X};\mathcal{Y})$, D_M be the domain of M operator equipped with the graph norm $\|\cdot\|_{D_M} := \|\cdot\|_{\mathcal{X}} + \|M\cdot\|_{\mathcal{Y}}$.

We define an L-resolvent set $\rho^L(M) := \{\mu \in \mathbb{C} : (\mu L - M)^{-1} \in \mathcal{L}(\mathcal{Y};\mathcal{X})\}$ of operator M and denote $R_\mu^L(M) := (\mu L - M)^{-1}L$, $L_\mu^L := L(\mu L - M)^{-1}$. Operator M is called (L,σ)-*bounded*, if

$$\exists a > 0 \quad \forall \mu \in \mathbb{C} \quad (|\mu| > a) \Rightarrow (\mu \in \rho^L(M)).$$

Under the condition that the operator M is (L,σ)-bounded, we define the projectors

$$P := \frac{1}{2\pi i}\int_\gamma R_\mu^L(M)\,d\mu \in \mathcal{L}(\mathcal{X}), \;\; Q := \frac{1}{2\pi i}\int_\gamma L_\mu^L(M)\,d\mu \in \mathcal{L}(\mathcal{Y}), \quad (3.1)$$

where $\gamma = \{\mu \in \mathbb{C} : |\mu| = r > a\}$ (see [22, p. 89, 90]). Let $\mathcal{X}^0 := \ker P$, $\mathcal{X}^1 := \mathrm{im}P$, $\mathcal{Y}^0 := \ker Q$, $\mathcal{Y}^1 := \mathrm{im}Q$. Let us denote by L_k (M_k) the constriction of the operator L (M) on $\mathcal{X}^k$ $(D_{M_k} := D_M \cap \mathcal{X}^k)$, $k = 0,1$.

Theorem 3. [22, p. 90, 91]. *Let an operator M be (L,σ)-bounded. Then*

(i) $M_1 \in \mathcal{L}(\mathcal{X}^1;\mathcal{Y}^1)$, $M_0 \in \mathcal{C}l(\mathcal{X}^0;\mathcal{Y}^0)$, $L_k \in \mathcal{L}(\mathcal{X}^k;\mathcal{Y}^k)$, $k = 0,1$;

(ii) *there exist operators* $M_0^{-1} \in \mathcal{L}(\mathcal{Y}^0;\mathcal{X}^0)$, $L_1^{-1} \in \mathcal{L}(\mathcal{Y}^1;\mathcal{X}^1)$.

We denote $G := M_0^{-1}L_0$. For $p \in \mathbb{N}_0 := \{0\} \cup \mathbb{N}$ an operator M is called (L,p)-bounded, if it is (L,σ)-bounded, $G^p \neq 0$, $G^{p+1} = 0$.

Consider the problem

$$x^{(k)}(t_0) = x_k,\ k = 0,1,\dots,r-1,\ (Px)^{(l)}(t_0) = x_l,\ l = r,\dots,m-1, \quad (3.2)$$

for the equation

$$D_t^\alpha Lx(t) = Mx(t) + N(t, D_t^{\alpha_1}x(t), D_t^{\alpha_2}x(t),\dots,D_t^{\alpha_n}x(t)), \quad (3.3)$$

where $0 \le \alpha_1 < \alpha_2 < \dots < \alpha_n < \alpha$, $m-1 \le \alpha < m \in \mathbb{N}$, $r-1 < \alpha_n \le r \in \mathbb{N}$, X is an open set in $\mathbb{R} \times \mathcal{X}^n$, $N : X \to \mathcal{Y}$ is a nonlinear operator.

Since $\ker L \neq \{0\}$, equation (3.3) is degenerate. In the case of an $(L,0)$-bounded operator M, we have $\ker P = \ker L$ [22], therefore, the degeneration subspace $\mathcal{X}^0$ in this case is minimal and the corresponding class of equations (3.3) is called weakly degenerate.

By a solution of problem (3.2), (3.3) on a segment $[t_0,t_1]$ we mean a function $x \in C([t_0,t_1];D_M) \cap C^{r-1}([t_0,t_1];\mathcal{X})$, such that

$$Lx \in C^{m-1}([t_0,t_1];\mathcal{Y}),\ J_t^{m-\alpha}\left(Lx - \sum_{k=0}^{m-1}(Lx)^{(k)}(t_0)\tilde{g}_{k+1}\right) \in C^m([t_0,t_1];\mathcal{Y}),$$

for all $t \in [t_0,t_1]$ $(t, D_t^{\alpha_1}x(t), D_t^{\alpha_2}x(t),\dots,D_t^{\alpha_n}x(t)) \in X$ and equalities (3.2), (3.3) hold.

By $[(I-Q)N]'_{x_n}(t,z_1,z_2,\dots,z_n)$ we denote the Frechet derivative of the operator $(I-Q)N$ at the point $(t,z_1,z_2,\dots,z_n) \in X$ by the last argument x_n. For brevity, we denote the projector along $\mathcal{X}^1$ on $\mathcal{X}^0$ as $P_0 := I - P$.

We denote $W = X \cap (\mathbb{R} \times (\mathcal{X}^0)^n)$,

$$\tilde{x} = x_0 + \frac{x_1}{1!}(t-t_0) + \frac{x_2}{2!}(t-t_0)^2 + \dots + \frac{x_{m-1}}{(m-1)!}(t-t_0)^{m-1},$$

for x_k, $k = 0,1,\dots,m-1$, from conditions (3.2).

Theorem 4. *Let $0 \le \alpha_1 < \alpha_2 < \dots < \alpha_n < \alpha$, $m-1 < \alpha \le m \in \mathbb{N}$, $r-1 < \alpha_n \le r \in \mathbb{N}$, an operator M be $(L,0)$-bounded, a set X be open in $\mathbb{R} \times \mathcal{X}^n$; $N \in C(X;\mathcal{Y})$, for all $(t,z_1,\dots,z_n) \in X$, such that $(t,P_0z_1,\dots,P_0z_n) \in X$, $N(t,z_1,\dots,z_n) = N_1(t,P_0z_1,\dots,P_0z_n)$ at some $N_1 \in C(W;\mathcal{Y})$, such that $(I-Q)N_1 \in C^1(W;\mathcal{Y})$; $x_1,x_2\dots x_{r-1} \in \mathcal{X}$, $x_r,x_{r+1},\dots,x_{m-1} \in \mathcal{X}^1$, the mapping $M_0^{-1}[(I-Q)N_1]'_{x_n}(t,z_1,\dots,z_n) : \mathcal{X}^0 \to \mathcal{X}^0$ be a bijection for all elements $(t,z_1,z_2,\dots,z_n)$ of the point $(t_0, D_t^{\alpha_1}|_{t=t_0}\tilde{x},\dots,D_t^{\alpha_n}|_{t=t_0}\tilde{x}) \in W$ neighborhood, herewith*

$$P_0x_0 + M_0^{-1}(I-Q)N(t_0, D_t^{\alpha_1}|_{t=t_0}P_0\tilde{x},\dots,D_t^{\alpha_n}|_{t=t_0}P_0\tilde{x}) = 0. \quad (3.4)$$

Then there exists such $t_1 > t_0$, that problem (3.2), (3.3) *has a unique solution on the segment* $[t_0,t_1]$.

Proof. Let us act on equation (3.3) by a continuous operator $M_0^{-1}(I-Q)$, the existence of which follows from Theorem 3. We get the equation

$$0 = w(t) + M_0^{-1}(I-Q)N_1\left(t, D_t^{\alpha_1}w(t), D_t^{\alpha_2}w(t), \dots, D_t^{\alpha_n}w(t)\right),$$

where $w(t) := P_0x(t)$. By the implicit function theorem, since there exists the inverse operator

$$\left(M_0^{-1}[(I-Q)N_1]'_{x_n}(t, D_t^{\alpha_1}w(t), D_t^{\alpha_2}w(t), \dots, D_t^{\alpha_n}w(t))\right)^{-1} \in \mathcal{L}(\mathcal{X}^0),$$

this equation can be solved with respect to $D_t^{\alpha_n}w$ at t from some interval $(t_0-\delta, t_0+\delta)$. Hence, we have the equation

$$D_t^{\alpha_n}w(t) = R(t, D_t^{\alpha_1}w(t), D_t^{\alpha_2}w(t), \dots, D_t^{\alpha_{n-1}}w(t)) \tag{3.5}$$

with a continuously differentiable mapping R. Theorem 2 implies the existence of a unique solution to the Cauchy problem $w^{(k)}(t_0) = P_0x_k$, $k = 0, 1, \dots, r-1$, for equation (3.5) on some segment $[t_0, \tilde{t}_1]$. Moreover, under the conditions of this theorem $L_0 = 0$, therefore, $Lw \equiv 0$.

Consider the Cauchy problem for the second equation obtained after the action by the operator $L_1^{-1}Q$ on equation (3.3),

$$D_t^{\alpha}v(t) = S_1v(t) + L_1^{-1}QN(t, D_t^{\alpha_1}w(t), D_t^{\alpha_2}w(t), \dots, D_t^{\alpha_n}w(t)),$$

$$v^{(k)}(t_0) = Px_k, \ k = 0, 1, \dots, m-1,$$

where $S_1 = L_1^{-1}M_1 \in \mathcal{L}(\mathcal{X}^1)$. The unique solvability of this problem on $[t_0, t_1]$ at some $t_1 \in (t_0, \tilde{t}_1]$ follows from Theorem 2. It completes the proof of the theorem. $\square$

Remark 1. Note that at $r = m$ (3.2) is the Cauchy problem

4. Initial-boundary value problem for a nonlinear integro-differential equation

In the region $(0,1)\times[t_0,\infty)$, $t_0 \in \mathbb{R}$, consider the initial-boundary value problem

$$\frac{\partial^l w}{\partial t^l}(s, t_0) = v_l(s), \ l = 0, 1, \dots, r-1, \ s \in (0,1), \tag{4.1}$$

$$\frac{\partial^k \Delta w}{\partial t^k}(s, t_0) = \Delta v_k(s), \ k = r, r+1, \dots, m-1, \ s \in (0,1), \tag{4.2}$$

$$w(0,t) = w(1,t), \quad \frac{\partial w}{\partial s}(0,t) = \frac{\partial w}{\partial s}(1,t), \quad t \geq t_0, \tag{4.3}$$

for a semilinear fractional-order equation

$$D_t^\alpha \Delta w + \left| \int_0^1 D_t^{\alpha_1} w(s,t)ds \right|^\beta \int_0^1 D_t^{\alpha_2} w(s,t)ds = 0, \ s \in (0,1), \ t \geq t_0. \quad (4.4)$$

Here $m-1 < \alpha \leq m$, $0 \leq \alpha_1 < \alpha_2 < \alpha$, $r-1 < \alpha_2 \leq r$, $\beta > 0$. Let us denote by $\langle \cdot, \cdot \rangle$ the inner product in the space $L_2(0,1)$. Let

$$\mathcal{X} = \{v \in H^2(0,1) : v(0) = v(1), v'(0) = v'(1)\}, \ \mathcal{Y} = L_2(0,1),$$

$$\tilde{v} = v_0 + \frac{v_1}{1!}(t-t_0) + \frac{v_2}{2!}(t-t_0)^2 + \dots + \frac{v_{m-1}}{(m-1)!}(t-t_0)^{m-1},$$

for v_k, $k = 0, 1, \dots, m-1$, from conditions (4.1), (4.2).

Theorem 5. *Suppose* $m-1 < \alpha \leq m$, $0 \leq \alpha_1 < \alpha_2 < \alpha$, $r-1 < \alpha_2 \leq r$, $\beta > 0$, $v_l \in \mathcal{X}$, $l = 0, 1, \dots, m-1$, $\langle v_k, 1 \rangle = 0$, $k = r, r+1, \dots, m-1$, $D_t^{\alpha_1}|_{t=t_0} \langle \tilde{v}, 1 \rangle \neq 0$, $D_t^{\alpha_2}|_{t=t_0} \langle \tilde{v}, 1 \rangle = 0$. *Then for some* $t_1 > t_0$ *problem* (4.1)–(4.4) *has a unique solution on the set* $(0,1) \times [t_0, t_1]$.

Proof. Let's take $L = \Delta$, $Mx = \langle x, 1 \rangle$ at $x \in \mathcal{X}$, then for $\mu \neq 0$, $x \in \mathcal{X}$, $y \in \mathcal{Y}$ we have

$$(\mu L - M)x = \mu \triangle x - \langle x, 1 \rangle = -\langle x, 1 \rangle + \sum_{k \in \mathbb{Z} \setminus \{0\}} 2\pi k \mu \langle x(s), e^{2\pi kis} \rangle e^{2\pi kis},$$

$$(\mu L - M)^{-1} y = -\langle y, 1 \rangle + \sum_{k \in \mathbb{Z} \setminus \{0\}} \frac{1}{2\pi k \mu} \langle y(s), e^{2\pi kis} \rangle e^{2\pi kis},$$

$$\|(\mu L - M)^{-1} y\|^2_{H^2(0,1)} = |\langle y, 1 \rangle|^2 + \sum_{k \in \mathbb{Z} \setminus \{0\}} \frac{1 + 4\pi^2 k^2}{4\pi^2 k^2 \mu^2} |\langle y(s), e^{2\pi kis} \rangle|^2 \leq$$

$$\leq C^2 \|y\|^2_{L_2(0,1)}.$$

Therefore, the operator M is (L, σ)-bounded. Wherein

$$R^L_\mu(M) = L^L_\mu(M) = \sum_{k \in \mathbb{Z} \setminus \{0\}} \frac{1}{\mu} \langle \cdot, e^{2\pi kis} \rangle e^{2\pi kis},$$

consequently, $P = Q = \sum\limits_{k \in \mathbb{Z} \setminus \{0\}} \langle \cdot, e^{2\pi kis} \rangle e^{2\pi kis}$, $\mathcal{X}^1$ is the closure of the linear span of the set $\{e^{\pm 2\pi is}, e^{\pm 4\pi is}, \dots\}$ in the space $\mathcal{X}$, $\mathcal{Y}^1$ is the closure of the same set in $L_2(0,1)$, and the subspaces $\mathcal{X}^0 = \mathcal{Y}^0 = \operatorname{span}\{1\}$ coincide and are one-dimensional. Insofar as $\ker L = \ker P$, then operator M is $(L, 0)$-bounded.

The nonlinear operator in the considered equation will have the form $N(x,y,z) = -\langle x,1\rangle - |\langle y,1\rangle|^\beta \langle z,1\rangle$, it is defined on $X = \mathbb{R}\times\mathcal{X}^3$, so $W = \mathbb{R}\times(\mathcal{X}^0)^3$. Let's show the action $N : X \to \mathcal{Y}$, for $x,y,z\in\mathcal{X}$ we have

$$\|N(x,y,z)\|_{L_2(0,1)} \leq |\langle x,1\rangle| + |\langle y,1\rangle|^\beta |\langle z,1\rangle|.$$

It's obvious that $N \in C^1(X; L_2(\Omega))$, $N(x,y,z) = N(P_0x, P_0y, P_0z)$, since, say, $P_0x = \langle x,1\rangle$, $\langle P_0x,1\rangle = \langle\langle x,1\rangle,1\rangle = \langle x,1\rangle$.

Note that conditions (4.2) are of the form $(Lx)^{(k)}(0) = Lx_k$ and therefore, in the case of an (L,σ)-bounded operator, by virtue of Theorem 3, they are equivalent to the conditions $(Px)^{(k)}(0) = x_k$, $k = r, r+1, \dots, m-1$.

The equalities $\langle v_k,1\rangle = 0$, $k = r, r+1, \dots, m-1$, entail the conditions $v_k(\cdot) = x_k \in \mathcal{X}^1$, $k = r, r+1, \dots, m-1$, of Theorem 4.

The operator M_0^{-1} is identical, for all $y\in\mathcal{Y}$ $(I-Q)y = \langle y,1\rangle \in \mathcal{Y}^0$, $P_0\tilde v(t_0,\cdot) = P_0 v_0 = \langle v_0,1\rangle$,

$$M_0^{-1}(I-Q)N(t_0, P_0\tilde v(t_0,\cdot), D_t^{\alpha_1}|_{t=t_0}P_0\tilde v, D_t^{\alpha_2}|_{t=t_0}P_0\tilde v) =$$

$$= (I-Q)(-\langle v_0,1\rangle - |D_t^{\alpha_1}|_{t=t_0}\langle\tilde v,1\rangle|^\beta D_t^{\alpha_2}|_{t=t_0}\langle\tilde v,1\rangle) =$$
$$= -\langle v_0,1\rangle - |D_t^{\alpha_1}|_{t=t_0}\langle\tilde v,1\rangle|^\beta D_t^{\alpha_2}|_{t=t_0}\langle\tilde v,1\rangle,$$

hence condition (3.4) from Theorem 4 has a form

$$|D_t^{\alpha_1}|_{t=t_0}\langle\tilde v,1\rangle|^\beta D_t^{\alpha_2}|_{t=t_0}\langle\tilde v,1\rangle = 0.$$

It is true in this case, since $D_t^{\alpha_2}|_{t=t_0}\langle\tilde v,1\rangle = 0$.

The Frechet derivative has the form

$$[(I-Q)N]'_z(x,y,z)h = |\langle y,1\rangle|^\beta|\langle h,1\rangle| = |\langle y,1\rangle|^\beta h$$

at $h\in\mathcal{X}^0$. Since by the hypothesis of this theorem $D_t^{\alpha_1}|_{t=t_0}\langle\tilde v,1\rangle \neq 0$, then the operator $M_0^{-1}[(I-Q)N]'_z(x,y,z)$ is a multiplication by a nonzero number $|\langle y,1\rangle|^\beta$ for all (x,y,z) from the neighborhood of the point

$$(v_0, D_t^{\alpha_1}|_{t=t_0}\langle\tilde v,1\rangle, D_t^{\alpha_2}|_{t=t_0}\langle\tilde v,1\rangle)$$

in the spase $\mathcal{X}^0$, therefore all conditions of Theorem4 are satisfied. □

Remark 2. For example conditions $D_t^{\alpha_1}|_{t=t_0}\langle\tilde v,1\rangle \neq 0$, $D_t^{\alpha_2}|_{t=t_0}\langle\tilde v,1\rangle = 0$ are met, if $\alpha_1 = r < \alpha_2 \notin \mathbb{N}$, $\langle v_r,1\rangle \neq 0$.

5. Initial-boundary value problem for a nonlinear system

Let $\Omega \subset \mathbb{R}^d$ be a bounded domain with a smooth boundary $\partial\Omega$. Consider the initial boundary value problem

$$\frac{\partial^k x_i}{\partial t^k}(s,t_0) = x_{ik}(s),\ k = 0,1,\dots,r-1,\ s \in \Omega,\ i = 1,2,3, \tag{5.1}$$

$$\frac{\partial^l x_1}{\partial t^l}(s,t_0) = x_{1l}(s),\ l = r, r+1,\dots,m-1,\ s \in \Omega, \tag{5.2}$$

$$x_i(s,t) = 0,\ s \in \partial\Omega,\ t \geq t_0,\ i = 1,2,3, \tag{5.3}$$

$$\begin{aligned} &D_t^\alpha \triangle x_1 = \triangle x_1 + h_1\left(s,t,D_t^{\alpha_1}x_2, D_t^{\alpha_1}x_3,\dots,D_t^{\alpha_n}x_2, D_t^{\alpha_n}x_3\right),\\ &0 = \triangle x_2 + h_2\left(s,t,D_t^{\alpha_1}x_2, D_t^{\alpha_1}x_3,\dots,D_t^{\alpha_n}x_2, D_t^{\alpha_n}x_3\right),\\ &0 = \triangle x_3 + h_3\left(s,t,D_t^{\alpha_1}x_2, D_t^{\alpha_1}x_3,\dots,D_t^{\alpha_n}x_2, D_t^{\alpha_n}x_3\right),\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad s \in \Omega,\ t \geq t_0, \end{aligned} \tag{5.4}$$

where $m-1 < \alpha \leq m \in \mathbb{N}$, $0 \leq \alpha_1 < \alpha_2 < \dots < \alpha_n < \alpha$, $r-1 < \alpha_n \leq r \in \mathbb{N}$, functions h_i are defined on $\mathbb{R}^{2n+2}$, $i = 1,2,3$.

Let A be the Laplace operator with the domain $H_0^2(\Omega) = \{z \in H^2(\Omega) : z(s) = 0,\ s \in \partial\Omega\} \subset L_2(\Omega)$, $\{\varphi_k\}$ be an orthonormal in $L_2(\Omega)$ system of its eigenfunctions corresponding to the eigenvalues of the operator A, numbered in the non-increasing order, taking into account their multiplicity.

We reduce problem (5.1)–(5.4) to abstract problem (3.2), (3.3), by choosing the spaces

$$\mathcal{X} = (H_0^{2+2j}(\Omega))^3, \quad \mathcal{Y} = (H^{2j}(\Omega))^3, \tag{5.5}$$

where $j > \frac{d}{4} - 1$, $H_0^{2+2j}(\Omega) = \{z \in H^{2+2j}(\Omega) : z(s) = 0,\ s \in \partial\Omega\}$, and the operators

$$L = \begin{pmatrix} \triangle & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in \mathcal{L}(\mathcal{X};\mathcal{Y}), \quad M = \begin{pmatrix} \triangle & 0 & 0 \\ 0 & \triangle & 0 \\ 0 & 0 & \triangle \end{pmatrix} \in \mathcal{L}(\mathcal{X};\mathcal{Y}). \tag{5.6}$$

Lemma 1. *Let spaces* (5.5) *and operators* (5.6) *be given. Then the operator M is $(L,0)$-bounded and the projectors have the form*

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5.7}$$

Proof. For $\mu \neq 1$ we have the operator

$$(\mu L - M)^{-1} = \sum_{k=1}^{\infty} \langle \cdot, \varphi_k \rangle \varphi_k \begin{pmatrix} (\mu-1)^{-1}\lambda_k^{-1} & 0 & 0 \\ 0 & -\lambda_k^{-1} & 0 \\ 0 & 0 & -\lambda_k^{-1} \end{pmatrix} \in \mathcal{L}(\mathcal{Y};\mathcal{X}),$$

so M (L,σ)-bounded,

$$R^L_\mu(M)=L^L_\mu(M)=\sum_{k=1}^{\infty}\langle\cdot,\varphi_k\rangle\varphi_k\begin{pmatrix}(\mu-1)^{-1} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}.$$

From these equalities, using formulas (3.1) and the residue theorem, we obtain the form of projectors (5.7). Since $\ker P=\ker L$, the operator M is $(L,0)$-bounded. □

It follows from this lemma that

$$\mathcal{X}^1=H_0^{2+2j}(\Omega)\times\{0\}\times\{0\},\ \mathcal{X}^0=\{0\}\times H^{2+2j}(\Omega)\times H_0^{2+2j}(\Omega),$$

$$\mathcal{Y}^1=H^{2j}(\Omega)\times\{0\}\times\{0\},\ \mathcal{Y}^0=\{0\}\times H^{2j}(\Omega)\times H^{2j}(\Omega).$$

From the form of the projector P it follows that the initial conditions (5.1), (5.2) for system (5.3), (5.4) can be written as (3.2). Let's construct according to the initial data elements

$$\tilde{x}_1=x_{10}+\frac{x_{11}}{1!}(t-t_0)+\frac{x_{12}}{2!}(t-t_0)^2+\cdots+\frac{x_{1(m-1)}}{(m-1)!}(t-t_0)^{m-1}$$

$$\tilde{x}_i=x_{i0}+\frac{x_{i1}}{1!}(t-t_0)+\frac{x_{i2}}{2!}(t-t_0)^2+\cdots+\frac{x_{i(r-1)}}{(r-1)!}(t-t_0)^{r-1},\ i=2,3.$$

Note that the functions $h_i=h_i(s,t,z_1,z_2,z_3,\dots,z_{2n})$ depend on the $2n$ phase variables $z_1,z_2,\dots,z_{2n}$. Let us introduce the notation

$$J(s,t,z_1,\dots,z_{2n})=\begin{pmatrix}\frac{\partial h_2}{\partial z_{2n-1}}(s,t,z_1,\dots,z_{2n}) & \frac{\partial h_2}{\partial z_{2n}}(s,t,z_1,\dots,z_{2n})\\ \frac{\partial h_3}{\partial z_{2n-1}}(s,t,z_1,\dots,z_{2n}) & \frac{\partial h_3}{\partial z_{2n}}(s,t,z_1,\dots,z_{2n})\end{pmatrix}.$$

Theorem 6. *Let* $m-1<\alpha\le m\in\mathbb{N}$, $0\le\alpha_1<\alpha_2<\cdots<\alpha_n<\alpha$, $r-1<\alpha_n\le r\in\mathbb{N}$, $h_i\in C^\infty(\mathbb{R}^{2n+2};\mathbb{R})$, $j>\frac{d}{4}-1$, $x_{ik},x_{1l}\in H_0^{2+2j}(\Omega)$, $i=1,2,3$, $k=0,1,\dots,r-1$, $l=r,r+1,\dots,m-1$, *for some* $c>0$ *for all* $s\in\Omega$

$$|\det J(s,t_0,D_t^{\alpha_1}|_{t=t_0}\tilde{x}_2,D_t^{\alpha_1}|_{t=t_0}\tilde{x}_3,\dots,D_t^{\alpha_n}|_{t=t_0}\tilde{x}_3)|\ge c>0, \tag{5.8}$$

$$\Delta x_{i0}+h_i(\cdot,t_0,D_t^{\alpha_1}|_{t=t_0}\tilde{x}_2,D_t^{\alpha_1}|_{t=t_0}\tilde{x}_3,\dots,D_t^{\alpha_n}|_{t=t_0}\tilde{x}_3)\equiv 0,\ i=2,3. \tag{5.9}$$

Then there exists such $t_1>t_0$ *that problem* (5.1)–(5.4) *has a unique solution on the set* $\Omega\times[t_0,t_1]$.

Proof. For the proof, we check the conditions of Theorem 4. First of all, note that $H_i(t)\in C^\infty((H^{2+2j}(\Omega))^{2n};H^{2+2j}(\Omega))$, where

$$H_i(t)(z_1,z_2,\dots,z_{2n}):=h_i(\cdot,t,z_1(\cdot),z_2(\cdot),\dots,z_{2n}(\cdot)),\ i=1,2,3,$$

for a fixed t by virtue of Proposition 1 [11, p. 197], since $2+2j > d/2$.

From the form of the projectors obtained in the previous lemma it follows that the nonlinear part of the equation depends only on the elements of the subspace $\mathcal{X}^0$, and the mapping $(I-Q)N$ is defined by two functions h_2, h_3. The bijectivity condition for the operators of the Frechet derivative follows from condition (5.8) of this theorem. Condition (3.4) for this problem is (5.9). By Lemma 1 the operator M is $(L,0)$-bounded and by Theorem 4 we obtain the required. □

6. Conclusion

A new class of initial value problems for degenerate evolution equations that are nonlinear with respect to the lowest fractional derivatives is investigated. In what follows, we will study the solvability of optimal control problems for systems whose state is described by equations of this class.

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Начальная задача для одного класса слабо вырожденных полулинейных уравнений с младшими дробными производными

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Аннотация. Исследована разрешимость одной начальной задачи для класса эволюционных уравнений со слабым вырождением, нелинейных относительно младших дробных производных Герасимова – Капуто. Линейная часть уравнения содержит относительно ограниченную пару операторов. Доказана однозначная локальная разрешимость в случае нелинейного оператора, зависящего только от элементов подпространства вырождения. Приведены примеры уравнения и системы уравнений в частных производных, начально-краевые задачи для которых сведены к начальной задаче для уравнений в банаховом пространстве изученного класса.

Ключевые слова: дробная производная Герасимова – Капуто, дифференциальное уравнение дробного порядка, вырожденное эволюционное уравнение, полулинейное уравнение.

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