



Серия «Математика»

2019. Т. 30. С. 73–82

Онлайн-доступ к журналу:

<http://mathizv.isu.ru>

ИЗВЕСТИЯ

Иркутского
государственного
университета

УДК 517.968; 519.642

MSC 45D05

DOI <https://doi.org/10.26516/1997-7670.2019.30.73>

Identification of Input Signals in Integral Models of One Class of Nonlinear Dynamic Systems *

S. V. Solodusha

Melentiev Energy Systems Institute of SB RAS, Irkutsk, Russian Federation

Abstract. The problem of restoring input signals is one of the intense developing research areas and is the intersection of the mathematical modeling theory, the automatic control theory and the inverse problems theory. This paper focuses on solving the identification problem of the input signal that corresponds to a given (desired) output in the case of no feedback. An approach to the approximate solution of polynomial Volterra equations of the first kind of the N th degree that arise when modeling nonlinear dynamics by the apparatus of Volterra integro-power series is described. These equations appear when a nonlinear dynamic process is modeled using the integro-power Volterra series. One class of nonlinear dynamical black box type systems is considered. Unlike a scalar input, the form of the integral model is complicated by the inclusion of terms that take into account the simultaneous change of individual components of the input signal vector. Integral models with constant Volterra kernels were considered earlier. This paper assumes the symmetric Volterra kernels are representable as the product of a finite number of continuous functions. The identification problem is solved using the Newton-Kantorovich method. A numerical solution of the corresponding linear integral Volterra equation of the first kind is proposed as an initial approximation. The obtained formulas for calculations are based on quadrature methods (right rectangles). The effectiveness of the proposed algorithms is illustrated for the reference dynamic system and confirmed by numerical results.

Keywords: Volterra polynomial equations of the first kind, the problem of restoring input signals, the Newton-Kantorovich method.

* The study was carried out in the framework of scientific project III.17.3.1 of the Basic Research Program of the SB RAS, reg. No. AAAA-A17-117030310442-8.

Introduction

The relevance of solving the problem of identifying input signals is due to the wide range of its practical applications [5]. The article considers one of the approaches to the numerical solution of this problem that arises when modeling the response of a nonlinear dynamical system $y(t)$ to an input signal $x(t)$ in the form of a Volterra polynomial (a segment of an integro-power series) [10]. So, if $x(t) = (x_1(t), \dots, x_p(t))^T$ – is a vector function of time, then the Volterra polynomial of the N -th degree has the form

$$\sum_{m=1}^N \sum_{1 \leq i_1 \leq \dots \leq i_m \leq p} \int_0^t \dots \int_0^t K_{i_1, \dots, i_m}(t, s_1, \dots, s_m) \prod_{j=1}^m x_{i_j}(s_j) ds_j = y(t), \quad (0.1)$$

$t \in [0, T]$, $y(t)$ – is a scalar function of time, $y'(t) \in C_{[0, T]}$, $y(0) = 0$, and Volterra kernels K_{i_1, \dots, i_m} are symmetric in variables s_{i_1}, \dots, s_{i_m} whose indices coincide. This apparatus is well known in the theory of mathematical modeling [3]. In what follows, for simplicity, we choose $p = 2$ in (0.1). An analysis of existing ways of applying Volterra polynomials to an automatic control problem (see, for example, [8]) allows us to consider the identification problem associated with the restoration of the control input signal $u(t) \equiv x_1(t)$, corresponding to known kernels K , a disturbance input signal $\zeta(t) \equiv x_2(t)$, and a given output $y(t)$. In mathematical terms, the problem is reduced to solving the polynomial equation (0.1), in which the Volterra kernels satisfy the following conditions

$$K_1'(t, s) \in C_\Delta, \quad \Delta = \{t, s : 0 \leq s \leq t \leq T\},$$

$$K_1(t, t) \neq 0 \quad \forall t \in [0, T].$$

The specificity of (0.1) for $N > 1$, in contrast to the linear case (given $N = 1$), consists in locality of T^* , i.e. the domain of existence of a (unique) continuous solution [1]. Equation (0.1) can be interpreted as a linear equation

$$\int_0^t K_1(t, s) u(s) ds = \tilde{y}(t)$$

with the disturbed right side:

$$\begin{aligned} \tilde{y}(t) = y(t) - \sum_{m=2}^N \sum_{i_1, \dots, i_m=1} \int_0^t K_{i_1, \dots, i_m}(t, s_1, \dots, s_m) \prod_{j=1}^m u(s_j) ds_j - \\ - \sum_{m=1}^N \sum_{i_1, \dots, i_m=2} \int_0^t K_{i_1, \dots, i_m}(t, s_1, \dots, s_m) \prod_{j=1}^m \zeta(s_j) ds_j - \end{aligned}$$

$$- \sum_{m=2}^N \sum_{\substack{\mu+\nu=m \\ i_1, \dots, i_\mu=1 \\ i_{\mu+1}, \dots, i_{\mu+\nu}=2}} \int_0^t K_{i_1, \dots, i_m}(t, s_1, \dots, s_m) \prod_{j=1}^{\mu} u(s_j) ds_j \prod_{j=1}^{\nu} \zeta(s_j) ds_j.$$

In [2], the correctness of (0.1) for $p = 1$ on a pair of spaces $(C_{[0,T]}, \overset{\circ}{C}_{[0,T]})$ given a sufficiently small $T < T^*$ is shown, which guarantees the existence, uniqueness, and stability of the solution in the space of continuous functions $C_{[0,T]}$. Thus, for given Volterra kernels, equation (0.1) is uniquely solvable in $C_{[0,T]}$, and the solution of $u^*(0)$ is determined by the following formula:

$$u^*(0) = \frac{y'(0) - K_2(0, 0)\zeta(0)}{K_1(0, 0)}. \tag{0.2}$$

The equality (0.2) determines the value of the solution of the equivalent Volterra equation of the second kind given $t = 0$ obtained by differentiating (0.1) with respect to t . In what follows, we use (0.2) in solving (0.1) by the Newton-Kantorovich iterative method: as an initial approximation, it is natural to choose the solution of the corresponding equation that is linear with respect to $u(t)$. Let us dwell on cases $N = 2, 3$ in (0.1) that are most common in practice. We study the specifics of the numerical solution (0.1) using the Newton-Kantorovich method [4]. The case of constant Volterra kernels was studied in [6]. In this study we consider the situation when

$$K_{11}(t, s_1, s_2) = \prod_{m=1}^2 \varphi(t, s_m), \quad K_{111}(t, s_1, s_2, s_3) = \prod_{m=1}^3 \varphi(t, s_m), \tag{0.3}$$

$$K_{112}(t, s_1, s_2, s_3) = \prod_{m=1}^2 \varphi(t, s_m)\psi(t, s_3),$$

where $\varphi(t, s) \in C_\Delta$, $\Delta = \{t, s : 0 \leq s \leq t \leq T\}$, $T < T^*$.

1. The numerical solution of the equation for $N = 2$

Let $N = 2$. Then instead of (0.1) we have

$$P(u) \equiv f(u(t)) - y(t) = 0, \tag{1.1}$$

where, taking into account (0.3),

$$f(u(t)) = I_1(u(t)) + I_2^2(u(t)) + p(t), \tag{1.2}$$

$$I_1(u(t)) = \int_0^t \left(K_1(t, s_1) + \int_0^t K_{12}(t, s_1, s_2)\zeta(s_2) ds_2 \right) u(s_1) ds_1,$$

$$I_2^2(u(t)) = \int_0^t \int_0^t K_{11}(t, s_1, s_2) u(s_1) u(s_2) ds_1 ds_2 = \left(\int_0^t \varphi(t, s) u(s) ds \right)^2, \quad (1.3)$$

$$p(t) = \int_0^t K_2(t, s_1) \zeta(s_1) ds_1 + \int_0^t \int_0^t K_{22}(t, s_1, s_2) \prod_{i=1}^2 \zeta(s_i) ds_i.$$

The iterative process of solving (1.1) by the Newton-Kantorovich method has the form

$$u_m = u_{m-1} - [P'(u_{m-1})]^{-1} (P(u_{m-1})), \quad m = 1, 2, \dots, \quad (1.4)$$

$$[P'(u_{m-1})] (u) = I_1(u) + 2I_2(u_{m-1})I_2(u).$$

Given (1.4) the sequence of approximate solutions $u_m(t)$ is found from the solution of the linear equation

$$I_1(u_m(t)) + 2I_2(u_{m-1}(t))I_2(u_m(t)) = I_2^2(u_{m-1}(t)) + y(t) - p(t). \quad (1.5)$$

As an initial approximation of $u_0(t)$ in (1.5), we choose a numerical solution of the equation

$$I_1(u_0(t)) = y(t) - p(t). \quad (1.6)$$

To solve (1.5), (1.6) numerically, we apply the quadrature formulas of the right (middle) rectangles that have the property of self-regularization [9].

In particular, approximation of $u^*(t_i)$ in the i -th node of mesh $t_i = ih$, $t_j = jh$, $i = \overline{1, n}$, $j = \overline{1, i}$, $nh = T$, $T < T^*$, obtained using the method of right rectangles has the following form:

$$u_m^h(t_i) = \frac{Z_{m-1}(t_i) - \sum_{j=1}^{i-1} u_m^h(t_j) \Psi_{m-1}(t_i, t_j)}{\Psi_{m-1}(t_i, t_i)}, \quad (1.7)$$

$$Z_{m-1}(t_i) = \left(h \sum_{j=1}^i \varphi^h(t_i, t_j) u_{m-1}^h(t_j) \right)^2 - p(t_i) + y(t_i),$$

$$\Psi_{m-1}(t_i, t_j) = hK_1^h(t_i, t_j) + h^2 \sum_{k=1}^i K_{12}^h(t_i, t_j, t_k) \zeta^h(t_k) + \quad (1.8)$$

$$+ 2h^2 \varphi^h(t_i, t_j) \sum_{k=1}^i \varphi^h(t_i, t_k) u_{m-1}^h(t_k)$$

with the initial approximation

$$u_0^h(t_i) = \frac{W(t_i)}{R(t_i)},$$

where

$$R(t_i) = hK_1^h(t_i, t_i) + h^2 \sum_{k=1}^i K_{12}^h(t_i, t_i, t_k) \zeta^h(t_k), \quad (1.9)$$

$$\begin{aligned} W(t_i) = & y(t_i) - h \sum_{j=1}^{i-1} K_1^h(t_i, t_j) u_0^h(t_j) - h^2 \sum_{j=1}^{i-1} \sum_{k=1}^i K_{12}^h(t_i, t_j, t_k) u_0^h(t_j) \zeta^h(t_k) - \\ & - h \sum_{j=1}^i K_2^h(t_i, t_j) \zeta^h(t_j) - h^2 \sum_{j=1}^i \sum_{k=1}^i K_{22}^h(t_i, t_j, t_k) \zeta^h(t_j) \zeta^h(t_k), \end{aligned} \quad (1.10)$$

where $p(t_i)$, $y(t_i)$ — are the values of the functions at the i -th node of the mesh, and at each iteration the corresponding conditions $\Psi_{m-1}(t_i, t_i) \neq 0$, $R(t_i) \neq 0$ must be satisfied in (1.8) and (1.9).

2. The numerical solution of the equation for $N = 3$

In what follows, let $N = 3$. Then, given (0.3), instead of (1.2) in (1.1) we have

$$f(u(t)) = I_1(u(t)) + q(t)I_2^2(u(t)) + I_2^3(u(t)) + p(t),$$

where

$$\begin{aligned} p(t) = & \int_0^t K_2(t, s_1) \zeta(s_1) ds_1 + \int_0^t \int_0^t K_{22}(t, s_1, s_2) \prod_{i=1}^2 \zeta(s_i) ds_i + \\ & + \int_0^t \int_0^t \int_0^t K_{222}(t, s_1, s_2, s_3) \prod_{i=1}^3 \zeta(s_i) ds_i, \\ I_1(u(t)) = & \int_0^t \left(K_1(t, s_1) + \int_0^t K_{12}(t, s_1, s_2) \zeta(s_2) ds_2 + \right. \\ & \left. + \int_0^t \int_0^t K_{122}(t, s_1, s_2, s_3) \prod_{i=2}^3 \zeta(s_i) ds_i \right) u(s_1) ds_1, \\ q(t) = & 1 + \int_0^t \psi(t, s_1) \zeta(s_1) ds_1. \end{aligned} \quad (2.1)$$

Taking into account the introduced notation (1.3), (2.1), the iterative process will take the form of (1.4), where

$$[P'(u_{m-1})](u) = I_1(u) + 2q(t)I_2(u_{m-1})I_2(u) + 3I_2^2(u_{m-1})I_2(u).$$

Hence,

$$I_1(u_m(t)) + I_2(u_m(t)) \left(2q(t)I_2(u_{m-1}(t)) + 3I_2^2(u_{m-1}(t)) \right) = \quad (2.2)$$

$$= I_2^2(u_{m-1}(t)) \left(q(t) + 2I_2(u_{m-1}(t)) \right) + y(t) - p(t),$$

where initial approximation $u_0(t)$ is a numerical solution of (1.6), (2.1). Using the method of right rectangles and taking into account (1.9), (1.10), we have:

$$u_0^h(t_i) = \frac{\widetilde{W}(t_i)}{R(t_i) + h^3 \sum_{k=1}^i \sum_{l=1}^i K_{122}^h(t_i, t_i, t_k, t_l) \zeta^h(t_k) \zeta^h(t_l)},$$

$$\widetilde{W}(t_i) = W(t_i) - h^3 \sum_{j=1}^{i-1} \sum_{k=1}^i \sum_{l=1}^i K_{122}^h(t_i, t_j, t_k, t_l) \zeta^h(t_k) \zeta^h(t_l) u_0^h(t_j) -$$

$$- h^3 \sum_{j=1}^i \sum_{k=1}^i \sum_{l=1}^i K_{222}^h(t_i, t_j, t_k, t_l) \zeta^h(t_j) \zeta^h(t_k) \zeta^h(t_l).$$

Approximating definite integrals in (2.2) by quadrature formulas, we obtain a calculation formula for $u_m^h(t_i)$ of the form (1.7) in which

$$Z_{m-1}(t_i) = y(t_i) - p(t_i) +$$

$$+ \left(h \sum_{j=1}^i \varphi^h(t_i, t_j) u_{m-1}^h(t_j) \right)^2 \left[q(t_i) + 2h \sum_{j=1}^i \varphi^h(t_i, t_j) u_{m-1}^h(t_j) \right],$$

$$\Psi_{m-1}(t_i, t_j) = hK_1^h(t_i, t_j) + h^2 \sum_{k=1}^i K_{12}^h(t_i, t_j, t_k) \zeta^h(t_k) +$$

$$+ h^3 \sum_{k=1}^i \sum_{l=1}^i K_{122}^h(t_i, t_j, t_k, t_l) \zeta^h(t_k) \zeta^h(t_l) +$$

$$+ h^2 \varphi^h(t_i, t_j) \left[2q(t_i) \sum_{k=1}^i \varphi^h(t_i, t_k) u_{m-1}^h(t_k) + 3h \left(\sum_{k=1}^i \varphi^h(t_i, t_k) u_{m-1}^h(t_k) \right)^2 \right].$$

Remark. Approximating definite integrals in (1.5), (2.2) using the quadrature formula of middle rectangles, it is easy to obtain a similar algorithm for calculating u_m^h , $m = 0, 1, 2, \dots$.

3. Results of the numerical experiment

Let us consider (0.1) given $N = 2$. We will illustrate the use of the obtained formulas on a test example. Let the kernels be $K_1 = -\frac{1}{4}$, $K_{22} = \frac{1}{16}$, $K_2 = -1$, $\varphi(t) = t$, $K_{12} = \frac{t}{2}$ and $\zeta(t) = t$. We choose

$$y(t) = \frac{t^{10}}{16} + \frac{t^7}{16} - \frac{3t^4}{64} - \frac{t^2}{2}, p(t) = -\frac{t^2}{2} + \frac{t^4}{64}.$$

Using formulas from [7], we obtain: $u^*(t) = t^3$, $T^* = 2^{-\frac{1}{3}} \approx 0,7937$. Taking into account the replacement $\mathcal{U}(t) = \int_0^t u(s)ds$, as per (1.6) we choose the initial approximation

$$\mathcal{U}_0(t) = \frac{t^4(t^6 + t^3 - 1)}{4(t^3 - 1)}.$$

According to (1.5),

$$\mathcal{U}_m(t) = \frac{t^2(16\mathcal{U}_{m-1}^2(t) - t^2 + t^5 + t^8)}{4(t^3 - 1 + 8\mathcal{U}_{m-1}(t))}.$$

The approximate solution of equation (1.1) obtained with double precision of calculations is presented in Table. 1, where $\tau_1 = 0,55$, $\tau_2 = 0,6$, $\tau_3 = 0,65$, $\tau_4 = 0,7$, $\|\varepsilon_m\|_{C_h} = \max_{0 \leq t_i \leq \tau_k} |u_m^h(t_i) - u^*(t_i)|$, m — is the iteration number, $m = \overline{1, 3}$.

Table 1

Errors of mesh solutions.

m	$\ \varepsilon_{\tau_1}\ _{C_h}$	$\ \varepsilon_{\tau_2}\ _{C_h}$	$\ \varepsilon_{\tau_3}\ _{C_h}$	$\ \varepsilon_{\tau_4}\ _{C_h}$
1	$0,895 \cdot 10^{-8}$	$0,769 \cdot 10^{-5}$	$0,616 \cdot 10^{-4}$	$0,488 \cdot 10^{-3}$
2	$0,125 \cdot 10^{-13}$	$0,123 \cdot 10^{-11}$	$0,112 \cdot 10^{-9}$	$0,110 \cdot 10^{-5}$
3	0	0	0	$0,232 \cdot 10^{-8}$

Table 2 shows the results of calculating $u_{m_i}^h = u_m^h(t_i)$ given that $m = 1, 3$ using (1.7) for the indicated data. Under $t_i \rightarrow 0,79$, due to violation of inequality $T < T^*$, a boundary layer appears.

If, for example, we limit ourselves to value $\tau_k = \tau_1$, then

$$\|\varepsilon_1^{h=0,01}\|_{C_h} = 0,004, \quad \|\varepsilon_1^{h=0,001}\|_{C_h} = 0,0004,$$

$$\|\varepsilon_3^{h=0,01}\|_{C_h} = 0,004, \quad \|\varepsilon_3^{h=0,001}\|_{C_h} = 0,0004.$$

Table 2

Values of functions $u_{m_i}^{h=0,01}$, $u_{m_i}^{h=0,001}$.

m	t_i	$u_{m_i}^{h=0,01}$	$u_{m_i}^{h=0,001}$	m	t_i	$u_{m_i}^{h=0,01}$	$u_{m_i}^{h=0,001}$
1	0,1	0,00086	0,00098	3	0,1	0,00086	0,00098
	0,2	0,00742	0,00794		0,2	0,00742	0,00794
	0,3	0,02568	0,02686		0,3	0,02568	0,02686
	0,4	0,06164	0,06376		0,4	0,06164	0,06376
	0,5	0,12129	0,12462		0,5	0,12129	0,12462
	0,6	0,21039	0,21514		0,6	0,21066	0,21546
	0,7	0,31911	0,32235		0,7	0,33572	0,34227

Here, $\|\varepsilon_m\|_{C_h} = \max_{0 \leq t_i \leq \tau_1} |u_m(t_i) - u^*(t_i)|$ and linear convergence takes place.

Conclusion

This study continues the line of research initiated in [6]. A numerical solution to the problem of identifying an input signal of one class of nonlinear dynamical systems, formulated in the form of a polynomial Volterra equation of the first kind, was considered. It is assumed that the symmetric Volterra kernels corresponding to the change of the desired input signal are represented as a product of the finite number of continuous functions. Numerical algorithms based on the application of the Newton-Kantorovich methods and the method of right rectangles were developed. As an initial approximation, taking into account the specifics of polynomial integral equations, a numerical solution of the corresponding linear Volterra equations of the first kind is used. The specifics of the algorithms are illustrated by the model example.

References

1. Apartsyn A.S. Polilineynnye uravneniya Vol'terra I roda: elementy teorii i chislennyye metody [Polilinear integral Volterra equations of the first kind: the elements of the theory and numeric methods]. *The Bulletin of Irkutsk State University. Series Mathematics*, 2007, vol. 1, no. 1, pp. 13-41. (in Russian)
2. Apartsyn A.S. Studying the polynomial volterra equation of the first kind for solution stability. *Automation and Remote Control*, 2011, vol. 72, no. 6, pp. 1229-1236. <https://doi.org/10.1134/S0005117911060099>
3. Cheng C.M., Peng Z.K., Zhang W.M., Meng G. Volterra-series-based nonlinear system modeling and its engineering applications: A state-of-the-art review. *Mechanical Systems and Signal Processing*, 2017, vol. 87, pp. 430-364. <https://doi.org/10.1016/j.ymssp.2016.10.029>

4. Kantorovich L.V., Akilov G.P. *Funktsional'nyy analiz v normirovannykh prostranstvakh* [Functional Analysis in Normed Spaces]. Moscow, Fizmatlit Publ., 1959. (in Russian)
5. Kleiman E.G. Identification of input signals in dynamical systems. *Automation and Remote Control*, 1999, vol. 60, no. 12, pp. 1675–1685.
6. Solodusha S.V. To the numerical solution of one class of systems of the Volterra polynomial equations of the first kind. *Num. Anal. Appl.*, 2018, vol. 11, no. 1, pp. 89–97. <https://doi.org/10.1134/S1995423918010093>
7. Solodusha S.V. Automatic control systems modeling by Volterra polynomials. *Model. Anal. Inform. Sist.*, 2012, vol. 19, no. 1, pp. 60–68. (in Russian)
8. *Spravochnik po teorii avtomaticheskogo upravleniya* [Handbook of Theory of Automatic Control]. Ed. A.A. Krasovskiy. Moscow, Nauka Publ., 1987. (in Russian)
9. Verlan' A.F., Sizikov V.S. *Integral'nyye uravneniya: metody, algoritmy, programmy* [Integral equations: methods, algorithms, programs]. Kiev, Naukova Dumka Publ., 1986. (in Russian)
10. Volterra V. *A Theory of Functionals, Integral and Integro-Differential Equations*. New York, Dover Publ., 1959.

Svetlana Solodusha, Candidate of Sciences (Physics and Mathematics), Assistant Professor, Leading Scientific Researcher, Melentiev Energy Systems Institute SB RAS, 130, Lermontov st., Irkutsk, 664033, Russian Federation, tel: (3952)500646, e-mail: solodusha@isem.irk.ru, ORCID iD <https://orcid.org/0000-0001-6162-7542>

Received 31.10.19

К идентификации входных сигналов в интегральных моделях одного класса нелинейных динамических систем

С. В. Солодуша

*Институт систем энергетики им. Л. А. Мелентьева СО РАН,
Иркутск, Российская Федерация*

Аннотация. Проблема восстановления входных сигналов является одним из интенсивно развивающихся научных направлений и находится на стыке теории математического моделирования, теории автоматического управления и теории обратных задач. Статья посвящена решению проблемы идентификации входного сигнала, которому соответствует заданный (желаемый) отклик при условии отсутствия обратной связи. Изложен подход к приближенному решению полиномиальных уравнений Вольтерра I рода N-й степени, возникающих при моделировании нелинейной динамики аппаратом интегро-степенных рядов Вольтерра. Рассматривается один класс нелинейных динамических систем типа черного ящика, входной сигнал которых является векторной функцией времени. В данном случае, в отличие от скалярного входа, интегральная модель усложняется за счет включения слагаемых, учитывающих одновременное изменение отдельных компонент вектора входного сигнала. Ранее рассмотрены интегральные модели с постоянными ядрами Вольтерра. В настоящей статье предполагается, что симметричные ядра Вольтерра представимы в виде произведения конечного числа непрерывных функций. Задача идентификации

решена с помощью метода Ньютона – Канторовича. В качестве начального приближения предложено численное решение соответствующего линейного интегрального уравнения Вольтерра I рода. Расчетные формулы получены на основе квадратурного метода (правых прямоугольников). Эффективность предлагаемых алгоритмов проиллюстрирована на эталонной динамической системе и подтверждена численными результатами.

Ключевые слова: полиномиальные уравнения Вольтерра I рода, задача восстановления входных сигналов, метод Ньютона-Канторовича.

Список литературы

1. Апарцин А. С. Полилинейные уравнения Вольтерра I рода: элементы теории и численные методы // Известия Иркутского государственного университета. Математика. 2007. Т. 1, № 1. С. 13-41.
2. Апарцин А. С. К исследованию устойчивости решения полиномиального уравнения Вольтерра I рода // Автоматика и телемеханика. 2011. № 6. С. 95-102. <https://doi.org/10.1134/S0005117911060099>
3. Volterra-series-based nonlinear system modeling and its engineering applications: A state-of-the-art review / C. M. Cheng, Z. K. Peng, W. M. Zhang, G. Meng // Mechanical Systems and Signal Processing. 2017. Vol. 87. P. 430-364. <https://doi.org/10.1016/j.ymssp.2016.10.029>
4. Канторович Л. В., Акилов Г. П. Функциональный анализ в нормированных пространствах. М. : Физматлит, 1959.
5. Клейман Е. Г. Идентификация входных сигналов в динамических системах // Автоматика и телемеханика. 1999. № 12. С. 3-15.
6. Солодуша С. В. К численному решению одного класса систем полиномиальных уравнений Вольтерра I рода // Сибирский журнал вычислительной математики. 2018. Т. 21, № 1. С. 117-126. <https://doi.org/10.1134/S1995423918010093>
7. Солодуша С. В. Моделирование систем автоматического управления на основе полиномов Вольтерра // Моделирование и анализ информационных систем. 2012. Т. 19, № 1. С. 60-68.
8. Справочник по теории автоматического управления / под ред. А. А. Красовского. М. : Наука, 1987.
9. Верлань А. Ф., Сизиков В. С. Интегральные уравнения: методы, алгоритмы, программы. Киев : Наукова думка, 1986.
10. Volterra V. A Theory of Functionals, Integral and Integro-Differential Equations. New York : Dover Publ., 1959.

Светлана Витальевна Солодуша, кандидат физико-математических наук, доцент, ведущий научный сотрудник, Институт систем энергетики им. Л. А. Мелентьева СО РАН, Российская Федерация, 664033, г. Иркутск, ул. Лермонтова, 130, тел.: (3952)500646, e-mail: solodusha@isem.irk.ru, ORCID iD <https://orcid.org/0000-0001-6162-7542>

Поступила в редакцию 31.10.19