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# The Optimal Rehedging Interval for the Options Portfolio within the RAPM, Taking into Account Transaction Costs and Liquidity Costs *

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**Abstract.** Using the approach of L.C.G. Rogers and S. Singh, we added liquidity costs accounting to the model with risk adjusted pricing methodology (RAPM), generalized by M. Jandačka and D. Ševčovič. This model minimizes the risk of transaction costs growth from the frequent delta hedging, and reduces the risk of the portfolio value changes (hedging error) due to rare rebalances. Numerical solution for price of option combination "short strangle" is found. An optimal interval of time for delta hedging is considered. Corresponding results are presented in the form of graphs characterizing the dependence of the interval on the current price of the underlying asset and on the time remaining until the expiration of options.

**Keywords:** rehedging interval, non-linear option pricing model, RAPM, transaction costs, liquidity cost, delta hedging.

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## 1. Introduction

When developing the classical model of options pricing, F. Black and M. Scholes [8] introduced several restrictions, such as the absence of transaction costs, an infinitely small number of stocks and cash, the possibility of continuous trading, etc. The model required continuous hedging, that is, maintaining a certain amount of the underlying asset (shares) so that the change in the value of the option sold is offset by a change in the value of the acquired shares. F. Black and M. Scholes developed a model for evaluating options in the form

$$u_t + \frac{1}{2}\sigma^2 x^2 u_{xx} + rxu_x - ru = 0, \tag{1.1}$$

where the price of the underlying asset $x \geqslant 0$ varies in the time interval $t \in [0, T]$, $r \geqslant 0$ is the risk-free interest rate, and $\sigma$ is the historical volatility of the price of the underlying asset.

But the presence of transaction costs violates the relationship established in the work [8], since the continuous portfolio adjustment (or "rebalancing", "rehedging") implies the continuous trading and, therefore, endless transaction costs. Discrete portfolio rebalancing, if transaction costs are taken into account, generates errors in the value of the replicating portfolio, which correlate with the market, and do not tend to zero during a more frequent adjustment.

The first work aimed at accounting for transaction costs was article of H. Leland [22]. Leland's idea was to replace the option pricing through the value of the replicating portfolio with the option pricing from the approximate hedge value. The suggestion of Leland was to rehedge the portfolio periodically, as in the Black — Scholes model, but using a modified volatility reflecting the presence of transaction costs. In the Leland model, the modified volatility $\hat{\sigma}_L$ has the form

$$\hat{\sigma}_L^2 = \sigma^2 \left[1 + \sqrt{2/\pi}\, k/\sigma\sqrt{\Delta t}\right], \tag{1.2}$$

where $\sigma$ is the volatility from the Black — Scholes model, $\Delta t$ is a small but not infinitesimal time interval between portfolio revisions, and $k$ is the cost of the two-way transaction, measured as the fraction of the volume of the transactions in buying and selling (for example, first we bought the asset, then sold):

$$k = \frac{x_{\text{ask}} - x_{\text{bid}}}{x} = 2\frac{x_{\text{ask}} - x_{\text{bid}}}{x_{\text{ask}} + x_{\text{bid}}}, \tag{1.3}$$

where $x_{\text{ask}}$ and $x_{\text{bid}}$ are the supply and the demand prices, respectively.

Leland's method proved to be convenient for practitioners. However, as it is shown by Y. M. Kabanov and M. M. Safarian [21], there are certain mathematical points in this approach. Even for the European call option,

the final cost of the replicating portfolio does not converge to the final payment on the option contract, if the transaction cost does not depend on the number of portfolio rebalances [21, Theorem 2]. The authors suggested that $k = k_n = k_0 n^{-\alpha}$, $\alpha \in [0, 1/2]$. Then the modified volatility $\hat{\sigma}_{KS}$, depending on the number of rebalances, will take the form

$$\hat{\sigma}_{KS}^2 = \sigma^2 \left(1 + \frac{\gamma_n}{\sigma}\right), \quad \gamma_n = \sqrt{\frac{8}{\pi}} k_n n^{1/2} = \sqrt{\frac{8}{\pi}} k_0 n^{1/2-\alpha}. \tag{1.4}$$

Much attention is paid to studying the impact of transaction costs on option prices. Here are just a few works on this topic [6; 10; 11; 13; 14; 20; 25–27].

Often authors uses the approximate hedging approach proposed by Leland [22], when instead of historical volatility, modified volatility is used that takes into account certain effects in the model.

M. Jandačka and D. Ševčovič [19] summarized and analyzed the risk adjusted pricing methodology (RAPM) model. The model takes into account the risk of a change in the value of the portfolio arising from an insufficiently frequent adjustment of the portfolio and the risk of an increase in transaction costs with frequent rebalancing. The modified volatility has a form

$$\hat{\sigma}_{JS}^2 = \sigma^2 \left(1 - q \left(x u_{xx}\right)^{\frac{1}{3}}\right), \tag{1.5}$$

where $q = 3\left(k^2 R/2\pi\right)^{\frac{1}{3}}$, $R \geq 0$ is the risk premium coefficient. It represents the marginal cost of investor exposure to risk (in other words, the premium to the price that the investor is willing to lose, if rebalancing does not occur often). Note that this model was further thoroughly numerically investigated in [3], and in [9] a family of exact solutions was found by the group analysis methods.

Another factor affecting prices is the liquidity. The current spread between the "bid" and "ask" prices in the limit order book is considered as a measure of transaction costs in most models. If you need to buy or sell a minimum lot, usually 1 or 100 units of the underlying asset, such a spread may reflect the future price. However, if you need to buy or sell more units of the underlying asset, then the existence of orders in order book and their volumes are important.

Two effects related to the liquidity are usually studied in the works [1; 2; 5; 7; 17; 23]. These are so-called *temporary effect on the price* and *permanent effect on the price*. The first of them arises in a short time as a result of the trade, directly during operations. The second one is a certain lasting effect on the price after operations with a some amount of an asset.

The article of L.Rogers and S. Singh [24] presents the model, in which the permanent impact on prices is eliminated. The idea is that there is an influence of the illiquidity, but this illiquidity does not affect the average

price of the underlying asset. This affects the price at which the trader bought or sold the asset in the order book. For example, if a trader wants to buy an asset faster, he will have to pay more, because the price of subsequent units of the acquired asset will be higher. However, once the fast transaction is completed, the model assumes that the order book is quickly filled again and that the fast transaction does not have a lasting effect on the average price of the underlying asset.

The aim of this work is to supplement the RAPM method, which already takes into account the risk of increased transaction costs and hedging errors. The risk of increasing the cost of illiquidity, which depends on the state of the limit order book, will be added. On the one hand, this approach allows one to naturally supplement the pricing models of nonlinear options with various factors that influence price changes. On the other hand, this approach allows to quickly switch to the practical use of models, providing the ability to calculate values, for example, the optimal interval for hedging.

The paper is structured as follows. Section 2 describes briefly the model of RAPM and the model of L. Rogers and S. Singh and their generalization. Also it was obtained a non-linear Black — Scholes equation and the optimal time interval for the delta-hedging. Numerical solution of the general model is discussed in Section 3. Also Section 3 illustrates the application of this model to the option combination "short strangle". Section 4 concludes the work.

## 2. The model and main results

### 2.1. The risk adjusted pricing methodology (RAPM) model

Briefly recall some assumptions and results of RAPM [19]. First of all, the assumption was made that the risk premium is added to the change of portfolio $\Pi$ for the time interval $\Delta t$:

$$\Delta\Pi = r\Pi\Delta t + (r_{TC} + r_{VP})x\Delta t, \tag{2.1}$$

where $r$ is the risk free interest rate, $r_{TC}$ is the transaction cost risk premium per unit asset price and $r_{VP}$ is the portfolio volatility risk premium. The first of these corresponds to the risk of increased transaction costs with frequent hedging. The second takes into account the risk from unprotected portfolio (an increase in hedging error) with insufficiently frequent hedging.

For the increasing of the transaction cost risk premium $r_{TC}$ the next expression is obtained:

$$r_{TC} = \frac{kx\sigma|u_{xx}|}{\sqrt{2\pi}\sqrt{\Delta t}}, \tag{2.2}$$

and for the portfolio volatility risk premium $r_{VP}$ we have

$$r_{VP} = \frac{1}{2} R\sigma^4 x^2 u_{xx}^2 \Delta t. \tag{2.3}$$

By minimizing the total risk function $r_T = r_{TC} + r_{VP}$ with respect to $\Delta t$, an expression is derived for the optimal rehedging interval $\Delta t_{\text{opt}}$:

$$\Delta t_{\text{opt}} = \left( \frac{k}{R} \frac{1}{\sqrt{2\pi}} \right)^{2/3} \frac{1}{\sigma^2 \left| x u_{xx} \right|^{2/3}}. \tag{2.4}$$

Risk-adjusted Black — Scholes equation was obtained in the form

$$u_t + \frac{1}{2}\sigma^2 \left( 1 - q(x u_{xx})^{1/3} \right) x^2 u_{xx} - r\left( u - x u_x \right) = 0, \tag{2.5}$$

where $q = 3(k^2 R/2\pi)^{1/3}$.

## 2.2. The model of L. C. G. Rogers and S. Singh

We also briefly present here some results of [24], which are necessary for further discussion. The authors present the cost of illiquidity that traders pay when they quickly buy or sell $h$ units of the underlying asset. This cost is the difference between the book value of holding ($h$ units at mid price $\bar{x}$) and real value (from order book, $h = \int_1^s \rho(\gamma) d\gamma$, with the density of orders $\rho(\gamma)$ and relative price $\gamma = x/\bar{x}$):

$$\bar{x} \int_1^s \gamma \rho(\gamma) d\gamma - h\bar{x} = \bar{x} \int_1^s (\gamma - 1)\rho(\gamma) d\gamma \equiv \bar{x} l(h), \tag{2.6}$$

where $s$ is the maximum of the relative price $\gamma$ (or the minimum, if trader sell asset). If a trader needs to buy or sell a certain amount of the underlying asset $h$ within a some time interval $\Delta t$, then the cost of illiquidity is $\bar{x} l(h) \Delta t$.

Subsequently, the authors used for the function $l(h)$ in form

$$l(h) = \int_1^s (\gamma - 1)\rho(\gamma) d\gamma = \frac{1}{2}\varepsilon h^2, \tag{2.7}$$

where $\varepsilon$ is a small parameter. For numerical solutions, the authors take $\varepsilon = 0.006$, based on practical measurements. This form of $l(h)$ was chose for reasons of the tractability of the HJB equation (see [24, Remark and equation (3.12)]).

### 2.3. Generalization of the RAPM model

A natural way to complement the RAPM method is to add new risk factors to (2.5). It is possible to obtain new coefficients $r_i$ of the corresponding new risk per unit of the underlying asset. In general, formula (2.1) can be represented as

$$\Delta\Pi = r\Pi\Delta t + x\Delta t\sum_{i=0}^{N} r_i, \tag{2.8}$$

where $N$ is an unknown (at this moment) number of all risks. If $N = 0$, then we have Black — Scholes model.

Expressions (2.2) and (2.3) take into account the risk of increasing transaction costs ($r_{TC}$) and the risk of portfolio volatility ($r_{VP}$). Consider now the risk of illiquidity ($r_{IL}$) from the model of Rogers and Singh.

Please note that $h$ corresponds to the amount of the underlying asset needed to rebalance the portfolio in the RAPM model. Since the delta hedging strategy is used, we have $\delta = -u_x$. Therefore, $h = \Delta\delta = \Delta(-u_x)$ at the adjustment of portfolio. And, as it can be seen in [19], $\Delta(-u_x) = -u_{xx}\sigma x\Delta W$.

Taking into account the Leland approximation,

$$|\Delta W| \approx \mathbb{E}\left[|\Delta W|\right] = \sqrt{2/\pi}\sqrt{\Delta t},$$

obtain the following formula of the required risk coefficient $r_{IL}$ per unit of the underlying asset:

$$r_{IL} = \frac{1}{h}l(h) = \frac{1}{2}\varepsilon h = \frac{\varepsilon x\sigma|u_{xx}|}{\sqrt{2\pi}}\frac{1}{\sqrt{\Delta t}}. \tag{2.9}$$

The resulting formula is similar to the formula for the risk premium for transaction costs, as expected. The absolute value of $u_{xx}$ is used, since, as in the case of transaction costs, there is always a risk of illiquidity, and it must be taken into account.

The total risk premium has the form

$$r_R = r_{TC} + r_{VP} + r_{IL} = \frac{(k+\varepsilon)x\sigma|u_{xx}|}{\sqrt{2\pi}\sqrt{\Delta t}} + \frac{1}{2}R\sigma^4x^2u_{xx}^2\Delta t. \tag{2.10}$$

Please note that $k$ characterizes the transaction cost with sufficient liquidity to buy or to sell the underlying asset at the best price ($x_{\text{ask}}$ or $x_{\text{bid}}$) in the market. The $\varepsilon$ characterizes the costs of acquiring or selling an underlying asset when a trader makes an operation for an amount exceeding the amount offered at the best prices. These latter costs are similar to transaction costs, but they are determined by the completeness of the order book, or, in other words, by the liquidity.

### 2.4. The optimal time interval and the non-linear equation of Black — Scholes type

Following [19], we find the minimum of the total risk premium by the differentiating with respect to $\Delta t$:

$$r'_R = \left( \frac{(k+\varepsilon)\sigma x}{\sqrt{2\pi}} |u_{xx}| \frac{1}{\sqrt{\Delta t}} + \frac{1}{2} R\sigma^4 x^2 u_{xx}^2 \Delta t \right)' = -\frac{1}{2} \frac{(k+\varepsilon)\sigma x}{\sqrt{2\pi}} |u_{xx}| \frac{1}{(\sqrt{\Delta t})^3} + \frac{1}{2} R\sigma^4 x^2 u_{xx}^2 = 0,$$

whence we get the optimal time interval for adjustment of portfolio

$$\Delta t_{\text{opt}} = \left( \frac{k+\varepsilon}{\sqrt{2\pi} R\sigma^3} \right)^{2/3} \left( \frac{1}{x|u_{xx}|} \right)^{2/3}. \tag{2.11}$$

The obtained interval, on the one hand, minimizes the risk of an increase in hedging errors due to rare rebalancing, and on the other hand, minimizes the risk of increased transaction costs and the cost of illiquidity due to too frequent re-hedging.

Similarly, using $k+\varepsilon$ instead of $k$ we get a non-linear Black — Scholes type equation (cf. (2.5) and [19, (2.19)–(2.20)])

$$u_t + \frac{1}{2}\sigma^2 \left(1 - q(xu_{xx})^{1/3}\right) x^2 u_{xx} - r\,(u - xu_x) = 0, \tag{2.12}$$

where $q = 3((k+\varepsilon)^2 R/2\pi)^{1/3}$.

## 3. Numeric example

### 3.1. Calibration of model

We will consider the futures-style options for the Brent crude from the Moscow Exchange. The underlying asset is the futures contract BR-11.19.

First of all, we need to calibrate the model. There are 3 parameters $k$, $R$ and $\varepsilon$, the values of which are necessary for the numerical solution of the equation (2.12). Based on the current spreads $x_{\text{ask}} - x_{\text{bid}}$, we get that the relative cost of a two-way transaction is $k = 0.0004$.

To find $\varepsilon$ the instant status of the order book was collected. The data allowed to build the distribution of the quantity of the underlying asset at prices in the order book. This allowed us to calculate the function $l(h)$ for each volume of the underlying asset using the data obtained.

Recall that we did not have a goal to investigate the function $l(h)$. Therefore, for its approximation, the most accessible tool was used. Using Microsoft Excel, two approximations were performed, the power-law and

the polynomial. The results presented on the Fig. 1 on p. 10. It show that the value $\varepsilon$ is non consistent with the value from the L. Rogers and S. Singh model. However, since the underlying asset under consideration is a futures contract, not a stock, for our own calculations we will use the approximate value $\varepsilon = 0.00006$.

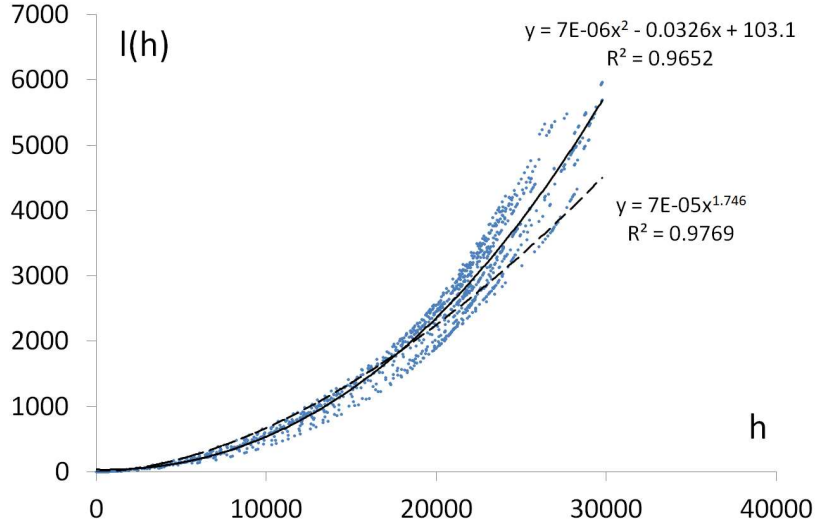


*Figure 1.* The function $l(h)$ for futures contracts BR-11.19

M. Jandačka and D. Ševčovič note the difficulty in calculating the $R$ metric. Therefore, for simplicity and possibility of comparison, we find it from the formula for the coefficient $q = 0.2$ without accounting of the cost of illiquidity, as in [19]. It should be noted that $R$ and $k$ are not directly used in numerical calculations, unlike $q$. With our numbers $R = 2\pi q^3/27k^2 = 11635$. Adding the value of $\varepsilon$ to $k$ in the formula for $q$, we obtain a new value $q$ that takes into account the cost of illiquidity $q = 3((k+\varepsilon)^2 R/2\pi)^{1/3} = 0.2195$.

### 3.2. Discretisation of non-linear Black — Scholes equation

Non-linear equations of the Black — Sholes type with a modified volatility are actively studied by numerical methods [3; 4; 12; 15; 18].

In this paper, the approach described in [16] was used to construct a numerical scheme. We briefly note that we used a two-layer implicitly explicit six-point stencil with weights, when the values of the desired function $u_n^{m+1}$ on the $(m+1)$-th layer have weight $\omega$, and the values from the previous $m$-th layer is counted with a weight of $1-\omega$.

The dependence of $\Delta t_{\text{opt}}$ on $x$ and $t$ is found numerically for the often used option combination "short strangle". This combination consists in

selling a put option with a lower strike price $K_1$ and selling a call option with a higher strike $K_2$ (i.e. $K_1 < K_2$). A trader who forms such a combination makes money, if the price of the underlying asset and the volatility do not change significantly before the options expire. otherwise, regardless of the direction of movement, the trader will take a loss.

The boundary conditions for the short strangle are determined by the sum of the boundary conditions for the options included therein:

$$\text{short put:} \quad u(0,t) = (-K_1 + p_1)e^{-r(T-t)}, \lim_{x\to\infty} u(x,t) = +p_1 e^{-r(T-t)},$$
$$u(x,T) = -\max\{K_1 - x, 0\} + p_1,$$
$$\text{short call:} \quad u(0,t) = +p_2 e^{-r(T-t)}, \quad \lim_{x\to\infty} \frac{u(x,t)}{x - (K_2 + p_2)e^{-r(T-t)}} = -1,$$
$$u(x,T) = -\max\{x - K_2, 0\} + p_2,$$
$$\text{short strangle:} \quad u(0,t) = (+p_1 + p_2 - K_1)e^{-r(T-t)},$$
$$\lim_{x\to\infty} \frac{u(x,t)}{x - (K_2 + p_1 + p_2)e^{-r(T-t)}} = -1,$$
$$u(x,T) = -(\max\{K_1 - x, 0\} + \max\{x - K_2, 0\}) + p_1 + p_2,$$

where $p_1$ and $p_2$ are the prices of sell of the options when forming the combination. These values are obtained for simplicity from the Black — Scholes model. We also assume that the risk-free interest rate is $r = 0$, that corresponds to the situation with the futures-style options.

### 3.3. The results of calculations

Fig. 2 on p. 12 presents difference of the combination price between the RAPM model and the RAPM model with the illiquidity cost, hereinafter: the model JSRS is the model of Jandačka — Ševčovič — Rogers — Singh. It can be seen that the price of the combination according to the RAPM model is slightly higher than the price according to the JSRS model.

Fig. 3 on p. 12 presents how the optimal time interval changes with reduction of time to expiration of options in the JSRS model. Please note that as the options expire, the price range of the underlying asset, where rebalancing is required, becomes narrower.

The calculations show that the interval value for the RAPM model is slightly less than for the JSRS model. In other words, a trader can rebalance less often if he uses the JSRS model.

## 4. Conclusion

We made an attempt to generalize the model RAPM and complement it with the cost of illiquidity. The calculations showed that, as expected,

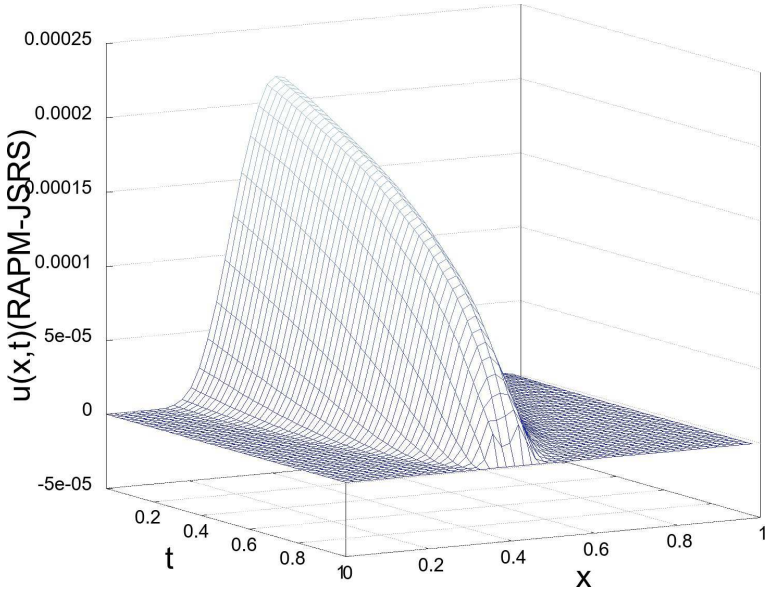


*Figure 2.* Difference of the combination price between the RAPM model (RAPM) and the model JSRS

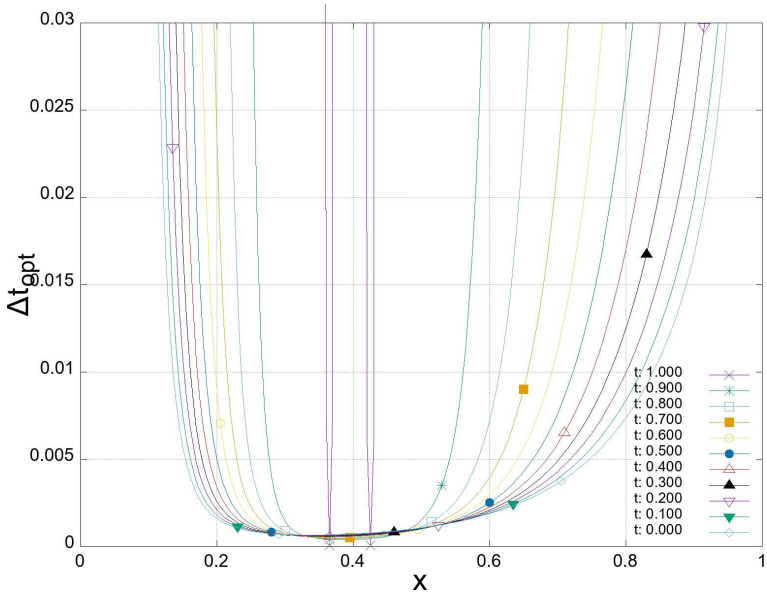


*Figure 3.* The optimal time interval for different time till expiration of the "short strangle"

taking into account the cost of illiquidity, the portfolio rebalancing in accordance with the delta hedging strategy should be somewhat less frequent. To determine the frequency of rebalancing, a method has been demonstrated

for obtaining the optimal time interval at which the risk of an increase in transaction costs, the risk of an increase in the cost of illiquidity and the risk of an increase in the hedging error are minimized.

The procedure for applying the obtained optimal rebalancing interval can be as follows. A trader who wants to hedge his portfolio of options can count the time from the last portfolio review and compare it with the interval defined by the formula (2.11). And if the calculated interval turned out to be less than the time elapsed after the previous adjustment, then the trader, buying or selling the underlying asset in accordance with the delta hedging strategy, can rebalance the portfolio. This method can be used in automatic systems for placing orders in the trading system.

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## Оптимальный интервал хеджирования для портфеля опционов в рамках RAPM с учетом операционных издержек и затрат на ликвидность

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**Аннотация**. Используя подход L. C. G. Rogers и S. Singh, мы добавили учет затрат на ликвидность в методологию ценообразования с поправкой на риск (RAPM), обобщенную M. Jandačka и D. Ševčovič (2005). Эта модель сводит к минимуму риск роста транзакционных издержек из-за частого дельта-хеджирования и снижает риск изменения стоимости портфеля (ошибка хеджирования) из-за редких перебалансировок. Найдено численное решение для цены комбинации опционов short strangle. Получен оптимальный интервал времени для дельта-хеджирования. Результаты исследования представлены в виде графиков, характеризующих зависимость интервала от текущей цены базового актива и от времени, оставшегося до истечения срока действия опционов.

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